3

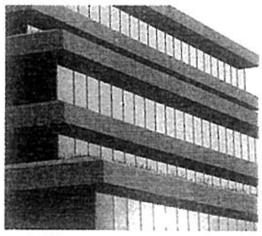
Parallel Lines and Planes

Rows of corn form parallel lines. Describe other real-world examples of parallel lines.

PARALLELISM IN LINES AND PLANES

OBJECTIVE: *Identify parallel planes, parallel lines, and skew lines. Name angle pairs formed when a transversal intersects two lines.*

3-1 Parallel Planes, Lines, and Transversals



In Chapter 1 you learned that two figures that intersect have a set of points in common. Figures that do not intersect have no points in common.

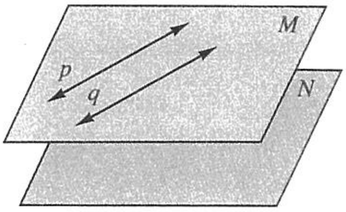
DEFINITION

Parallel planes are planes that do not intersect.

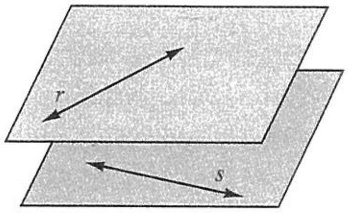
DEFINITION

Parallel lines are coplanar and do not intersect.

Lines that are not coplanar and do not intersect are called **skew lines.**



Planes M and N are parallel. ($M \parallel N$)
Lines p and q are parallel. ($p \parallel q$)



Lines r and s are skew.

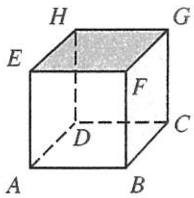
Segments and rays are parallel if they are contained in parallel lines.

Example 1

Refer to the box on the right.
a. Name a pair of parallel planes.
b. Name a pair of parallel lines.

Solution

a. plane $ADHE \parallel BCGF$ **b.** $\overline{AB} \parallel \overline{HG}$



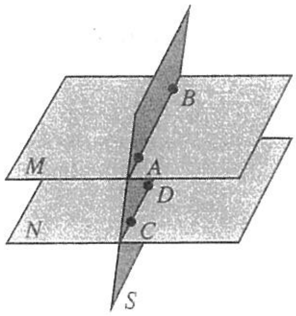
Try This

Name a pair of skew lines.

The following theorem concerns three-dimensional relationships among lines and planes.

◆ THEOREM 3.1

If two parallel planes are cut by a third plane, the lines of intersection are parallel.



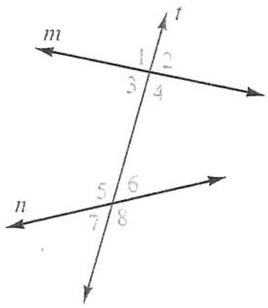
Given: Plane M is parallel to plane N; plane S intersects plane M in $\overline{AB}$; plane S intersects plane N in $\overline{CD}$.

Prove: $\overline{AB} \parallel \overline{CD}$

Proof

Statements	Reasons
1. $M \parallel N$; S and M intersects in $\overline{AB}$; S and N intersect in $\overline{CD}$.	1. Given
2. $\overline{AB}$ and $\overline{CD}$ do not intersect.	2. Def. of $\parallel$ planes
3. $\overline{AB}$ and $\overline{CD}$ are coplanar.	3. Both lines are in plane S.
4. $\overline{AB} \parallel \overline{CD}$	4. Def. of $\parallel$ lines

A line that intersects two coplanar lines in two different points is called a **transversal**. A transversal that intersects two coplanar lines forms eight angles. In the figure to the right, transversal t intersects lines m and n.



Angles 3, 4, 5, and 6 are interior angles.
Angles 1, 2, 7, and 8 are exterior angles.

The pairs of angles have special names.

Alternate interior angles are two nonadjacent interior angles on opposite sides of the transversal. There are two pairs of alternate interior angles: $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$.

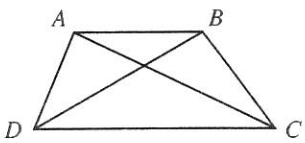
Alternate exterior angles are two nonadjacent exterior angles on opposite sides of the transversal. There are two pairs of alternate exterior angles: $\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$.

Same-side interior angles are two interior angles on the same side of the transversal. There are two pairs of same-side interior angles: $\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$.

Corresponding angles are two nonadjacent angles on the same side of the transversal such that one is an exterior angle and the other is an interior angle. There are four pairs of corresponding angles: $\angle 1$ and $\angle 5$, $\angle 3$ and $\angle 7$, $\angle 2$ and $\angle 6$, $\angle 4$ and $\angle 8$.

Example 2

a. Name two lines in the figure that appear to be parallel.
b. Name two pairs of alternate interior angles for lines AB and CD.



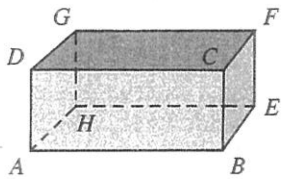
Solution

a. $\overline{AB}$, $\overline{CD}$
b. $\angle BAC$ and $\angle ACD$, $\angle ABD$ and $\angle BDC$

Try This

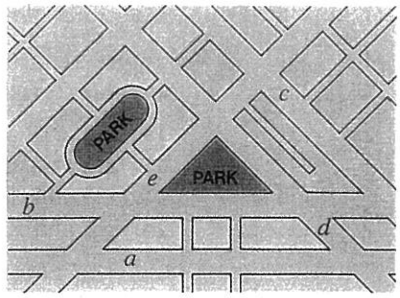
Name two pairs of alternate interior angles for lines AD and BC.

Class Exercises



Short Answer

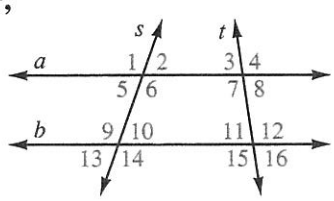
1. Name three pairs of parallel planes.
2. Name all the lines that are parallel to $\overline{EF}$.
3. How many lines are parallel to each line in the figure?
4. Name all the lines that are skew to $\overline{HE}$.
5. How many lines are skew to each line in the figure?



6. Name two pairs of streets that appear to be parallel.
7. Which streets are transversals for streets *a* and *b*?
8. Which streets are transversals for streets *d* and *e*?
9. Name the only street that is a transversal for streets *a* and *d*.

Sample Exercises

Identify each angle pair as alternate interior, alternate exterior, same-side interior, corresponding, or none of these.



10. $\angle 13$ and $\angle 5$
11. $\angle 1$ and $\angle 3$
12. $\angle 7$ and $\angle 12$
13. $\angle 3$ and $\angle 16$
14. $\angle 7$ and $\angle 10$
15. $\angle 13$ and $\angle 4$

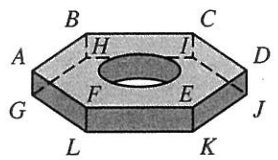
Discussion

16. Draw a diagram to illustrate the statement:
If plane *M* is parallel to plane *N*, not every line in *M* is parallel to a given line *AB* in plane *N*.
17. Draw and label a pair of lines and a transversal. Label the figure so that $\angle 7$ and $\angle 8$ are corresponding angles. Must their supplements be corresponding angles? Give a convincing argument for your answer.

Exercises

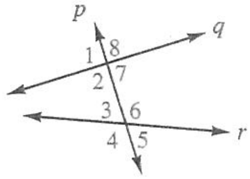
A

Refer to the figure to name the plane that is parallel to the given plane.



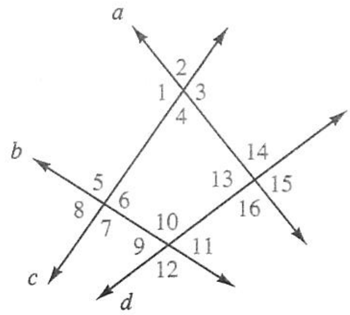
1. plane *ABHG*
2. plane *CDJI*
3. plane *BCIH*
4. Is plane *ABHG* parallel to plane *FEKL*? Explain.
5. Name four parallel planes.
6. Name three lines parallel to $\overline{CD}$.
7. Which lines in plane *AGLF* are skew to $\overline{CD}$?
8. Name four lines that are skew to both $\overline{EK}$ and $\overline{BH}$.
9. Name two lines that are parallel to $\overline{AB}$ and skew to $\overline{BC}$.
10. Are any lines both parallel and skew to $\overline{FE}$?

Name the indicated angles determined by intersecting lines *q* and *r* cut by transversal *p*.



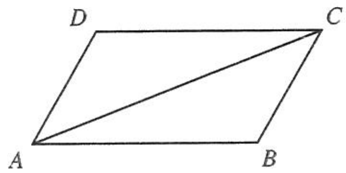
11. two pairs of alternate interior angles
12. two pairs of alternate exterior angles
13. two pairs of same-side interior angles
14. four pairs of corresponding angles
15. a pair of corresponding angles that includes $\angle 6$
16. a pair of alternate interior angles that includes $\angle 6$

Identify each pair of angles as alternate interior, alternate exterior, same-side interior, corresponding, or none of these.



17. $\angle 1$ and $\angle 13$ **18.** $\angle 6$ and $\angle 10$
19. $\angle 12$ and $\angle 14$ **20.** $\angle 4$ and $\angle 10$
21. $\angle 11$ and $\angle 13$ **22.** $\angle 8$ and $\angle 15$
23. $\angle 1$ and $\angle 15$ **24.** $\angle 5$ and $\angle 2$

25. Name two pairs of alternate interior angles for lines *a* and *b* cut by transversal *c*.
26. Name two pairs of corresponding angles for lines *c* and *d* cut by transversal *a*.

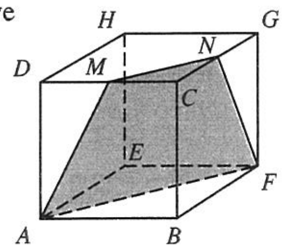


27. For $\overline{AB}$ and $\overline{DC}$ cut by transversal $\overline{AC}$, name the angle that is the alternate interior angle to $\angle BAC$.
28. For $\overline{BC}$ and $\overline{AD}$ cut by transversal $\overline{AC}$, name the angle that is the alternate interior angle to $\angle BCA$.

B

Determine whether each statement is always, sometimes, or never true.

29. If two lines in space are parallel to the same line, then they are parallel to each other.
30. If a plane intersects one of two parallel planes, then it intersects the other parallel plane.
31. If two planes are parallel to the same line, then they are parallel to each other.
32. If a plane is parallel to one of two skew lines, then it intersects the other skew line.
33. If two lines do not intersect, then they are parallel.
34. If a line and a plane do not intersect, then they are parallel.
35. If $\angle 1$ and $\angle 2$ are alternate interior angles, then the vertical angles relative to them are alternate exterior angles.
36. If $\angle 3$ and $\angle 4$ are same-side interior angles, then their supplements are also same-side interior angles.
37. If $\angle 7$ and $\angle 8$ are corresponding angles, then the vertical angles relative to them are alternate interior angles.



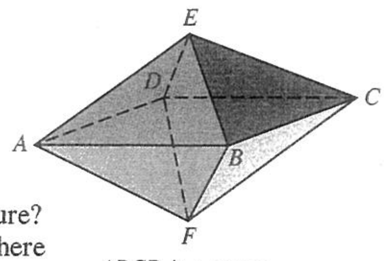
38. In the box to the right, opposite sides are parallel. Suppose a plane cuts the box at *A*, *F*, *N*, and *M*. Explain why $\overline{MN} \parallel \overline{AF}$.

Indicate which of the following are examples of intersecting, parallel, or skew lines.

39. railroad tracks
40. picture frames
41. bowling alleys
42. airplane flight paths
43. rows of corn
44. a picket fence
45. lines on a football field
46. a TV antenna

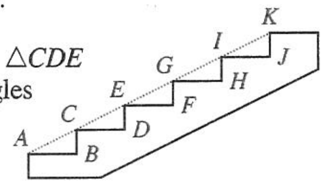
Name each of the following, if it exists, in the crystal.

47. one line that appears to be parallel to $\overline{BE}$
48. the line that appears to be parallel to $\overline{BC}$
49. a plane parallel to *ABCD*
50. a plane parallel to *ABE*
51. one line that appears to be skew to $\overline{AD}$
52. How many edges are skew to $\overline{AD}$?
53. How many segments are shown in the figure?
54. How many pairs of parallel segments do there appear to be?

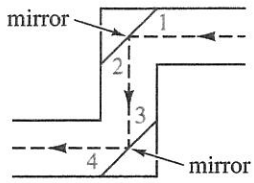


ABCD is a square.

55. A carpenter builds a stairway by cutting triangles like $\triangle ABC$ and $\triangle CDE$ from a piece of lumber. $\angle DCE$ and $\angle FEG$ are corresponding angles relative to what pair of parallel lines and what transversal?

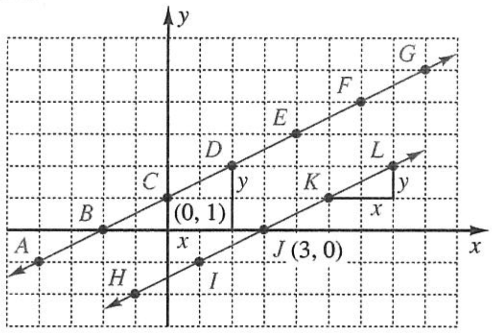


56. In a periscope a pair of mirrors are mounted parallel to each other, as shown. The path of light becomes a transversal. Name a pair of alternate interior angles.



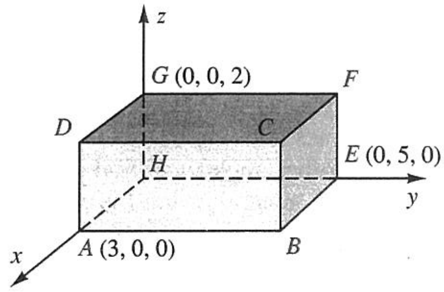
C

In the coordinate graph shown, $\overline{AG}$ and $\overline{HL}$ are parallel.



57. Find the coordinates of each labeled point on $\overline{AG}$ and on $\overline{HL}$.
58. For each pair of points on $\overline{HL}$ construct a triangle like the one shown and calculate the ratio $\frac{y}{x}$ where x and y are the lengths of the sides of the triangle. Do the same for $\overline{AG}$.
59. Write a generalization that is based on your results in Exercise 58

60. Just as any point in an x-y coordinate plane can be named by a pair of coordinates, any point in an x-y-z coordinate space can be named by a triple of coordinates. In the box shown, the x-y-z coordinates of three of the points have been given. Find the coordinates for each of the remaining corners of the box.
61. How many pairs of parallel segments exist in the box to the right?



Draw each figure

62. Line $l \parallel m$, lines l and n are skew, lines m and n are skew.
63. Line $s \parallel t$, line $s \parallel w$, lines t and w are skew.
64. Line $a \perp b$, lines a and c are skew, lines b and c are skew.
65. Line $d \parallel$ plane P, plane $P \parallel$ plane Q, d is $\nparallel$ to Q.

Critical Thinking

Some statements that are true about lines in a plane are not necessarily true about lines in space. Determine whether each statement is always true about lines in space.

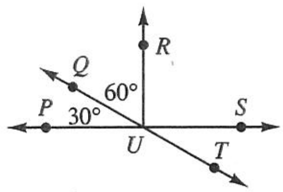
66. Two lines parallel to a third line are parallel to each other
67. If a line is perpendicular to one of two parallel lines, then it is perpendicular to the other parallel line.

68. The number of points that are formed by intersections of a given number of lines depends upon their positions relative to one another. Three lines form 0, 1, 2, or 3 points of intersection. Investigate the number of points of intersection that are possible for four coplanar lines.

0 points — 1 point — 2 points — 3 points

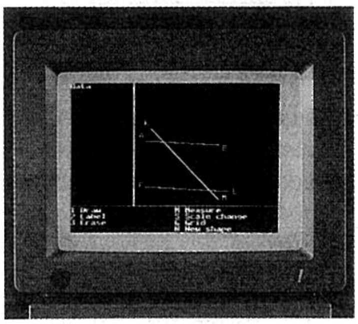
Mixed Review

1. Name a pair of vertical angles.
2. Name an angle that forms a linear pair with $\angle RUS$.
3. Name a pair of adjacent, complementary angles.
4. Name a pair of complementary angles that are not adjacent.
5. Name an angle that is the supplement of $\angle QUR$.
6. Name an angle that is the supplement of $\angle TUS$.



Computer Activity

Parallel Lines and Transversals



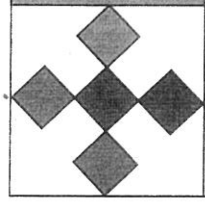
Use computer software to draw two parallel lines and a transversal.

1. Measure pairs of corresponding angles in the figure you have drawn. What do you notice? Do you think this is true for any pair of parallel lines and any transversal? State a generalization.
2. Measure other angles and find pairs that are congruent. State an appropriate generalization for each pair.
3. Measure pairs of same-side interior angles. Find the sum of the measures of each pair. What do you notice? State a generalization.

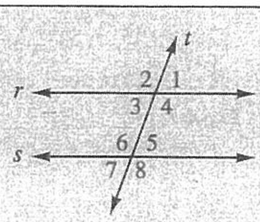
OBJECTIVE: *State and apply theorems about angles formed when parallel lines are cut by a transversal.*

3-2 Properties of Parallel Lines

EXPLORE



Use a ruler to draw two parallel lines, one on each side of the ruler. Draw a transversal and number the angles as shown. Use tracing paper or a protractor to find pairs of congruent angles.



In the last lesson, special pairs of angles formed by two lines and a transversal were defined. You may have discovered while completing the Explore in this lesson, that when the two lines cut by a transversal are parallel, certain pairs of angles are congruent. This observation is the basis for the following postulate.

POSTULATE 13

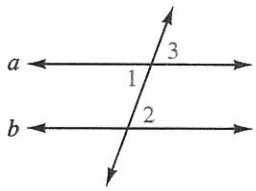
If two parallel lines are cut by a transversal, then corresponding angles are congruent.

Postulate 13 is used to prove the next theorem.

THEOREM 3.2

If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

Given: $a \parallel b$
Prove: $\angle 1 \cong \angle 2$

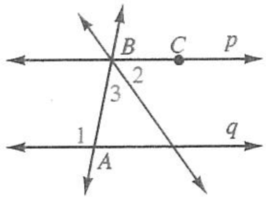


Proof

Statements	Reasons
1. $a \parallel b$	**1.** Given
2. $\angle 2 \cong \angle 3$	**2.** If two parallel lines are cut by a transversal, then corresponding $\angle$s are $\cong$.
3. $\angle 3 \cong \angle 1$	**3.** Vertical $\angle$s are $\cong$.
4. $\angle 1 \cong \angle 2$	**4.** Congruence of $\angle$s is transitive.

Example 1

$p \parallel q$, $m\angle 1 = 100$, $m\angle 2 = 55$ Find $m\angle 3$.



Solution

$m\angle 1 = m\angle ABC$ *If two parallel lines are cut by a transversal, then alternate interior angles are congruent. (Theorem 3.2)*

$m\angle 1 = m\angle 2 + m\angle 3$
$100 = 55 + m\angle 3$ *Substitution*
$45 = m\angle 3$

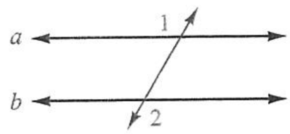
Try This

$p \parallel q$, $m\angle 1 = 135$, $m\angle 3 = 75$ Find $m\angle 2$.

Two additional relationships you might have discovered in the Explore are stated in Theorems 3.3 and 3.4, which you will be asked to prove in Exercises 30–31.

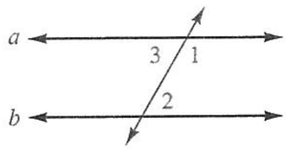
THEOREM 3.3

If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.



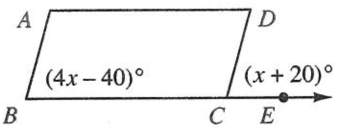
THEOREM 3.4

If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.



Example 2

$\overline{AB} \parallel \overline{CD}$ Find $m\angle B$.



Solution

$m\angle ABC = m\angle DCE$ *If two parallel lines are cut by a transversal, then corresponding angles are congruent. (Postulate 13)*

$4x - 40 = x + 20$ *Substitution*
$3x = 60$
$x = 20$ $m\angle B = 40$

Try This

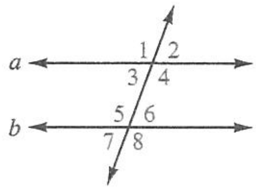
$\overline{AD} \parallel \overline{BC}$, $m\angle D = 3x - 25$, $m\angle DCE = x + 20$ Find $m\angle D$.

Properties of Parallel Lines Cut by a Transversal

1. Corresponding angles are congruent.
2. Alternate interior angles are congruent.
3. Alternate exterior angles are congruent.
4. Same-side interior angles are supplementary.

Class Exercises

Short Answer

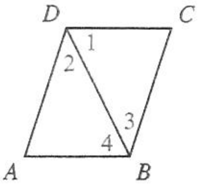


$a \parallel b$ State the postulate or theorem that justifies each conclusion.

1. $\angle 1 \cong \angle 8$
2. $\angle 4 \cong \angle 5$
3. $m\angle 4 + m\angle 6 = 180$
4. $\angle 3 \cong \angle 7$
5. $\angle 2 \cong \angle 7$
6. $\angle 6 \cong \angle 3$

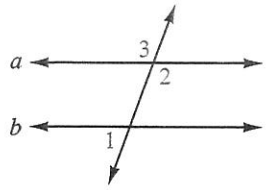
Sample Exercises

$a \parallel b$, $m\angle 1 = 65$



7. Find $m\angle 5$.
8. Find $m\angle 8$.
9. Find $m\angle 6$.
10. If $\overline{AB} \parallel \overline{CD}$, name a pair of congruent angles.
11. If $\overline{AD} \parallel \overline{BC}$, name two pairs of supplementary angles.

Discussion

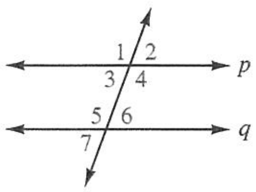


12. If $a \parallel b$, give a convincing argument that $\angle 1$ and $\angle 3$ are supplementary.
13. If $a \parallel b$, give a convincing argument that $\angle 1$ and $\angle 2$ are supplementary.

Exercises

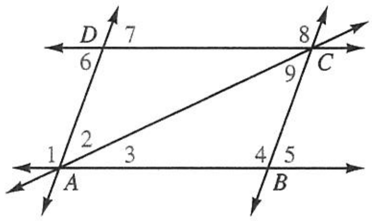
A

$p \parallel q$, $m\angle 3 = 65$



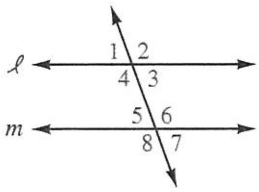
1. Find $m\angle 1$.
2. Find $m\angle 2$.
3. Find $m\angle 4$.
4. Find $m\angle 5$.
5. Find $m\angle 6$.
6. Find $m\angle 7$.

$\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{BC}$, $m\angle ADC = 110$, $m\angle ACD = 28$



7. Find $m\angle 1$.
8. Find $m\angle 2$.
9. Find $m\angle 3$.
10. Find $m\angle 4$.
11. Find $m\angle 5$.
12. Find $m\angle 6$.
13. Find $m\angle 7$.
14. Find $m\angle 8$.
15. Find $m\angle 9$.
16. Name all the angles congruent to $\angle 1$.
17. Name all the angles congruent to $\angle 3$.
18. Name all the angles supplementary to $\angle 1$.
19. Name all the angles supplementary to $\angle 3$.

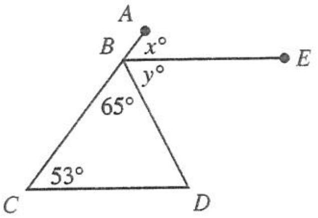
$l \parallel m$ Find x.



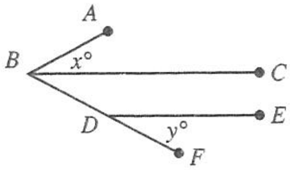
20. $m\angle 1 = 3x + 7$, $m\angle 5 = 5x - 3$
21. $m\angle 4 = 8x + 12$, $m\angle 6 = 2x + 54$
22. $m\angle 3 = 4x + 7$, $m\angle 6 = 2x + 23$
23. $m\angle 4 = x^2 + 5x$, $m\angle 8 = 9x + 12$
24. $m\angle 2 = x^2 - 6x$, $m\angle 5 = 7x + 220$

Find the values of x and y.

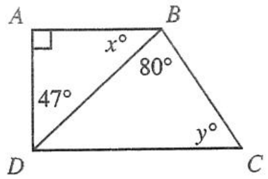
25. $\overline{BE} \parallel \overline{CD}$



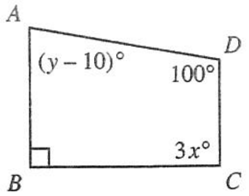
26. $\overline{BC}$ bisects $\angle ABD$.
$m\angle ABD = 64$, $\overline{BC} \parallel \overline{DE}$



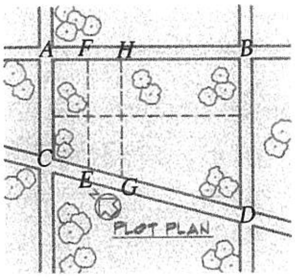
27. $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \perp \overline{BC}$



28. $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \perp \overline{AD}$



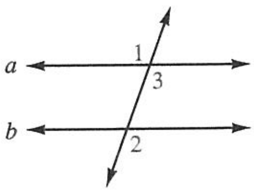
29. A developer is dividing a plot of land into lots so that the sides of each lot are parallel to the street AC. If $m\angle ACD = 105$, find $m\angle CEF$ and $m\angle CGH$.



30. Complete the proof of Theorem 3.3.

Given: $a \parallel b$

Prove: $\angle 1 \cong \angle 2$



Proof

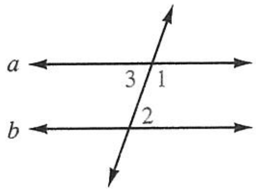
Statements	Reasons
1. $a \parallel b$	1. ___
2. $\angle 1 \cong \angle 3$	2. ___
3. $\angle 3 \cong \angle 2$	3. ___
4. $\angle 1 \cong \angle 2$	4. ___

31. Write a two-column proof for Theorem 3.4.

Given: $a \parallel b$

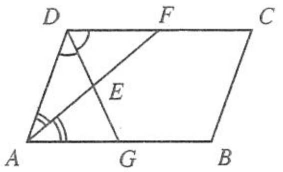
Prove: $\angle 1$ and $\angle 2$ are supplementary.

Plan First show that $\angle 2 \cong \angle 3$. Then use the fact that $\angle 1$ is the supplement of $\angle 3$.



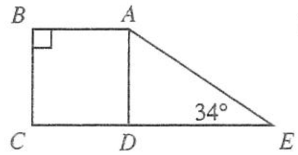
B

$\overline{AB} \parallel \overline{CD}$, $\overline{DG}$ bisects $\angle ADC$.
$\overline{AF}$ bisects $\angle DAB$. $m\angle ADC = 124$

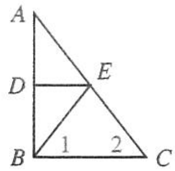


32. Find $m\angle ADG$.
33. Find $m\angle DAF$.
34. Find $m\angle DGB$.

35. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$
Find $m\angle EAD$.

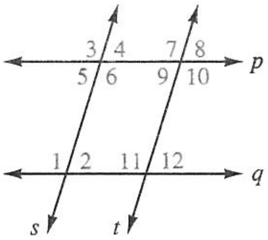


36. **Given:** $m\angle 1 = m\angle 2$, $\overline{DE} \parallel \overline{BC}$
Prove: $\overline{ED}$ bisects $\angle AEB$.



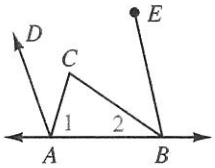
37. **Given:** $p \parallel q$, $s \parallel t$
Prove: $\angle 1 \cong \angle 7$

38. **Given:** $p \parallel q$, $s \parallel t$
Prove: $\angle 2 \cong \angle 9$

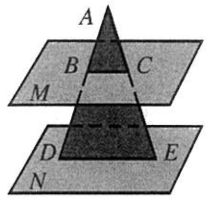


C

39. **Given:** $\overline{CA}$ bisects $\angle DAB$. $\overline{AD} \parallel \overline{BE}$
$\angle 1$ and $\angle 2$ are complementary.
Prove: $\overline{BC}$ bisects $\angle ABE$.



40. **Given:** $\overline{BC}$ and $\overline{DE}$ are coplanar.
plane $M \parallel$ plane N
Prove: $\angle ABC \cong \angle ADE$, $\angle ACB \cong \angle AED$



41. Draw a diagram and prove the statement: If $\overline{BD}$ bisects $\angle ABC$ and $\overline{AM} \parallel \overline{BD}$ with point M on $\overrightarrow{CB}$, then $m\angle ABD = m\angle AMB$.

Critical Thinking

42. If a theorem is accepted as a postulate, then the statement earlier accepted as a postulate might be proven as a theorem. Suppose Theorem 3.2 is accepted as a postulate. Prove Postulate 13 as a theorem.
43. Assume Theorem 3.4 is a postulate. Prove Postulate 13 as a theorem.

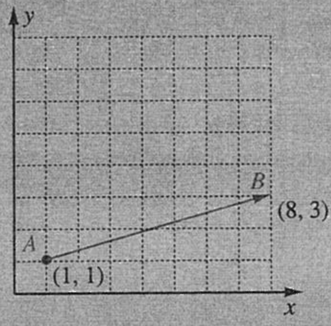
Algebra Review

Solve each system of equations.

1. $x + y = 10$, $y = x + 8$
2. $y = x - 6$, $x + y = -2$
3. $x - y = 5$, $x + 2y = 7$
4. $x + 2y = 10$, $3x + 4y = 8$

CONNECTIONS

Vectors



Representing quantities that have both direction and magnitude is often done through the use of directed segments called vectors. The vector AB (written $\overrightarrow{AB}$) is a vector whose direction is about 15 degrees above horizontal and whose magnitude, or length, is $\sqrt{53}$.

Vectors can also be represented by an ordered pair of numbers. The vector $\overrightarrow{AB}$ can also be written as $(7, 2)$. The point A is the origin of the vector and the point B is the terminal point of $\overrightarrow{AB}$. The ordered pair notation for a vector results from subtracting the components of the ordered pair representing the origin from the components of the ordered pair representing the terminal point. In the case of $\overrightarrow{AB}$, the origin is $A(1, 1)$ and the terminal point is $B(8, 3)$. Thus, $\overrightarrow{AB} = (8 - 1, 3 - 1)$ or $(7, 2)$.

Two vectors (a, b) and (c, d), or the lines or line segments containing them, are parallel if $\frac{a}{b} = \frac{c}{d}$. Two vectors (a, b) and (c, d), or the lines or line segments containing them, are perpendicular if $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} = -1$.

Example

a. Test to see if the vectors $(1, 3)$ and $(17, 51)$ are parallel.

b. Test to see if the vectors $(1, 3)$ and $(-12, 4)$ are perpendicular.

Solution

a. They are parallel since $\frac{1}{3} = \frac{17}{51}$.

b. They are perpendicular since $\frac{1}{3} \cdot \frac{-12}{4} = \frac{-12}{12} = -1$.

Try This

a. Test to see if the vectors $(1, 7)$ and $(13, 91)$ are parallel.

b. Test to see if the vectors $(4, 3)$ and $(-6, 8)$ are perpendicular.

Exercises

Find the ordered pairs representing the following vectors if $A(5, 3)$, $B(4, 0)$, $C(1, 2)$, and $D(2, 5)$.

1. $\overrightarrow{AB}$ **2.** $\overrightarrow{AC}$ **3.** $\overrightarrow{CD}$ **4.** $\overrightarrow{DC}$

5. Is $\overrightarrow{CD}$ the same as $\overrightarrow{DC}$? Explain the relationship between these vectors.

6. Use the distance formula on page 27 to find the magnitude of vectors $\overrightarrow{CD}$ and $\overrightarrow{DC}$.

7. Two vectors having the same ordered pair representation, and therefore the same magnitude, are called equal vectors. What can you say about how equal vectors are related geometrically?

8. What can you say about two vectors that are contained in the same line and are pointed in the same direction?

Indicate which pairs of vectors among the following are parallel and which are perpendicular.

9. $(3, 4)$ **10.** $(2, -4)$ **11.** $(-8, 6)$ **12.** $(6, 8)$ **13.** $(6, -12)$

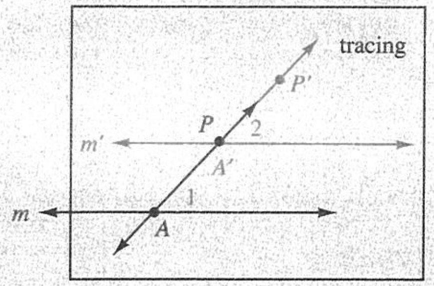
OBJECTIVE: *State and apply theorems that can be used to prove lines parallel.*

3-3 Proving Lines Parallel

EXPLORE

Draw line m and point A on the line and point P above the line. Then draw a transversal through P that intersects line m at A.

Locate and draw the translation image in the direction of vector $\overrightarrow{AP}$ of this figure using tracing paper. (Trace the figure and slide the tracing the distance and direction of $\overrightarrow{AP}$.) Since translations preserve angle measure $m\angle 1 = m\angle 2$. Do you think line m and its translation image are parallel?



In the previous lesson, the hypothesis of each theorem stated that two lines were parallel and the conclusion concerned angle measures. In this lesson, the hypothesis of each theorem is a statement about angle measures and the conclusion states that two lines are parallel.

POSTULATE 14

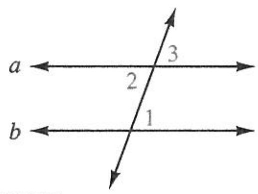
If two lines are cut by a transversal and a pair of corresponding angles are congruent, then the lines are parallel.

THEOREM 3.5

If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the lines are parallel.

Given: $\angle 1 \cong \angle 2$

Prove: $a \parallel b$

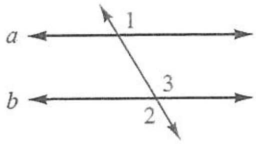


Proof

Statements	Reasons
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $\angle 2 \cong \angle 3$	**2.** Vertical angles are $\cong$.
3. $\angle 1 \cong \angle 3$	**3.** Congruence of $\angle$s is transitive.
4. $a \parallel b$	**4.** If two lines are cut by a transversal and corr. $\angle$s are $\cong$, the lines are $\parallel$.

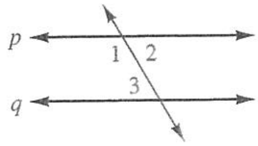
◆ THEOREM 3.6

If two lines are cut by a transversal and a pair of alternate exterior angles are congruent, then the lines are parallel.



◆ THEOREM 3.7

If two lines are cut by a transversal and a pair of same-side interior angles are supplementary, then the lines are parallel

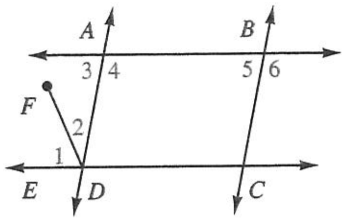


You will be asked to prove Theorems 3.6 and 3.7 in Exercises 20 and 25.

Example 1

Use the given information to determine which lines are parallel. State the theorem or postulate that justifies your answer.

a. $\angle 3 \cong \angle 5$ **b.** $m\angle 1 + m\angle 2 = m\angle 4$



Solution

a. $\overline{AD} \parallel \overline{BC}$ *If two lines are cut by a transversal and a pair of corr. ∠s are ≅, then the lines are ∥.*

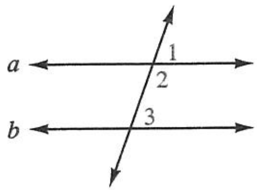
b. $\overline{AB} \parallel \overline{CD}$ *If two lines are cut by a transversal and a pair of alternate interior ∠s are ≅, then the lines are ∥.*

Try This

Write a statement about a pair of same-side interior angles that makes $\overline{AD} \parallel \overline{BC}$.

Example 2

$m\angle 2 = 12x + 7$, $m\angle 3 = 8x - 7$ Find the value of x for which $a \parallel b$.



Solution

$(12x + 7) + (8x - 7) = 180$
$20x = 180$
$x = 9$

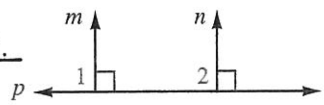
If same-side interior angles are supplementary, then the lines are parallel.

Try This

$m\angle 1 = 2x + 6$, $m\angle 3 = 3x - 5$ Find the value of x for which $a \parallel b$.

◆ THEOREM 3.8

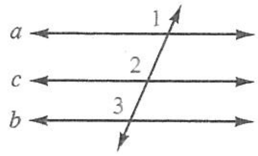
In a plane, two coplanar lines perpendicular to the same line are parallel.



You will be asked to prove Theorems 3.8 and 3.9 in Exercises 30–31.

◆ THEOREM 3.9

Two lines parallel to a third line are parallel to each other.



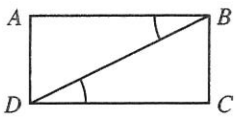
Ways to Prove Two Lines Parallel Using a Transversal

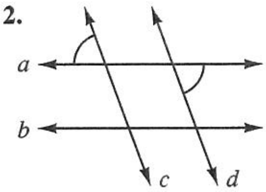
1. Show that a pair of corresponding angles are congruent.
2. Show that a pair of alternate interior angles are congruent.
3. Show that a pair of alternate exterior angles are congruent.
4. Show that a pair of same-side interior angles are supplementary.
5. Show that two coplanar lines are perpendicular to the same line.

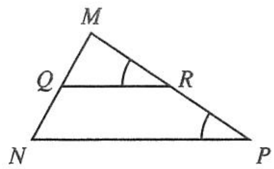
Class Exercises

Short Answer

Congruent angles are marked. State which lines are parallel. Give the theorem or postulate that justifies your answer.

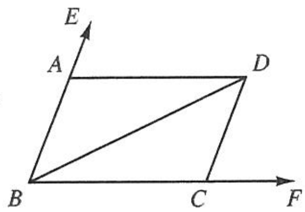
1. 

2. 

3. 

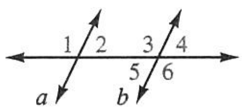
Sample Exercises

Use the given information to state which lines are parallel. Give the theorem or postulate that justifies your answer.



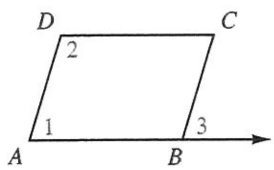
4. $m\angle BAD = 113$, $m\angle ABC = 67$, $m\angle BCD = 103$
5. $m\angle EAD = 57$, $m\angle ABC = 57$, $m\angle DCF = 67$

Find the value of x for which $a \parallel b$.



6. $m\angle 2 = 2x$, $m\angle 3 = 4x$
7. $m\angle 1 = 2x$, $m\angle 6 = 136$
8. $m\angle 1 = 3x$, $m\angle 5 = 60$

Discussion

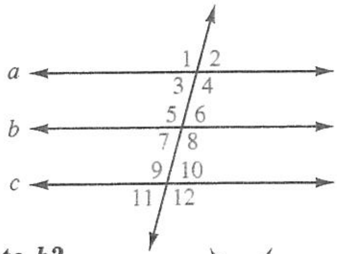


9. If $\overline{AD} \parallel \overline{BC}$ and $m\angle 1$ is doubled, how must $m\angle 3$ change if $\overline{AD}$ is to remain parallel to $\overline{BC}$?
10. If $\overline{AB} \parallel \overline{DC}$ and $m\angle 1$ is increased by 30 degrees, how must $m\angle 2$ change if $\overline{AB}$ is to remain parallel to $\overline{DC}$?

Exercises

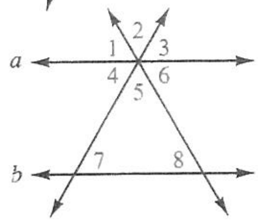
A

Use the given information to state which lines are parallel. Give the theorem or postulate that justifies your answer.



1. $\angle 1 \cong \angle 9$ 2. $\angle 3 \cong \angle 6$ 3. $m\angle 8 + m\angle 10 = 180$
4. $\angle 4 \cong \angle 9$ 5. $\angle 8 \cong \angle 12$ 6. $\angle 1 \cong \angle 8$

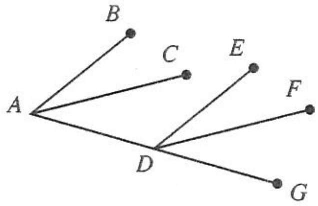
Does the given information allow you to conclude that a is parallel to b? If so, state the theorem or postulate that justifies your answer.



7. $m\angle 4 + m\angle 5 + m\angle 8 = 180$
8. $m\angle 1 + m\angle 2 + m\angle 7 = 180$

9. Name two pairs of corresponding angles that, if congruent, would allow you to conclude that a is parallel b.

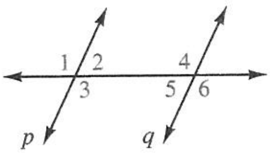
Use the given information to determine if $\overline{AC}$ is parallel to $\overline{DF}$.



10. $m\angle BAD = 42$, $m\angle BAC = 25$, $m\angle FDG = 17$
11. $m\angle BAD = 68$, $m\angle FDG = 34$, $\overrightarrow{AC}$ bisects $\angle BAD$.
12. $m\angle EDF = 28$, $m\angle EDA = 100$, $m\angle CAD = 54$
13. $m\angle BAC = 35$, $m\angle BAD = 61$, $m\angle ADE = 119$, $m\angle ADF = 144$

Find the value of x for which p is parallel to q.

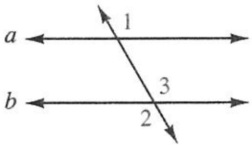
14. $m\angle 1 = 3x + 13$, $m\angle 4 = 4x - 6$
15. $m\angle 3 = 4x + 2$, $m\angle 5 = 8x - 2$
16. $m\angle 2 = \frac{2}{3}x - 16$, $m\angle 5 = 24$
17. $m\angle 1 = 2x + 12$, $m\angle 6 = \frac{3}{4}x + 27$
18. $m\angle 2 = x^2 + 7x$, $m\angle 5 = 9x + 3$
19. $m\angle 3 = x^2 + 8x$, $m\angle 5 = 4x + 20$



20. Complete the proof of Theorem 3.6.

Given: $\angle 1 \cong \angle 2$

Prove: $a \parallel b$

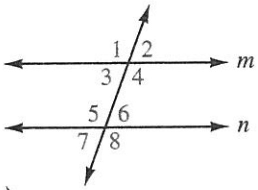


Proof

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. ___
2. $\angle 2 \cong \angle 3$	2. ___
3. $\angle 1 \cong \angle 3$	3. ___
4. $a \parallel b$	4. ___

B

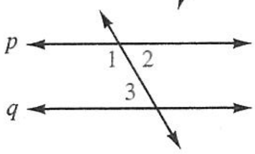
Determine the values of x and y for which $m \parallel n$.



21. $m\angle 1 = x + y$, $m\angle 4 = x - y$, $m\angle 8 = 118$
22. $m\angle 3 = 2x - y$, $m\angle 6 = x + 2y$, $m\angle 7 = 75$
23. $m\angle 1 = 6x - 3y$, $m\angle 4 = 2x - 8y$, $m\angle 5 = 126$
24. $m\angle 2 = 63$, $m\angle 3 = x - 3y$, $m\angle 6 = 7x - 2y + 2$
25. Write a two-column proof for Theorem 3.7.

Given: $\angle 1$ and $\angle 3$ are supplementary.

Prove: $p \parallel q$



26. In figure $ABCD$, $\overline{AD} \perp \overline{BA}$ and $\overline{CB} \perp \overline{AB}$. Draw the figure and determine which lines, if any, are parallel.

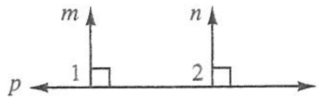
27. In figure $ABCD$, $\overline{CD} \perp \overline{BC}$, $\overline{AB} \perp \overline{BC}$, and $m\angle DAB = 115$. Draw the figure and find the measure of $\angle ADC$.

28. In figure $ABCD$, $m\angle A = 115$, $m\angle ABD = 35$, $m\angle DBC = 30$, and $m\angle BDC = 30$. Draw the figure and determine which lines are parallel.

29. Write a proof for Theorem 3.8.

Given: $m \perp p$
$n \perp p$

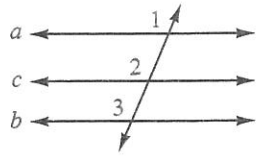
Prove: $m \parallel n$



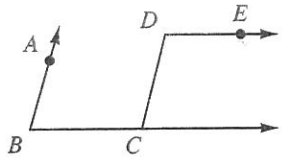
30. Write a proof for Theorem 3.9.

Given: $a \parallel c$, $b \parallel c$

Prove: $a \parallel b$



31. Given: $\angle BCD \cong \angle CDE$, $m\angle B + m\angle D = 180$
Prove: $\overline{AB} \parallel \overline{DC}$



32. Draw two parallel lines, a and b, and a transversal t. Use a protractor to bisect a pair of alternate exterior angles. Write a statement about these bisectors. Prove the statement.

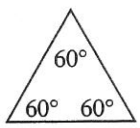
Critical Thinking

33. Consider the relation "is parallel to." Determine whether the reflexive, symmetric, and transitive properties hold for this relation for **a.** lines in a plane **b.** lines in space.

Mixed Review

Classify each triangle as acute, right, or obtuse.

1.



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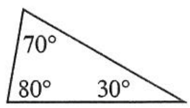

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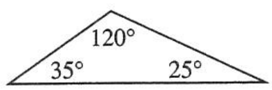

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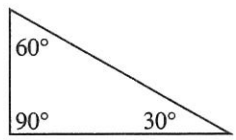
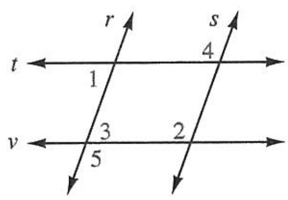


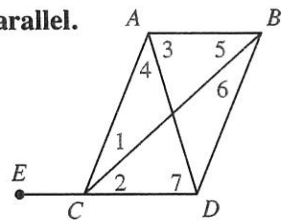
Quiz

Determine whether each statement is true or false.



1. $\angle 1$ and $\angle 3$ are alternate interior angles.
2. $\angle 3$ and $\angle 2$ are corresponding angles.
3. If $t \parallel v$, then $m\angle 2 + m\angle 5 = 180$.
4. If $\angle 2 \cong \angle 4$, then $t \parallel v$.
5. If $t \parallel v$, $r \parallel s$, and $m\angle 4 = 24$, then $m\angle 3 = 24$.

Use the given information to name the lines, if any, that are parallel.



6. $\angle 3 \cong \angle 7$ **7.** $\angle 2 \cong \angle 6$
8. $\angle 2 \cong \angle 3$ **9.** $\angle ACE \cong \angle BDC$
10. $m\angle ACD + m\angle BDC = 180$

11. $m\angle 2 = 3x + 10$, $m\angle 5 = x + 28$.
Find the value of x for which $\overline{AB} \parallel \overline{CD}$.

COMPARING APPROACHES IN GEOMETRY

Synthetic and Transformation Geometry

Theorem 3.5

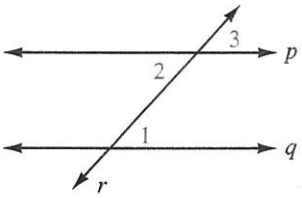
If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the lines are parallel.

Theorem 3.5 is proven on page 117 using what is called a synthetic approach, as are all other proofs based on the set of axioms that you are learning.

Transformation proofs are usually based on the properties that are preserved by the transformation being used. For example, on page 31, you learned that translations, reflections, and rotations preserve segment length and angle measure. A property of 180° rotations used below is that a line and its rotation image are parallel.

Compare the synthetic and transformation approaches to a proof of Theorem 3.5.

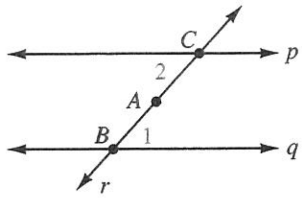
Synthetic Approach



Plan for Proof

It is given that $\angle 1 \cong \angle 2$. By the vertical angles theorem you know that $\angle 2 \cong \angle 3$. Finally use the fact that congruence of angles is transitive to conclude that $\angle 1 \cong \angle 3$. Since corresponding angles are congruent, the lines must be parallel.

Transformation Approach

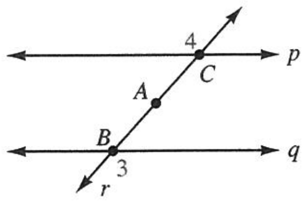


Plan for Proof

Consider the 180° rotation with center A—the midpoint of $\overline{BC}$. The rotation image of point B is C and of line r is again line r. Since $\angle 1 \cong \angle 2$, the rotation image of $\angle 1$ must be $\angle 2$. Therefore, the rotation image of p is q. It follows that $p \parallel q$.

Exercises

A is the midpoint of $\overline{BC}$ and the alternate exterior angles $\angle 3$ and $\angle 4$ are congruent. Consider the 180° rotation image centered at A.



1. Find the rotation image of point B.
2. Find the rotation image of line r.
3. How does the rotation image of $\angle 3$ compare with $\angle 3$ in measure?
4. Find the rotation image of line q.
5. Since the 180° rotation image of line q is parallel to q itself, you can conclude that line p is parallel to _?_.
6. The statements in Exercises 1–5 outline a transformation proof to what theorem?

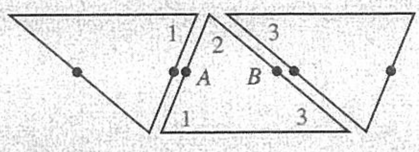
ANGLE SUM THEOREMS

OBJECTIVE: *Apply the Angle Sum Theorem for Triangles and related corollaries.*

3-4 Angles of a Triangle

EXPLORE

Draw any triangle. Use tracing paper to draw the 180° rotation image of the triangle centered at points A and B—the midpoints of two sides of the triangle. What can you conclude about $m\angle 1 + m\angle 2 + m\angle 3$?



You may have discovered in the Explore that you need the following postulate to prove the theorem.

POSTULATE 15

Given a line l and a point P not on l, there exists one and only one line through P parallel to l.

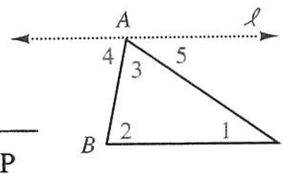
A line added to a figure is called an **auxiliary line.** An auxiliary line is added in the proof of the following theorem.

THEOREM 3.10 Angle Sum Theorem for Triangles

The sum of the measures of the angles of a triangle is 180.

Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$



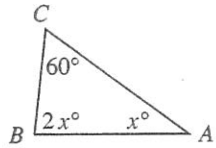
Proof

Statements	Reasons
1. Let l be the only line through A parallel to $\overline{BC}$.	1. Given a line l and a point P not on l, there exists one and only one line through P parallel to l. (Postulate 15)
2. $m\angle 4 = m\angle 2$, $m\angle 1 = m\angle 5$	2. If two ∥ lines are cut by a transversal, then the alternate interior ∠s are ≅.
3. $m\angle 5 + m\angle 4 + m\angle 3 = 180$	3. Linear Pair Postulate
4. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	4. Substitution

Example 1

Find the measure of each angle.

a. $\angle A$ **b.** $\angle B$



Solution

a. $x + 2x + 60 = 180$ *The sum of the measures of the angles of a triangle is 180.*

$3x = 120$

$x = 40$

$m\angle A = 40$

b. $m\angle B = 2x$

$m\angle B = 2(40)$

$m\angle B = 80$

Try This

In $\triangle ABC$, $\angle B \cong \angle C$ and $m\angle CAB = 100$. Find $m\angle B$.

Example 2

The measures of the angles of $\triangle ABC$ are in the ratio $1:2:3$. Classify the triangle as acute, right, or obtuse.

Solution

Let $x = m\angle A$

$2x = m\angle B$

$3x = m\angle C$

$x + 2x + 3x = 180$ *The sum of the measures of the angles of a triangle is 180.*

$6x = 180$

$x = 30$

$m\angle A = 30$, $m\angle B = 60$, $m\angle C = 90$

Therefore $\triangle ABC$ is a right triangle.

Try This

The measures of the angles of $\triangle PQR$ are in the ratio of $2:3:4$. Classify the triangle as acute, right, or obtuse.

A statement that is a direct consequence of a particular theorem is called a **corollary** of the theorem. Below are two corollaries of Theorem 3.10. They are corollaries because they are easily derived from the Angle Sum Theorem for Triangles.

COROLLARY 3.10a

The angles of an equiangular triangle each have a measure of 60.

COROLLARY 3.10b

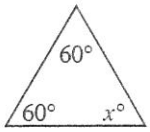
The acute angles of a right triangle are complementary.

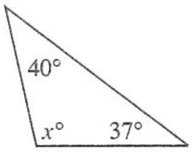
You will be asked to prove these corollaries in Exercises 24 and 25.

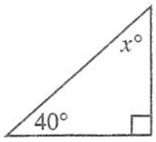
Class Exercises

Short Answer

Find the value of x.

1.


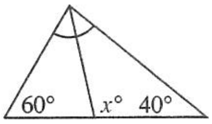
2.


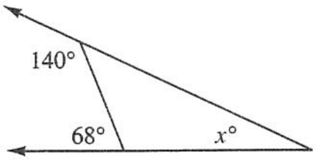
3.


4. Two angles of a triangle have measures of 50 and 100. Find the measure of the third angle.
5. Two angles of a triangle have measures of 25 and 55. Classify the triangle as acute, right, or obtuse.
6. Two angles of a right triangle are congruent. Find the measures of the angles.

Sample Exercises

Find the value of x. Congruent angles are marked.

7.


8.


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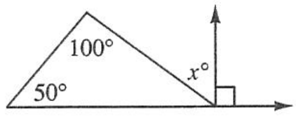


10. $\triangle ABC$ has $\angle B \cong \angle C$. If $m\angle A$ is increased by 20 and $\angle B$ remains congruent to $\angle C$, by how much are $m\angle B$ and $m\angle C$ decreased?
11. $\triangle ABC$ has $\angle B \cong \angle C$. If $m\angle B$ is increased by 15 and $\angle C$ is altered to remain congruent to $\angle B$, what is the effect on $m\angle A$?

Draw the indicated triangle, or write *not possible*.

12. acute equiangular triangle
13. obtuse equiangular triangle

Discussion

14. What is the greatest number of right angles a triangle can have?
15. What is the greatest number of acute angles a triangle can have?

Exercises

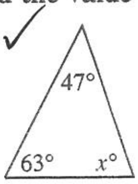
A

Draw the indicated triangle, or write *not possible*.

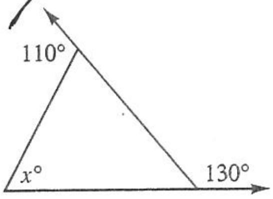
1. right triangle with two congruent angles
2. right equiangular triangle
3. obtuse right triangle
4. acute scalene triangle
5. acute triangle with two congruent angles
6. obtuse equiangular triangle
7. right scalene triangle
8. scalene isosceles triangle

Find the value of x. Congruent angles are marked.

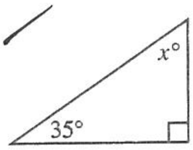
9.



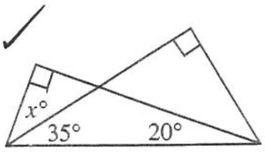
10.



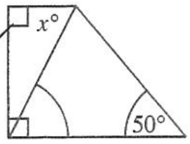
11.



12.



13.



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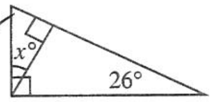
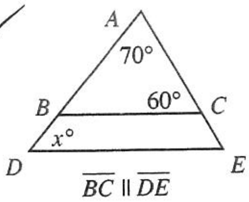
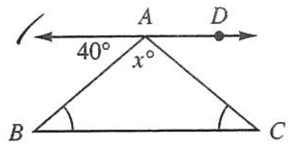


15.



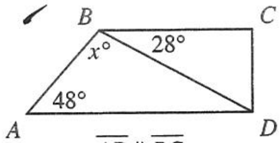
$\overline{BC} \parallel \overline{DE}$

16.



$\overleftrightarrow{AD} \parallel \overline{BC}$

17.



$\overline{AD} \parallel \overline{BC}$

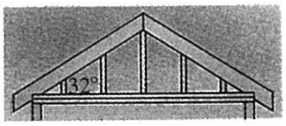
18. The ratio of the measures of the angles of a triangle is 4 : 5 : 6. Find the measures of the angles and classify the triangle as acute, right, or obtuse.

19. The ratio of the measures of the angles of a triangle is 1 : 3 : 5. Find the measures of the angles and classify the triangle as acute, right, or obtuse.

20. The ratio of the measures of the acute angles of a right triangle is 2 : 3. Find the measures of the two acute angles.

21. The measure of each of two angles of a triangle is four times that of the third angle. Find the measures of the angles.

22. The rafters of a roof form an angle of 32° at the base. What angle do the afters form at the ridge?



23. Write a two-column proof for Corollary 3.10a.

Given: $\triangle ABC$ is equiangular.

Prove: $m\angle A = m\angle B = m\angle C = 60$

24. Write a two-column proof for Corollary 3.10b.

Given: $\triangle ABC$, $\angle A$ is a right angle.

Prove: $\angle B$ and $\angle C$ are complementary.

B

25. The measures of the angles of a triangle are x, x, and $\frac{4}{3}x$. If the largest angle is doubled and the triangle remains isosceles, how would you represent the measures of the angles in the new triangle?

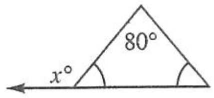
26. The measures of the angles of a triangle are represented by $2x + 15$, $x + 20$, and $3x + 25$. Find the measures of the angles.

27. The measures of the angles of a triangle are x, x, and $3x$. If the measure of the largest angle is increased by ten and the triangle remains isosceles, how would you represent the measures of the angles in the new triangle?

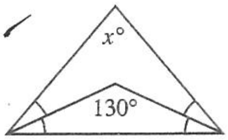
28. In a right triangle, the measure of one acute angle is four times the measure of the other. Find the measures of the angles of $\triangle MNO$.

Find the value of x. Congruent angles are marked.

29.



30.



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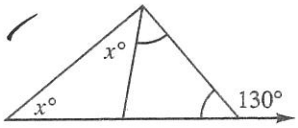

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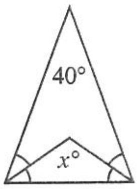

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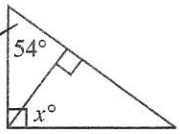

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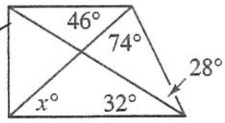
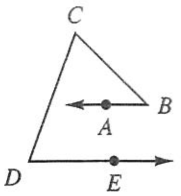


35. Given: $\overline{AB} \parallel \overline{DE}$

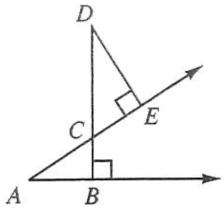
Prove: $m\angle B + m\angle C + m\angle D = 180$

36. Given: $\overline{DE} \perp \overline{AE}$, $\overline{BD} \perp \overline{AB}$

Prove: $m\angle CAB = m\angle CDE$

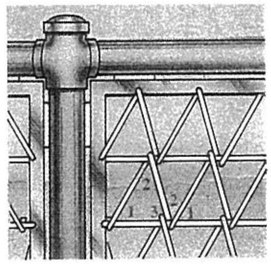


Ex. 35



Ex. 36

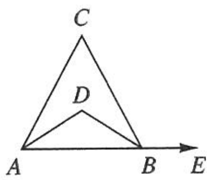
37. The pattern of triangles shown, a chain-link fence, illustrates the Angle Sum Theorem for Triangles. Use the figure to demonstrate the Angle Sum Theorem for Triangles.



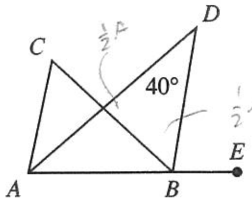
C

38. Given: $\overline{AD}$ bisects $\angle CAB$.
$\overline{BD}$ bisects $\angle CBA$.
$m\angle CAB = m\angle CBA$

Prove: $\angle ADB \cong \angle CBE$



39. $\overline{AD}$ bisects $\angle CAB$.
$\overline{BD}$ bisects $\angle EBC$. $m\angle D = 40$
Find $m\angle C$.



Critical Thinking

40. Accept as a postulate that the acute angles of a right triangle are complementary. Use this to prove that the sum of the measures of the angles of triangles is 180. (HINT: Draw a perpendicular from A to $\overline{BC}$.)

Mixed Review

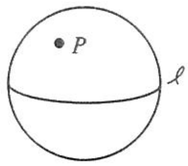
1. The measure of the supplement of an angle is three times the measure of the angle. Find the measure of the angle and its supplement.
2. The measure of the complement of an angle is five times the measure of the angle. Find the measure of the angle and its complement.

Enrichment

Non-Euclidean Geometry

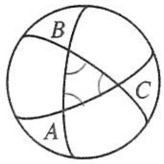
In this book you are studying Euclidean geometry, in which it is assumed that through a given point P not on a given line l there is exactly one line through P that is parallel to line l. But there are other geometries that differ from Euclidean geometry in their assumptions about the number of lines that contain P and are parallel to line l. Two such assumptions are (1) there are no lines through P parallel to line l, and (2) there are more than one line through P parallel to line l. (See the Historical Note on page 134.)

The "No-Parallel" Geometry



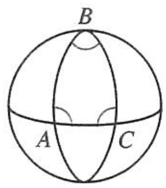
In this geometry, sometimes called "spherical" geometry, points are points on a sphere and lines are "great circles." A great circle on a sphere is a circle whose center is also the center of the sphere. Consider the equator as line l and New York as point P. Are there lines (great circles) through P that do not intersect the equator? Why or why not?

Exercises

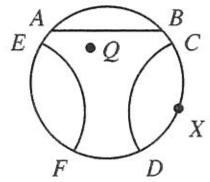


Use a string and a globe to answer these questions.

1. Use string to identify three lines (great circles) that intersect to form a spherical triangle ABC. Estimate the measure of each angle of $\triangle ABC$. Does the sum of the measures of $\triangle ABC$ appear to be equal to, less than, or greater than 180?
2. Consider the equator as one line and two other lines that pass through the north and south poles to form a triangle. How can BA and BC be placed to maximize the sum of the measures of the angles of $\triangle ABC$?



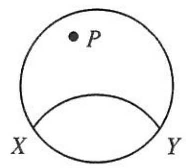
The "Many-Parallels" Geometry



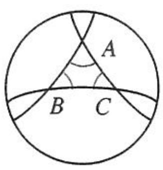
In this geometry, points consist of only those points that are in the interior of a given circle. Lines consist of arcs of circles that form "right angles" with the given circle. Thus CD is a line but AB is not a line, since AB is not the arc of a circle. Line CD is parallel to line EF since they do not intersect. Point Q is a point in this geometry, but point X is not a point, since it is not in the interior of the circle.

Exercises

1. How many lines (arcs of circles) can be drawn through P that are parallel to line (arc) XY?



2. Draw a "triangle" consisting of three arcs. Estimate the measure of each of the three angles indicated. Estimate the sum of the measures of the angles of the triangle. Does the sum of the measures appear to be equal to, less than, or greater than 180?

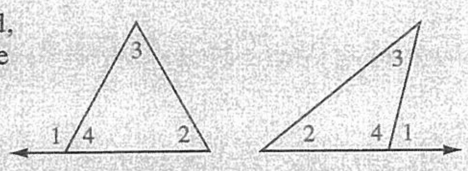


OBJECTIVE: *Use the Exterior Angle Theorem to find the measure of an angle of a triangle.*

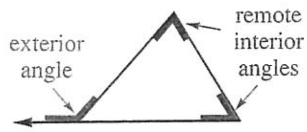
3-5 Theorems Related to the Angle Sum Theorem for Triangles

EXPLORE

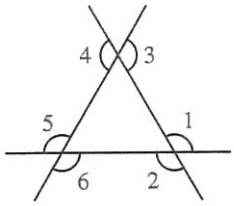
Draw triangles with one side extended, as shown. Use a protractor to measure $\angle 1$, $\angle 2$, and $\angle 3$. Compare $m\angle 1$ with $m\angle 2 + m\angle 3$. What do you discover?



If you extend one side of a triangle, an exterior angle is formed. An **exterior angle** of a triangle forms a linear pair with the adjacent interior angle of the triangle. The two angles of the triangle that are not adjacent to the exterior angle are called the **remote interior angles** of that exterior angle.



Every triangle has six exterior angles that form three pairs of vertical angles. Each angle in such a pair of vertical angles has the same remote interior angles. For example, $\angle 3$ and $\angle 4$ have the same remote interior angles.



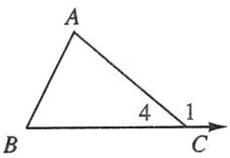
The following theorem is a consequence of the Angle Sum Theorem for Triangles and states a relationship between an exterior angle of a triangle and its remote interior angles.

THEOREM 3.11 The Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

Given: $\triangle ABC$ with exterior $\angle 1$

Prove: $m\angle 1 = m\angle A + m\angle B$

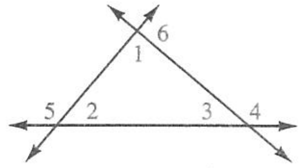


Proof

Statements	Reasons
1. $\triangle ABC$ with exterior $\angle 1$	1. Given
2. $\angle 1$ and $\angle 4$ form a linear pair.	2. Definition of linear pair
3. $m\angle 1 + m\angle 4 = 180$	3. Two angles that form a linear pair are supplementary.
4. $m\angle 1 = 180 - m\angle 4$	4. Addition Property of Equations
5. $m\angle A + m\angle B + m\angle 4 = 180$	5. Angle Sum Theorem for Triangles
6. $m\angle A + m\angle B = 180 - m\angle 4$	6. Addition Property of Equations
7. $m\angle A + m\angle B = m\angle 1$	7. Substitution Property

Example 1

$m\angle 5 = 130$, $m\angle 3 = 38$ Find $m\angle 1$.



Solution

$m\angle 5 = m\angle 1 + m\angle 3$ *Exterior Angle Theorem*

$130 = m\angle 1 + 38$

$m\angle 1 = 130 - 38$

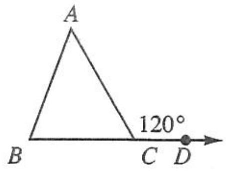
$m\angle 1 = 92$

Try This

$m\angle 6 = 87$, $m\angle 2 = 42$ Find $m\angle 3$.

Example 2

$m\angle B$ is twice $m\angle A$. Find $m\angle ABC$.



Solution

$x + 2x = 120$ *$m\angle A + m\angle B = m\angle ACD$ by the Exterior Angle Theorem*

$3x = 120$

$x = 40$

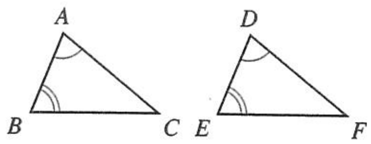
$2x = 80$ $m\angle ABC = 80$

Try This

$m\angle B$ is five times $m\angle A$. Find $m\angle B$.

◆ **THEOREM 3.12**

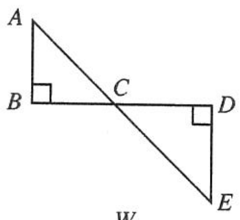
If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.



You will be asked to prove this theorem in Exercise 16.

Example 3

Identify which angles of $\triangle ABC$ are congruent to angles of $\triangle CDE$. State reasons why the angles are congruent.



Solution

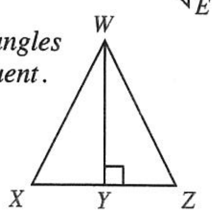
$\angle ABC \cong \angle CDE$ *All right angles are congruent.*

$\angle ACB \cong \angle ECD$ *Vertical angles are congruent.*

$\angle BAC \cong \angle CED$ *If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.*

Try This

$\overline{WY}$ bisects $\angle XWZ$. Identify which angles of $\triangle WYX$ are congruent to angles of $\triangle WYZ$. State reasons why the angles are congruent.

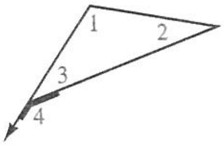


Class Exercises

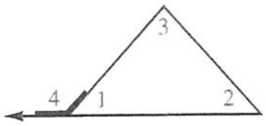
Short Answer

Name the remote interior angles related to ∠4.

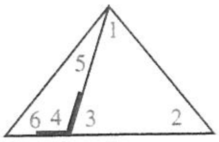
1.



2.

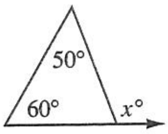


3.



Find the value of x.

4.



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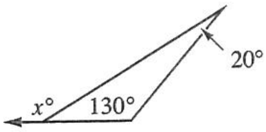

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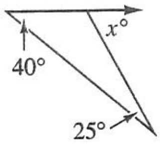
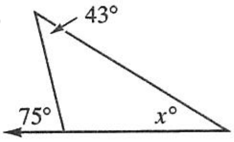


Sample Exercises

Find the value of x.

7.



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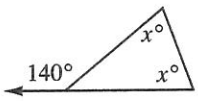

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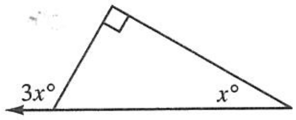

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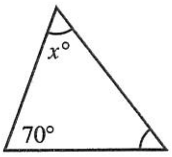

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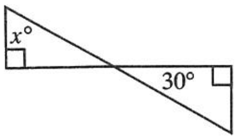

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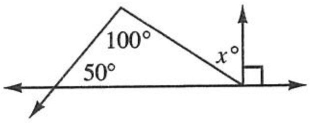
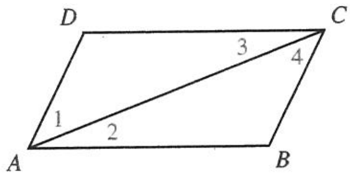


13. Complete the proof.

Given: $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \parallel \overline{CD}$

Prove: $\angle B \cong \angle D$



Proof

Statements	Reasons
1. $\overline{AD} \parallel \overline{BC}$	**1.** Given
2. $\overline{AB} \parallel \overline{CD}$	**2.** Given
3. $\angle 2 \cong \angle 3$	**3.** ______
4. $\angle 1 \cong \angle 4$	**4.** ______
5. $\angle B \cong \angle D$	**5.** ______

Discussion

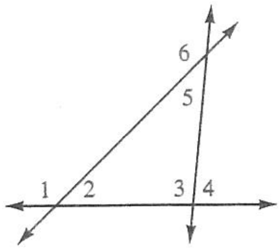
14. If two angles of one triangle are congruent to two angles of another triangle, is it possible for the third angles not to be congruent? Give a convincing argument to support your answer.

15. If two sides of one triangle are congruent to two sides of another triangle, is it possible for the third sides not to be congruent? Give a convincing argument to support your answer.

Exercises

A

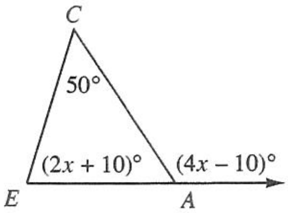
Find the measure of the indicated angle.



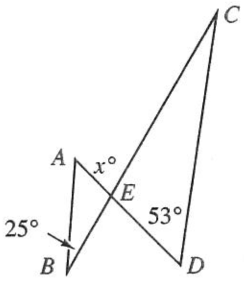
1. If $m\angle 4 = 75$ and $m\angle 2 = 40$, find $m\angle 5$.
2. If $m\angle 1 = 125$ and $m\angle 3 = 90$, find $m\angle 5$.
3. If $m\angle 5 = 40$ and $m\angle 1 = 115$, find $m\angle 4$.
4. If $m\angle 6 = 150$ and $m\angle 2 = m\angle 3$, find $m\angle 2$.
5. If $m\angle 4 = 80$ and $m\angle 2 = m\angle 5$, find $m\angle 2$.
6. If $m\angle 2 = m\angle 5$ and $m\angle 4 = 80$, find $m\angle 6$.
7. If $m\angle 1 = 100$ and $m\angle 4 = 90$, find $m\angle 6$.
8. If $m\angle 4 = 100$ and $m\angle 2$ is three times $m\angle 5$, find $m\angle 2$.
9. If $m\angle 5 = 30$ and $m\angle 4$ is twice $m\angle 2$, find $m\angle 4$.

$\overline{AB} \parallel \overline{CD}$. **Find $m\angle CEA$.**

10.



11.



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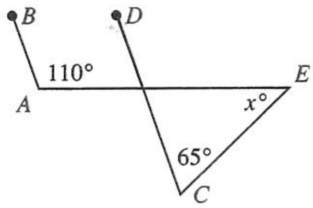
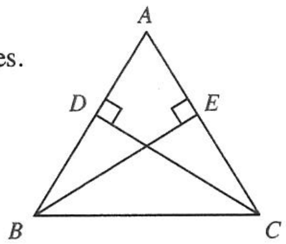


13. Complete the proof.

Given: $\angle ADC$ and $\angle AEB$ are right angles.

Prove: $\angle ABE \cong \angle ACD$

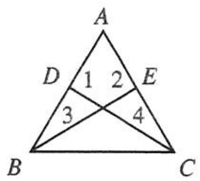


Proof

Statements	Reasons
1. $\angle ADC$ is a right angle.	1. Given
2. $\angle AEB$ is a right angle.	2. Given
3. $\angle ADC \cong \angle AEB$	3. _____
4. $\angle A \cong \angle A$	4. _____
5. $\angle ABE \cong \angle ACD$	5. _____

14. Given: $\angle 1 \cong \angle 2$

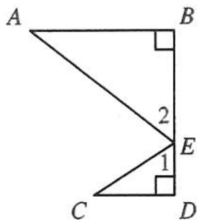
Prove: $\angle 3 \cong \angle 4$



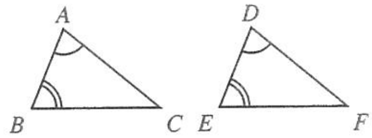
15. Given: $\overline{AB} \perp \overline{BD}$, $\overline{CD} \perp \overline{BD}$

$\angle 1 \cong \angle 2$

Prove: $\angle A \cong \angle C$



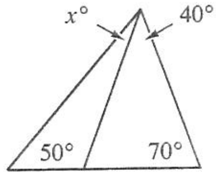
16. Write a two-column proof for Theorem 3.12.
Given: $\angle A \cong \angle D$, $\angle B \cong \angle E$
Prove: $\angle C \cong \angle F$



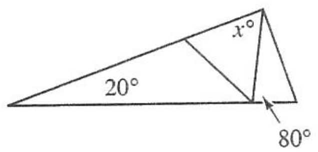
B

Find the value of *x*.

17.



18.



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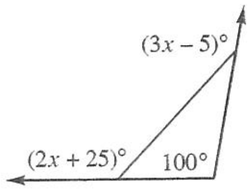

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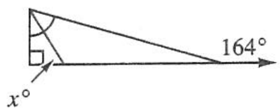

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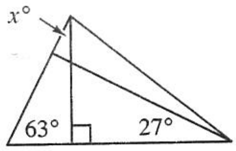

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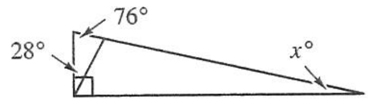
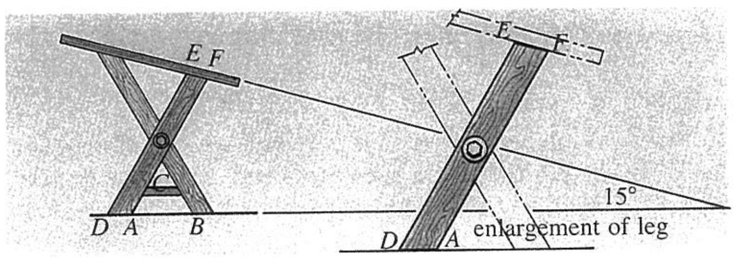


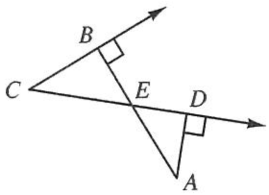
23. If the measures of the exterior angles of the acute angles of a right triangle are $2x + 15$ and $3x + 5$, find the measures of the acute angles.

24. If the ratio of the measures of the angles of a triangle is $1:2:3$, find the ratio of the measures of the exterior angles of the triangle.

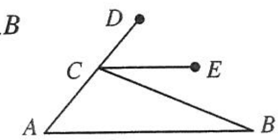
25. The design for a drafting table is shown. If the line of the slant of the table top is extended to the floor a 15° angle is formed. If $\triangle ABC$ is equiangular, what are the four angles of the table leg *ADEF*?



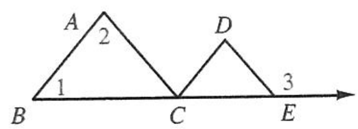
26. Given: $\overline{AB} \perp \overline{BC}$, $\overline{AD} \perp \overline{CD}$
Prove: $\angle A \cong \angle C$



27. Given: $\overline{CE}$ bisects $\angle BCD$. $\angle A \cong \angle B$
Prove: $\overline{CE} \parallel \overline{AB}$

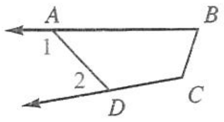


28. Given: $\overline{AC} \parallel \overline{DE}$
Prove: $m\angle 3 = m\angle 1 + m\angle 2$

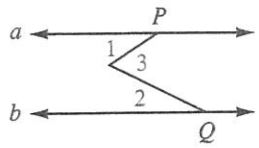


C

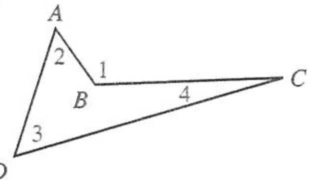
29. Given: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$ as shown
Prove: $m\angle 1 + m\angle 2 = m\angle C + m\angle B$



30. Given: $a \parallel b$
Prove: $m\angle 3 = m\angle 1 + m\angle 2$

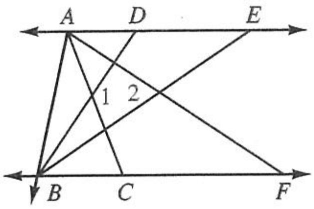


31. Given: figure $ABCD$
Prove: $m\angle 1 = m\angle 2 + m\angle 3 + m\angle 4$



Critical Thinking

32. $\overline{AE} \parallel \overline{BF}$, $\overline{BD}$ and $\overline{BE}$ trisect $\angle ABF$. $\overline{AC}$ and $\overline{AF}$ trisect $\angle BAE$.
Find $m\angle 1$ and $m\angle 2$ if
a. $m\angle ABF = 45$
b. $m\angle ABF = 60$
c. $m\angle ABF = 75$
What seems to be true about $\angle 1$ and $\angle 2$?
Prove your conjecture.



Algebra Review

Translate to a system of equations and solve.

1. Find two integers whose sum is 26 and whose difference is 8.
2. There are 37 students in geometry class. There are 9 more boys than girls. How many girls and boys are in class?

Historical Note

The Parallel Postulate

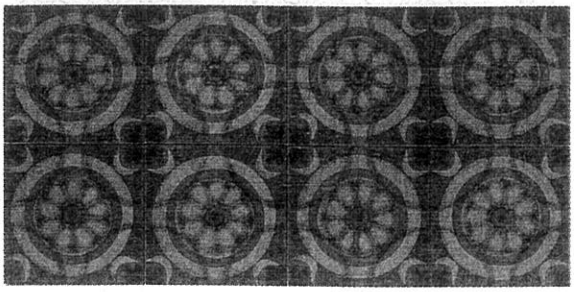
One of the most famous postulates in Euclidean geometry is Euclid's fifth postulate, known as the parallel postulate. It appears on page 123 of this book in this revised form:

Given a line *l* and a point *P* not on *l*, there exists one and only one line through *P* parallel to *l*.

For hundreds of years mathematicians tried without success to prove the postulate as a theorem, that is, to deduce it from Euclid's other four postulates. It was not until the last century that three mathematicians, Bolyai, Lobachevsky, and Gauss, working independently, discovered that Euclid's parallel postulate could not be proven from his other postulates. Their discovery paved the way for the development of other kinds of geometry, called non-Euclidean geometries.

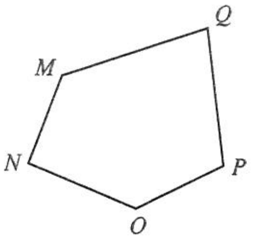
OBJECTIVE: *Apply angle sum theorems for the interior and exterior angles of a polygon.*

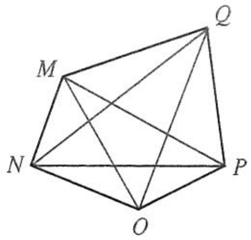
3-6 Angles of a Polygon



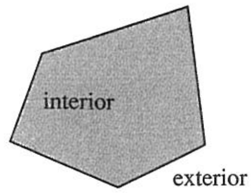
Some of the terms you have been using with triangles apply to other polygons as well. A polygon is a union of segments that meet only at endpoints such that

(1) at most two segments meet at one point
(2) each segment meets exactly two other segments.

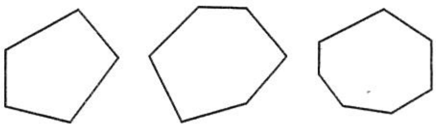




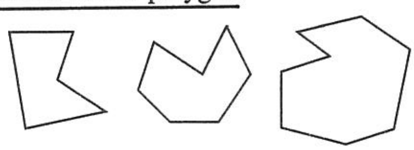
Points M, N, O, P, and Q are vertices of polygon $MNOPQ$.
$\overline{MN}$, $\overline{NO}$, $\overline{OP}$, $\overline{PQ}$, and $\overline{QM}$ are sides of the polygon.
$\overline{MO}$, $\overline{MP}$, $\overline{NP}$, $\overline{NQ}$, and $\overline{OQ}$ are diagonals of polygon $MNOPQ$.



A polygon divides a plane into three parts: the interior of the polygon, the exterior of the polygon, and the polygon itself. A polygon is convex if all the diagonals of the polygon are in the interior of the polygon.



Convex polygons

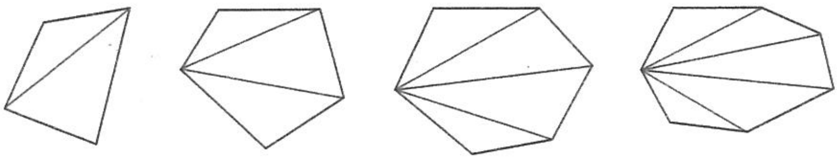


Nonconvex polygons

A polygon is named by the number of sides it has.

Number of sides	Polygon
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
.	.
.	.
.	.
n	n-gon

The sum of the measures of the interior angles of a convex polygon can be determined by drawing diagonals from only one vertex.



Sides	4	5	6	7
Triangles	2	3	4	5
Angle sum	2(180) = 360	3(180) = 540	4(180) = 720	5(180) = 900

Observe that in each case the number of triangles formed is two less than the number of sides. This suggests the following theorem.

THEOREM 3.13

The sum of the measures of the angles of a convex polygon of n sides is $(n - 2)\,180$.

Example 1

Find the sum of the measures of the angles of a nonagon.

Solution

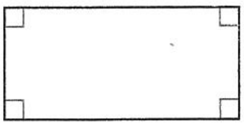
$(n - 2)\,180$ — *Theorem 3.13; a nonagon has nine sides.*

$(9 - 2)\,180 = 1260$ — $n = 9$

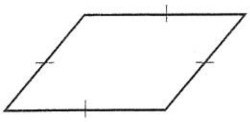
Try This

The sum of the measures of the angles of a polygon is 1620. How many sides does the polygon have?

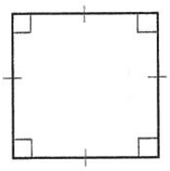
A polygon is *equilateral* if all of its sides are congruent, *equiangular* if all of its angles are congruent. If it is both equilateral and equiangular, it is a *regular polygon*.



Equiangular polygon



Equilateral polygon



Regular polygon

If the sum of the angle measures of a regular polygon is divided by the number of its sides, the resulting quotient is the measure of each angle.

COROLLARY 3.13a

The measure of each angle of a regular polygon of n sides is $\frac{(n-2)180}{n}$.

Example 2

Find the measure of each angle of a regular decagon.

Solution

$m\angle A = \frac{(n-2)180}{n}$ *Let $m\angle A$ be the measure of one angle of a regular decagon.*

$m\angle A = \frac{(10-2)180}{10}$ $n = 10$

$m\angle A = 144$

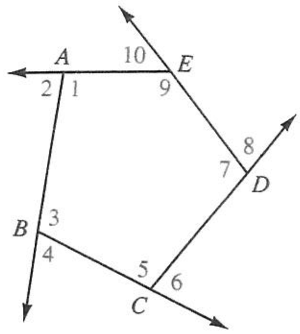
Try This

Each angle of a regular polygon has a measure of 140. How many sides does the polygon have?

A property of the exterior angles of a polygon can be developed by the following method.

At each vertex of pentagon *ABCDE* there is a linear pair of angles.

Use the Linear Pair Postulate.



$$m\angle 1 + m\angle 2 = 180$$
$$m\angle 3 + m\angle 4 = 180$$
$$m\angle 5 + m\angle 6 = 180$$
$$m\angle 7 + m\angle 8 = 180$$
$$m\angle 9 + m\angle 10 = 180$$

Add the left members and the right members of the above equations to obtain

$$\text{(exterior sum)} + \text{(interior sum)} = 5(180)$$
$$\text{(exterior } \angle \text{ sum)} + (5-2)180 = 5(180)$$
$$\text{(exterior } \angle \text{ sum)} = 5(180) - 3(180)$$
$$\text{(exterior } \angle \text{ sum)} = 2(180)$$
$$\text{(exterior } \angle \text{ sum)} = 360$$

If five sides is replaced by *n* sides, the sum will still be 360.

◆ **THEOREM 3.14**

The sum of the measures of the exterior angles of a convex polygon is 360.

Example 3

The measure of an exterior angle of a regular polygon is 24. How many sides does the polygon have?

Solution

$24x = 360$ *Let x = the number of sides.*

$x = 15$

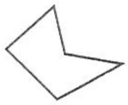
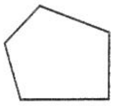
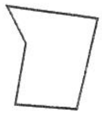
Try This

The sum of the measures of four exterior angles of a pentagon is 300. What is the measure of the fifth exterior angle?

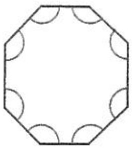
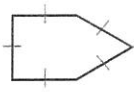
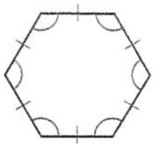
Class Exercises

Short Answer

Indicate which of the polygons are convex.

1.  2.  3. 

Name each polygon and indicate if it is a regular polygon.

4.  5.  6. 

Sample Exercises

Complete the table below for regular polygons.

7. Number of sides	6	8	10		20	
8. Measure of each interior ∠				150	162	
9. Measure of each exterior ∠						12

Discussion

10. As the number of sides of a convex polygon increases, what happens to the sum of the measures of its interior angles? What is the greatest possible sum of the measures of the interior angles of a polygon?

11. As the number of sides of a regular polygon increases, what happens to the measure of each exterior angle? What is the least possible measure of an exterior angle of a polygon?

Exercises

A

The number of sides of a convex polygon is given. Find the sum of the measures of the interior angles of each polygon.

1. 8 **2.** 12 **3.** 14 **4.** 16 **5.** p

The sum of the measures of the interior angles of a convex polygon is given. Find the number of sides of each polygon.

6. 7020 **7.** 1980 **8.** 6120 **9.** 1800 **10.** 3420

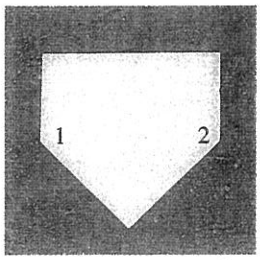
The number of sides of a regular polygon is given. Find the measure of each interior angle of each polygon.

11. 7 **12.** 9 **13.** 11 **14.** 15 **15.** 17

Find the measure of each exterior angle of the regular polygon.

16. pentagon **17.** heptagon **18.** decagon **19.** 18-gon **20.** 20-gon

21. Home plate on a baseball field has three right angles and two congruent angles shown as $\angle 1$ and $\angle 2$ in the figure. Find $m\angle 1$ and $m\angle 2$.



22. The sum of the measures of seven angles of an octagon is 1000. Find the measure of the eighth angle.

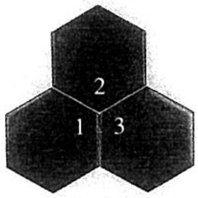
23. How many sides does a regular polygon have if each exterior angle has a measure of 15?

24. How many sides does a regular polygon have if each interior angle has a measure of 108?

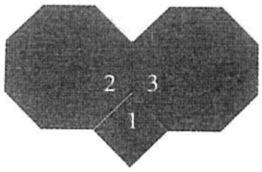
B

25. Draw a diagram to show that a floor can be tiled with equilateral triangles. Give a convincing argument that the sum of the measures of the angles sharing a common vertex is 360.

26. The tiling pattern to the right consists of regular hexagons. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ and verify that their sum is 360.



27. The tiling pattern to the right consists of regular octagons and a square. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ and verify that their sum is 360.



28. Explain why three regular pentagons cannot form a tiling pattern.

29. Explain why two regular pentagons and a regular 10-gon will fit around a point.

30. Explain why an equiangular triangle, a regular 7-gon, and a regular 42-gon will fit around a point.

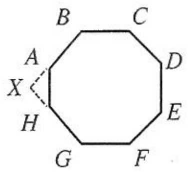
31. Find the number of sides of a polygon if the sum of the measures of its interior angles is twice the sum of the measures of its exterior angles.

32. Find the number of sides of a polygon if the sum of the measures of its interior angles is four times the sum of the measures of its exterior angles.

33. The measure of each interior angle of a regular polygon is eight times that of an exterior angle. How many sides does the polygon have?

34. In quadrilateral *ABCD* the measures of $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are in the ratio of 1 : 2 : 3 : 4, respectively. Find the measures of the four angles and determine which segments are parallel.

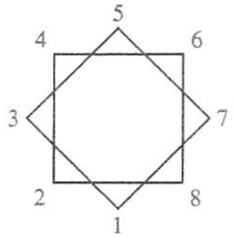
35. *ABCDEFGH* is a regular octagon. If sides $\overline{AB}$ and $\overline{GH}$ are extended to meet at *X*, find $m\angle BXG$.



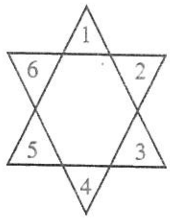
C

The numbered angles in each figure are congruent. Find the sum of the numbered angles.

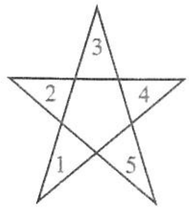
36.



37.



38.



39. Write a generalization about the sums you found in Exercises 36–38.

Critical Thinking

40. Complete the table below by drawing each polygon and its diagonals.

Number of sides	3	4	5	6	7	8	9
Number of diagonals	0	2	5				

41. Predict how many diagonals a decagon has.
Write a formula expressing the relationship between the number of sides (n) and the number of diagonals (d). Use it to check Exercise 40.

Mixed Review

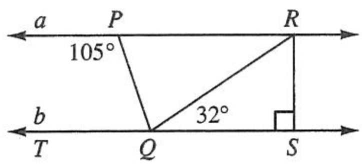
1. The midpoint of $\overline{SV}$ is T. Name the congruent segments formed.
2. Line l bisects $\overline{MN}$ at P. Name the congruent segments formed.
3. $\overrightarrow{BC}$ is the bisector of $\angle ABD$. Name the congruent angles formed.

Quiz

Determine whether each statement is true or false.

1. An obtuse triangle may have a right angle.
2. The acute angles of a right triangle are complementary.
3. A triangle may have an obtuse exterior angle.
4. In any triangle, the remote interior angles are acute.
5. If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.
6. The sum of the measures of the angles of a nonagon is 1440.

$a \parallel b$ **Find the measure of each angle.**



7. $m\angle QRS$ **8.** $m\angle PQT$ **9.** $m\angle PQR$

10. In an octagon, the sum of the measures of the interior angles is _?_ and the sum of the measures of the exterior is _?_.

CRITICAL THINKING

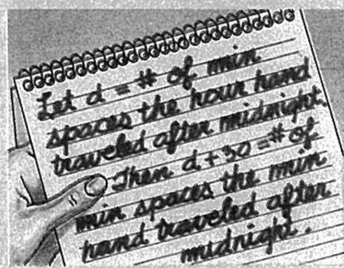
Introducing the Process

Critical Thinking has been described as a thinking process used to decide what to believe or do. Consider the following situation.

Inspector Glueso arrived at the scene of a murder at Motley Motel early in the morning. The first thing he found was a broken pocket watch clutched in the victim's hand. Other evidence led Glueso to determine that the murder was committed at the time shown on the watch and that the murderer was one of three people at the motel: Mae East, Mr. Clean, or Joe Monitor. The hotel operator verified that Mae East, an actress, had been talking on the telephone from 12:32 AM to 12:47 AM. Joe Monitor, the motel manager, had been seen using his computer from about 12:20 AM until he shut it off sometime after 12:30 AM. The computer log showed that it was shut off at 12:34 AM. Mr. Clean, the night custodian, had punched out at 12:31 AM and had left promptly. The only fact that Glueso could remember about the watch was that the hour and minute hands were in a straight line when he picked it up. Who should Glueso arrest as the prime suspect?

The major points below outline the steps in Critical Thinking. Answer each question as you work, with Glueso, to solve his problem.

1. **Understand the Situation**
   a. What is the situation about?
   b. What conclusion, decision, or solution is required?
   c. Are there any questions you could ask to clarify the situation?
2. **Deal with the Data/Evidence/Assumptions**
   a. Make a list of the most important data/evidence. Is more needed?
   b. Which evidence is opinion, not fact? Which is not relevant?
   c. What assumptions are made? Can other assumptions be made?
3. **Go Beyond the Data/Evidence/Assumptions**—Read the notes Glueso made in his notebook to the right. How could you use this and the evidence to solve the mystery?
   (HINT: Since the hour hand travels through 5-minute spaces while the minute hand travels through 60-minute spaces, the distance the hour hand had traveled was equal to $\frac{1}{12}$ the distance the minute hand had traveled.)



4. **State and Support Your Conclusion**—State your solution to the mystery in writing. Write a paragraph supporting your conclusion.
5. **Apply the Conclusion/Decision/Solution**—The solution of a similar mystery depended upon the fact that a crime occurred the first time after midnight that the hands of a clock were both at the same place. At what time (to the nearest second) did this crime take place?

CHAPTER SUMMARY

Vocabulary

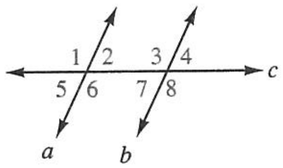
alternate interior angles (3-1)
alternate exterior angles (3-1)
auxiliary line (3-4)
convex polygon (3-6)
corresponding angles (3-1)
diagonal of a polygon (3-6)
exterior angle of a triangle (3-5)
parallel lines (3-1)
parallel planes (3-1)
regular polygon (3-6)
remote interior angles (3-5)
same-side interior angles (3-1)
skew lines (3-1)
transversal (3-1)

Key Ideas

1. Two coplanar lines that do not intersect are called parallel lines.
2. When two parallel lines are cut by a transversal,
   a. corresponding angles are congruent.
   b. alternate interior angles are congruent.
   c. alternate exterior angles are congruent.
   d. same-side interior angles are supplementary.
3. The following are ways to prove two lines parallel.
   a. Show that corresponding angles are congruent.
   b. Show that alternate interior angles are congruent.
   c. Show that alternate exterior angles are congruent.
   d. Show that same-side interior angles are supplementary.
4. The sum of the measures of the angles of a triangle is 180. The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.
5. The sum of the measure of the angles of a convex polygon with n sides is $(n - 2)\,180$. The sum of the exterior angles is 360.

CHAPTER REVIEW

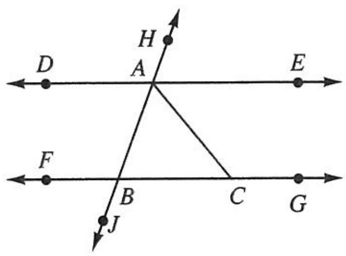
3-1



1. Name two pairs of alternate interior angles.
2. Name two pairs of same-side interior angles.
3. Name two pairs of alternate exterior angles.
4. Name four pairs of corresponding angles.

3-2

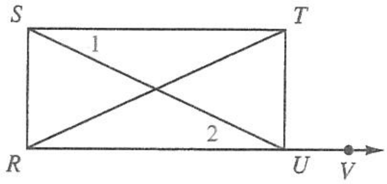
$\overleftrightarrow{DE} \parallel \overleftrightarrow{FG}$ **State the postulate or theorem that justifies each conclusion.**



5. $\angle EAC \cong \angle ACB$
6. $m\angle EAC + m\angle ACG = 180$
7. $\angle DAH \cong \angle FBH$
8. $\angle HAE \cong \angle JBF$
9. $\overline{DE} \parallel \overline{FG}$, $m\angle DAB = 5x - 22$, $m\angle ABC = 2x + 5$. Find $m\angle DAH$.

3-3

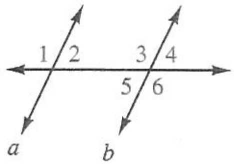
Use the given information to state which segments are parallel. Identify the postulate or theorem that justifies your answer.



10. $\angle 1 \cong \angle 2$ **11.** $m\angle SRU + m\angle RUT = 180$

12. $\angle SRU \cong \angle TUV$ **13.** $\overline{ST} \perp \overline{TU}$, $\overline{RU} \perp \overline{TU}$

Find the value of x for which $a \parallel b$.



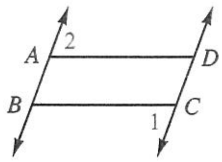
14. $m\angle 2 = 5x + 12$, $m\angle 4 = 2x + 18$

15. $m\angle 2 = 4x + 6$, $m\angle 5 = 14$

16. $m\angle 1 = \frac{3}{4}x + 42$, $m\angle 6 = x + 2$

17. Given: $\overline{AB} \parallel \overline{CD}$, $\angle 1 \cong \angle 2$

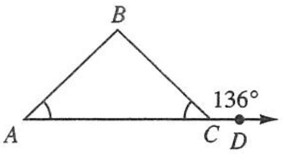
Prove: $\overline{AD} \parallel \overline{BC}$



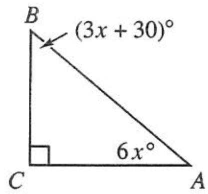
3-4

Find the measure of $\angle B$.

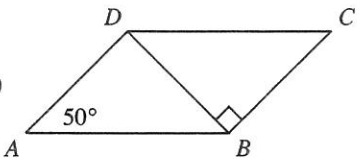
18.



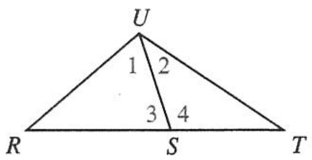
19.



20. $\overline{BD}$ bisects $\angle ADC$.
$\angle A \cong \angle DBA$
$\overline{BC} \perp \overline{BD}$, $m\angle A = 50$
Find $m\angle C$.



3-5



21. Name an exterior angle of $\triangle RUS$.

22. If $m\angle R = 35$ and $m\angle 1 = 20$, find $m\angle 4$.

23. If $m\angle 3 = 5x + 10$, $m\angle 2 = 50$, and $m\angle T = 3x + 10$, find $m\angle T$.

24. $\angle 3$ is an exterior angle for $\triangle$ _?_ and its remote interior angles are $\angle$ _?_ and $\angle$ _?_ .

3-6

25. Find the sum of the measures of the angles of a decagon.

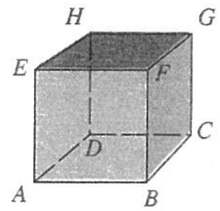
26. Find the number of sides of a convex polygon if the sum of the measures of its interior angles is 4140.

27. Find the measure of an interior angle of a regular hexagon.

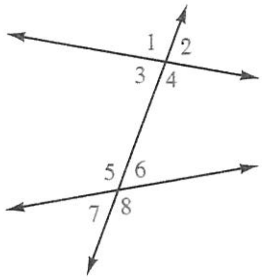
28. Find the measure of an exterior angle of a regular octagon.

CHAPTER TEST

Determine whether each statement is true or false.



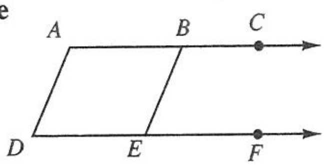
1. $\overline{AB}$ and $\overline{HG}$ are parallel.
2. $\overline{AB}$ and $\overline{CD}$ are skew.
3. Plane $ABEF$ and plane $BCFG$ are parallel.
4. If two parallel lines are cut by a transversal, then corresponding angles are congruent.
5. If two parallel lines are cut by a transversal, then same-side interior angles are congruent.
6. If two coplanar lines are perpendicular to the same line, then they are perpendicular to each other.
7. In a triangle, the measure of an exterior angle is equal to the sum of the measures of its remote interior angles.
8. The sum of the angles of a heptagon is 900°.



Identify each pair of angles as alternate interior, alternate exterior, same-side interior, corresponding, or none of these.

9. $\angle 4$ and $\angle 8$
10. $\angle 3$ and $\angle 6$
11. $\angle 1$ and $\angle 7$

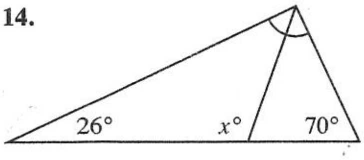
Use the given information to determine which segments are parallel.



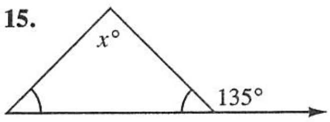
12. $\angle D \cong \angle BEF$
13. $m\angle ABE + m\angle BAD = 180$

Find the value of x. Congruent angles are marked.

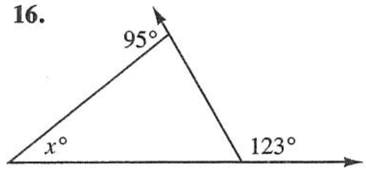
14.

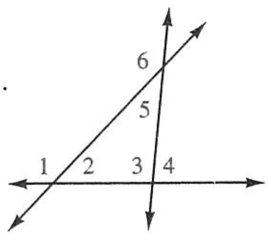


15.



16.





17. If $m\angle 4 = 85$ and $m\angle 2 = 38$, find $m\angle 5$.
18. If $m\angle 1 = 118$ and $m\angle 3 = 95$, find $m\angle 5$.
19. Find the measure of each interior angle of an 18-sided regular polygon.
20. Find the number of sides of a convex polygon if the measures of its interior angles have a sum of 5940.
21. **Given:** $\overline{AB} \parallel \overline{CE}$, $\angle 2 \cong \angle 3$
    Prove: $\angle 1 \cong \angle 3$