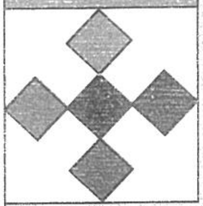
OBJECTIVE: *Apply the Pythagorean Theorem.*

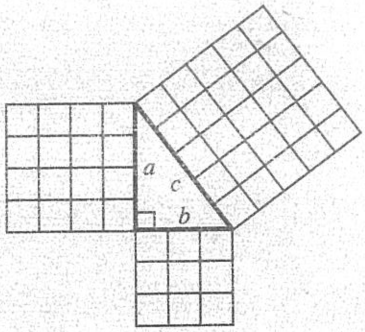
7-2 The Pythagorean Theorem



EXPLORE

How does the sum of the areas of the squares on the legs of this right triangle compare to the area of the square on the hypotenuse? Make a generalization.

Use graph paper to decide if this generalization is true for right triangles with legs $a = 5$, $b = 12$, and $a = 8$, $b = 15$.



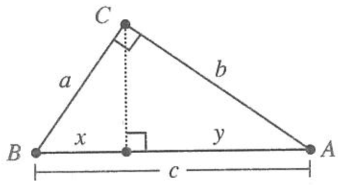
The Pythagorean Theorem is one of the most famous and useful theorems in plane geometry. It can be used to find the length of the third side of a right triangle when the lengths of two of the sides are known. It was named after the Greek mathematician Pythagoras who is thought to have given the first proof of the theorem around 500 BC. In the Explore, you may have "discovered" this famous theorem.

THEOREM 7.4 Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.

Given: $\triangle ABC$ is a right triangle.
$\angle C$ is a right angle.

Prove: $a^2 + b^2 = c^2$



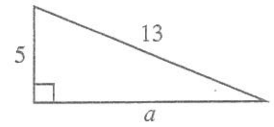
Proof

Statements	Reasons
1. $\triangle ABC$ is a right triangle. $\angle C$ is a right angle.	1. Given
2. Draw a perpendicular from C to $\overline{AB}$.	2. Theorem 2.9
3. $\frac{c}{a} = \frac{a}{x}$, $\frac{c}{b} = \frac{b}{y}$	3. Theorem 7.3
4. $a^2 = cx$, $b^2 = cy$	4. Product of mean equals product of extremes.
5. $a^2 + b^2 = cx + cy$	5. Addition Property
6. $a^2 + b^2 = c(x + y)$	6. Distributive Property
7. $a^2 + b^2 = c^2$	7. Segment Addition Postulate, Substitution

The following example shows how to use the Pythagorean Theorem.

Example

Find the value of a.



Solution

$5^2 + a^2 = 13^2$ *Use the Pythagorean Theorem to write this equation.*

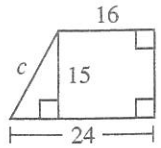
$25 + a^2 = 169$

$a^2 = 144$

$a = 12$ *Note that $a = -12$ is another solution to the equation, but the length of a segment must be a positive number.*

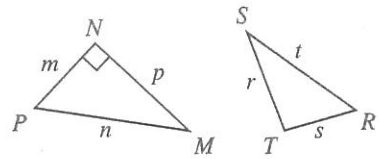
Try This

Find the value of c in the figure to the right.



Class Exercises

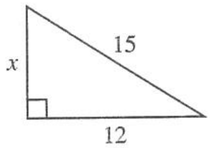
Short Answer

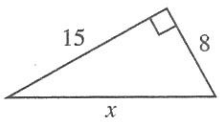


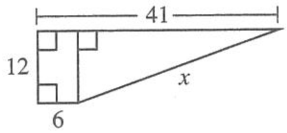
1. State the Pythagorean Theorem in your own words.
2. What equation gives the relationship of the sides in $\triangle MNP$?
3. What information must you know about $\triangle RST$ to conclude that $r^2 + s^2 = t^2$.
4. Give an equation that can be solved to find the length of the hypotenuse of a right triangle with a 10-cm leg and a 24-cm leg.

Sample Exercises

Find the value of x.

5. 

6. 

7. 

8. Find the length of a diagonal of a square with side length $2\sqrt{2}$.
9. Find the perimeter of a rhombus with diagonals of 10 and 24 inches.

Discussion

10. If $\triangle ABC$ has sides with lengths 3, 4, and 5, why can you not conclude directly from the Pythagorean Theorem that it is a right triangle?

Determine whether each statement about the Pythagorean Theorem is always, sometimes, or never true.

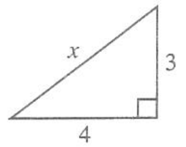
11. It states a relationship among the sides of an obtuse triangle.
12. It states a relationship among the sides of an isosceles triangle.
13. It states a relationship among the sides of a scalene triangle.
14. It states a relationship among the sides of a right triangle.

Exercises

A

Find the value of x.

1.



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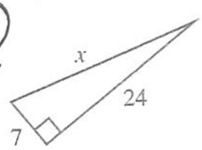

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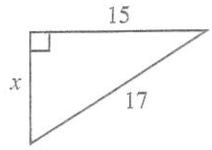

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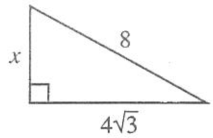

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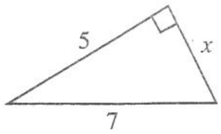

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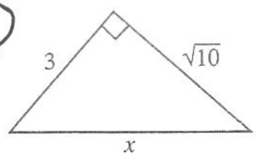
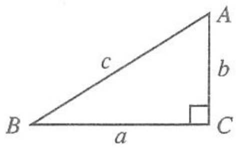
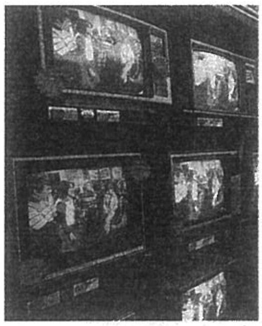


Find each missing length. Express radicals in simplest form.



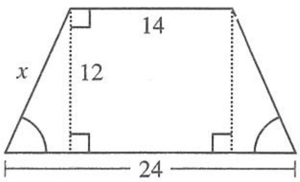
7. If $a = 6$ and $b = 8$, then $c =$ ___.
8. If $c = 15$ and $a = 9$, then $b =$ ___.
9. If $b = 2$ and $a = 2$, then $c =$ ___.
10. If $c = \sqrt{15}$ and $a = \sqrt{10}$, then $b =$ ___.
11. If $b = \sqrt{2}$ and $a = \sqrt{3}$, then $c =$ ___.
12. If $a = 2\sqrt{3}$ and $c = 6$, then $b =$ ___.

13. If the legs of an isosceles right triangle are 6 units long, find the length of the hypotenuse.
14. The length of a rectangle is 24 cm and the width is 10 cm. How long is the diagonal?
15. A television screen measures approximately 15.5 in. high and 19.5 in. wide. A television is advertised by giving the approximate length of the diagonal of its screen. How should this television be advertised?

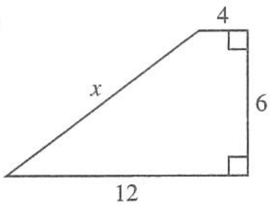


Find each missing length x.

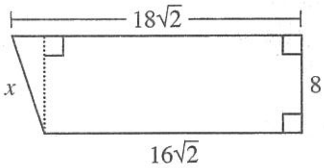
16.



17.



18.

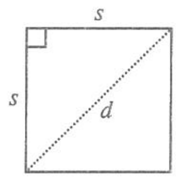


The length of the side of a square is given. Find the length of the diagonal of the square. Express radicals in simplest form.

19. $s = 2$ 20. $s = 3$ 21. $s = 4$ 22. $s = 5$

B

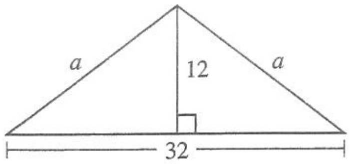
23. Look for a pattern in the answers to Exercises 19–22. Predict the formula for finding the diagonal of any square with side length s. Use the Pythagorean Theorem to show that your prediction is correct.



24. Find the length of each leg of an isosceles right triangle with hypotenuse 30 cm long.

Find the value of each variable.

25.



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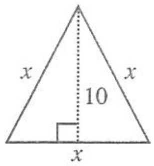

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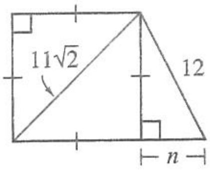

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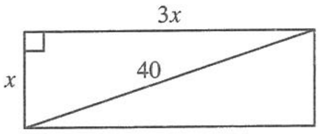

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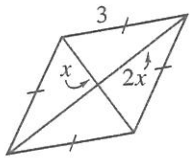

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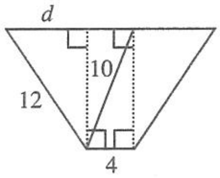
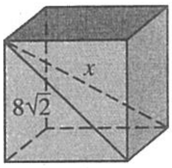
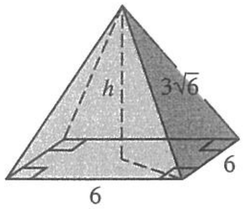


31.

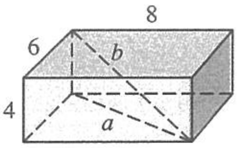


Cube

32.

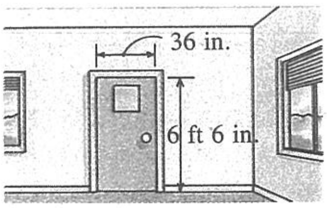


33.



Rectangular prism

34. The base of an isosceles triangle is $2x$ cm long. The altitude to the base is $3x$ cm long. Find the length of one other side of the triangle.
35. The hypotenuse of a right triangle is three times the length of a leg. The sum of the sides of the triangle is between 6 and 8. How long are the legs and the hypotenuse of the triangle if the length of the hypotenuse is an integer?
36. Find the altitude of an equilateral triangle with side length ten.
37. Find the perimeter of a rectangle that has diagonal length eight and a side of length five.
38. Find the perimeter of an isosceles trapezoid that has a base with length 10, another base with length 18, and height 8.
39. A 6-ft ladder is placed against a wall with its base 2 ft from the wall. How high above the ground is the top of the ladder?
40. A person travels 8 mi due north, 3 mi due west, 7 mi due north, and 11 mi due east. How far is that person from the starting point?
41. A door is 6 ft 6 in. high and 36 in. wide. Can a thin piece of plywood 7 ft wide be carried through the door?

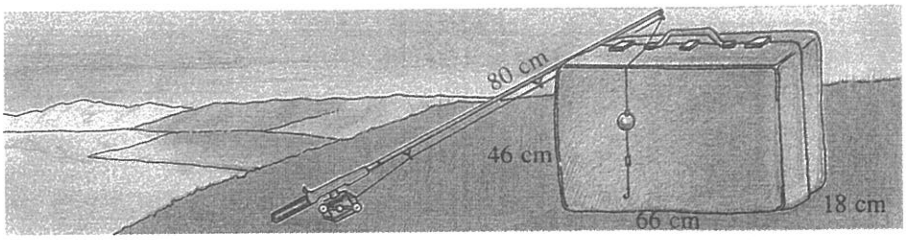


C

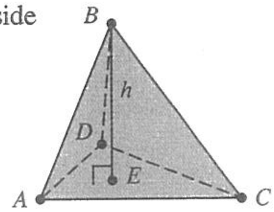
42. In $\triangle ABC$, a, b, and c are the lengths of the sides opposite vertices A, B, and C. If $a = 7$, $b = 8$, and $c = 9$, find the length of the altitude of the triangle from vertex C.

43. In isosceles $\triangle DEF$, $DE = EF = 25$ and $DF = 30$. Find the length of the altitude of the triangle from vertex F.

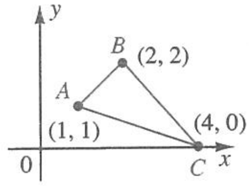
44. Will a fishing rod that collapses to a length of 80 cm fit into a suitcase with the dimensions shown?



45. The faces of this triangular pyramid are equilateral triangles with side length 6. Find the altitude h of the pyramid. (HINT: Point E is the intersection of the medians of $\triangle ADC$.)

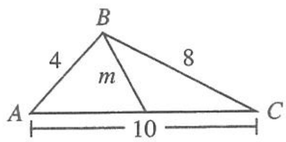


46. $\triangle ABC$ is a right triangle. Show that $AC^2 = AB^2 + BC^2$.



47. Mark and connect the points $(2, 4)$, $(-2, 2)$, $(4, 0)$, and $(-4, 4)$ on a coordinate grid. Find the perimeter of this quadrilateral and express it as simply as possible.

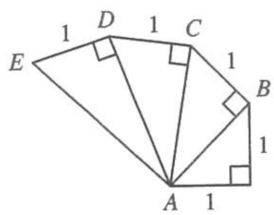
48. Find the length, to the nearest tenth, of the median m of this triangle. (HINT: Draw the altitude of the triangle from B.)



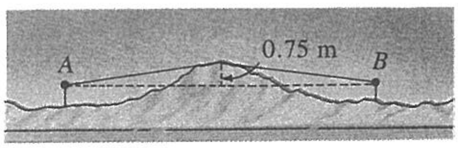
49. Use a straightedge and compass to construct a right triangle with hypotenuse equal to the length of this segment.

50. The 3-mi road from Cisco to Rockton forms a right angle with the 4-mi road from Rockton to Bayville. Emerson is on the straight road from Bayville to Cisco 2 mi from Bayville. Find the distance from Emerson to Rockton.

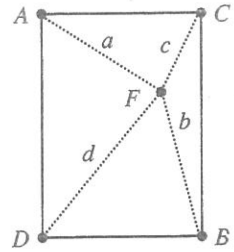
51. Find the lengths of $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, and $\overline{AE}$. Then, use a compass and straightedge to construct segments with lengths $\sqrt{6}$ and $\sqrt{7}$.



52. A surveyor wants to measure the distance between points A and B on rough land. She wants to find the actual horizontal distance AB. If the land is 0.75 m higher midway between the two stakes, and the measuring tape reads 27.0 m, use a calculator to find the actual distance AB.

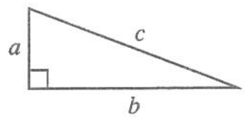


Critical Thinking

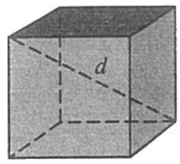


53. Four cities A, B, C, and D are located at the corners of a rectangle. If a factory is built at any point F inside the rectangle, the builders claim that the following relationship between the distances from the factory to the cities holds. $a^2 + b^2 = c^2 + d^2$

    Draw two or three accurate rectangles and test this conjecture by measuring to the nearest millimeter. If you think the conjecture is true, give a convincing argument to support your conclusion.



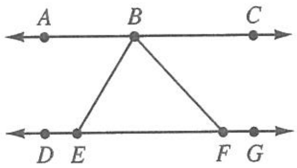
54. A student looked at a right triangle and stated "b^2 can be found by multiplying the sum of the other leg and the hypotenuse by the difference of that leg and the hypotenuse." Test several cases and decide if this generalization is true. If you think it is, give a convincing argument to support your conclusion.



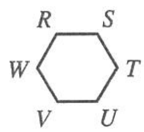
55. Give a general rule for finding the length of the diagonal of a cube when you know the length of one of its sides.

Mixed Review

Complete each statement.



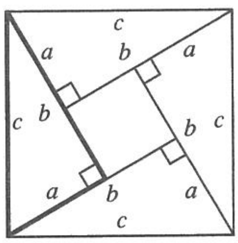
1. $\angle ABE$ and $\angle BEF$ are ___ angles.
2. If $\overleftrightarrow{AC} \parallel \overleftrightarrow{DG}$, then $\angle DEB \cong$ ___.
3. If $\overleftrightarrow{AC} \parallel \overleftrightarrow{DG}$, $m\angle ABE = 60$, and $m\angle EBF = 50$, then $m\angle BFG =$ ___.
4. If $\angle CBF$ is supplementary to ___, then $\overleftrightarrow{AC} \parallel \overleftrightarrow{DG}$.
5. $m\angle EBF + m\angle BFE =$ ___



6. Find the sum of the measures of the angles of $RSTUVW$.
7. If $RSTUVW$ is a regular hexagon, find the measure of each of its angles.
8. Find the sum of the measures of the exterior angles of $RSTUVW$.

Enrichment (Read only)

Another Proof of the Pythagorean Theorem



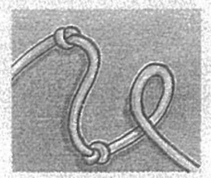
The Hindu mathematician Bhaskara (1114–1185) was reported to have been very excited when he used this figure to discover a proof of the Pythagorean Theorem. He placed three copies of the right triangle as shown to form a large square with a small square in the center.

1. Use the variables given to express the side length and area of the small square.
2. Write an equation relating the areas of the small and the four triangles to the area of the large square.
3. Simplify the equation to prove the theorem!

7-3 The Converse of the Pythagorean Theorem

EXPLORE

Cut a string or piece of yarn 12 in. long. At what two locations could you tie knots in the string so that it could be used to accurately form a right angle? Why does your method work?



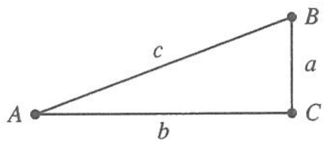
When using the Pythagorean Theorem, you are given a right triangle and conclude a relationship among its side lengths. When using the *converse* of the Pythagorean Theorem, you are given a relationship among the side lengths and conclude that the given triangle is a right triangle. You may have used this idea in the Explore.

THEOREM 7.5 Converse of the Pythagorean Theorem

If a triangle has side lengths a, b, and c, and $a^2 + b^2 = c^2$, then the triangle is a right triangle with right angle opposite the side of length c.

Given: $\triangle ABC$ with $a^2 + b^2 = c^2$

Prove: $\triangle ABC$ is a right triangle.



Plan Draw a right triangle $\triangle PQR$ with $\angle Q$ a right angle, legs of lengths a and b, and hypotenuse length x. Use the Pythagorean Theorem to show that $x = c$. Then prove that the two triangles are congruent and use this to show that $\angle C$ is a right angle.

You will be asked to prove this theorem in Exercise 24.

Example 1

Is a triangle with sides 8, 15, and 17 a right triangle?

Solution

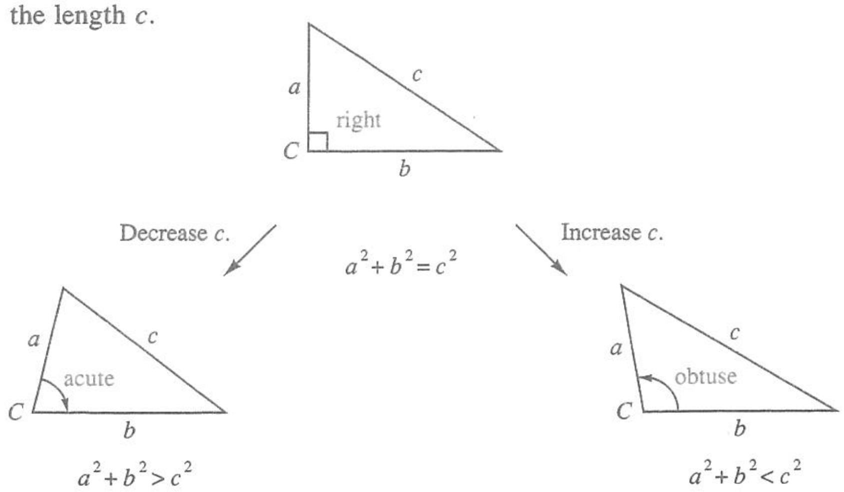
$8^2 + 15^2 = 289$, $17^2 = 289$

Since $8^2 + 15^2 = 17^2$, a triangle with sides 8, 15, and 17 is a right triangle. *Theorem 7.4*

Try This

Is a triangle with sides 5, 6, and 8 a right triangle?

Consider the diagram below showing the result of increasing or decreasing the length c.



This diagram suggests the following convincing argument that the next theorem is true. If c is decreased, while keeping a and b the same, $a^2 + b^2$ will be greater than c^2 and $\angle C$ will be smaller and acute. If c is increased, while keeping a and b the same, $a^2 + b^2$ will be less than c^2 and $\angle C$ will be larger and obtuse.

These ideas are summarized in Theorem 7.6

◆ THEOREM 7.6

If $a < b < c$ are lengths of the sides of a triangle and

a. $a^2 + b^2 < c^2$, then the triangle is an obtuse triangle.

b. $a^2 + b^2 > c^2$, then the triangle is an acute triangle.

Example 2

Is a triangle with sides of lengths 2, 3, and 4 acute, right, or obtuse?

Solution

$2^2 + 3^2 = 4 + 9 = 13$

$4^2 = 16$

$2^2 + 3^2 < 4^2$, so the triangle is obtuse. *Theorem 7.6*

Try This

Is a triangle with sides $\sqrt{3}$, $\sqrt{4}$, and $\sqrt{5}$ a right triangle? Explain.

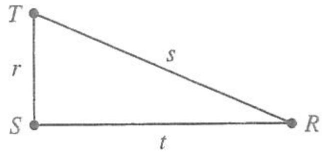
A **Pythagorean Triple** is any three whole numbers a, b, and c that satisfy the equation $a^2 + b^2 = c^2$. Theorem 7.5 allows you to conclude that the numbers in a Pythagorean Triple are sides of a right triangle. Some commonly used Pythagorean Triples are (3, 4, 5), (5, 12, 13), (8, 15, 17), and (7, 24, 25). Any multiple of a Pythagorean Triple is also a Pythagorean Triple. For example, (6, 8, 10), (9, 12, 15), and so on, are multiples of (3, 4, 5) and are all Pythagorean Triples.

Class Exercises

Short Answer

1. State the Pythagorean Theorem in your own words.
2. State the converse of the Pythagorean Theorem in your own words.

Complete each statement.



3. If $r^2 + t^2 < s^2$, then $\angle S$ is ___.
4. If $r^2 + t^2 > s^2$, then $\angle S$ is ___.
5. If $r^2 + t^2 = s^2$, then $\angle S$ is ___.
6. The converse of the Pythagorean Theorem helps you tell if a triangle is ___.
7. If x, y, and z are a Pythagorean Triple, they satisfy the equation ___.

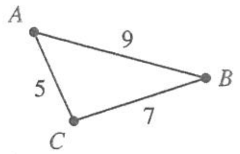
Sample Exercises

Which of these triples are sides of a right triangle?

8. (2, 3, 4)
9. (6, 8, 10)
10. (0.1, 0.4, 0.5)
11. (1, 1, 2)
12. $(\sqrt{2}, \sqrt{3}, \sqrt{5})$
13. $(\frac{1}{3}, \frac{1}{4}, \frac{1}{5})$

Classify each triangle as acute, right, or obtuse.

14.



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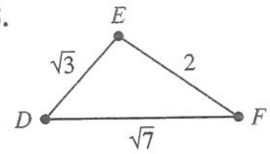

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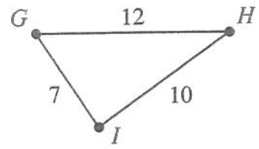
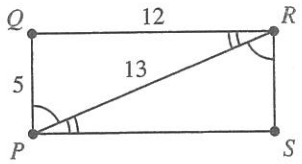


17. A rhombus has side length 5 and diagonal length $5\sqrt{2}$. Is the quadrilateral a square? Why or why not?
18. Certain side lengths and pairs of congruent angles are marked in the figure. How do you know that $\angle S$ is a right angle?



Discussion

Determine whether each statement is true or false. Give a convincing argument to support your decision.

19. The numbers $\sqrt{3}$, $\sqrt{4}$, and $\sqrt{7}$ do not form a Pythagorean Triple.
20. The converse of the Pythagorean Theorem allows you to conclude that the sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse.
21. The numbers 0.3, 0.4, and 0.5 form a Pythagorean Triple.
22. If the sum of the squares of the lengths of the two shorter sides of a triangle is less than the square of the length of the longer side, the triangle is acute.
23. A student claimed that if each number in a Pythagorean Triple is multiplied by the same number, another Pythagorean Triple results. Use the triple (3, 4, 5) and discuss whether you think this is true. Give another example of a "family of triples."

Exercises

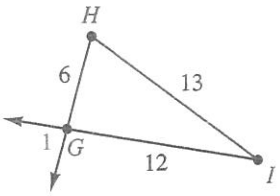
A

Classify each triangle with the given side lengths as acute, right, or obtuse.

1. 4, 5, 7
2. 6, 8, 10
3. 0.4, 0.5, 0.6
4. 0.9, 4.0, 4.1
5. $\sqrt{3}, \sqrt{2}, \sqrt{5}$
6. $\sqrt{2}, \sqrt{3}, \sqrt{4}$
7. 9, 10, 12
8. $\frac{3}{5}, \frac{4}{5}, 1$

Decide if $\angle 1$ is acute, right or obtuse. Give reasons for your decision.

9.



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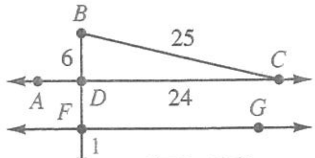
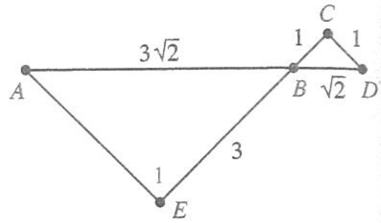


$\overleftrightarrow{AC} \parallel \overleftrightarrow{FG}$

11.



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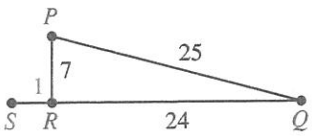

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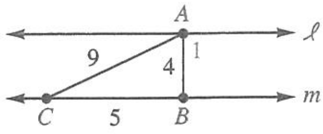
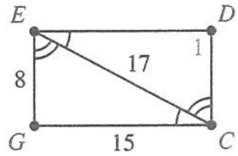


$\ell \parallel m$

14.



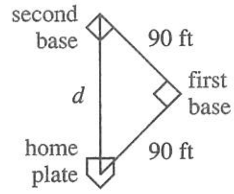
15. The sides of a triangle are 9, 40, and 41. Is the triangle a right triangle? Is a triangle with side lengths twice these a right triangle? Why or why not?

B

Classify each triangle with the given side lengths as acute, right or obtuse, or state that no conclusion can be made. Assume that n is a nonzero whole number.

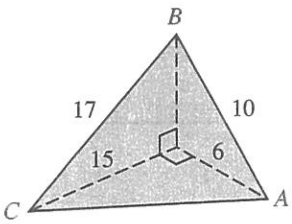
16. $3n, 4n, 5n$
17. $8n, 15n, 17n+1$
18. $11n, 60n-1, 61n$

19. The shortest side of a triangle has length 14. The other sides have lengths $x + 1$ and $x + 3$. Find the value of x that would make the triangle a right triangle and give the length of each side.
20. The shortest side of a triangle has length 4. The other sides have lengths x and $x + 1$. Find the value of x that would make the triangle a right triangle and give the length of each side.
21. A grounds crew was laying out a baseball diamond. They measured the distance from home plate to second base to check that the baseline angle at first base was a right angle. Find distance d.

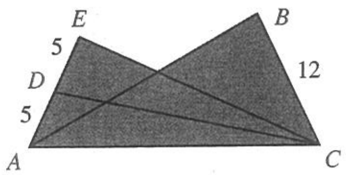


Decide if $\triangle ABC$ is right, acute, or obtuse. Explain.

22.



23.



$AB = 23$, $CD = 24$, $\overline{EC} \cong \overline{AC}$