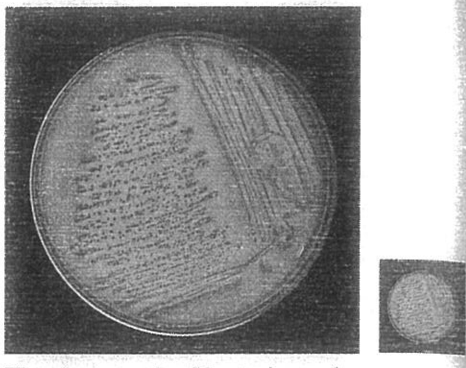
OBJECTIVE: *Deduce information about the measures of sides and angles of two similar figures.*

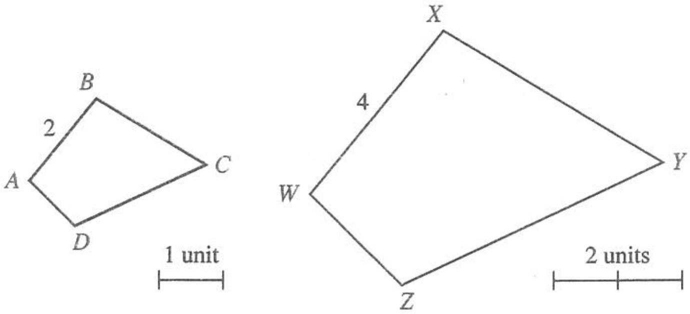
6-3 Similar Polygons



The photograph of bacteria on the left has been enlarged by a factor of 10 to make it easier for scientists to study.

You learned in Chapter 3 that triangles having the same size and shape are called congruent figures. In this lesson you will learn about triangles and other polygons that have the same shape but not necessarily the same size.

Polygon $WXYZ$ is the same shape as polygon $ABCD$. Each side of $WXYZ$ is double the length of the corresponding side of $ABCD$. That is, the ratio of the length of each side of $WXYZ$ to its corresponding side of $ABCD$ is 2. A characteristic of figures that have the same shape is that ratios of corresponding sides are equal. Two polygons that have the same shape are called **similar**.



DEFINITION

Two polygons are similar (~) if their vertices can be matched so that

a. corresponding angles are congruent, and

b. ratios of lengths of corresponding sides are equal.

To indicate that polygon $ABCD$ is similar to polygon $WXYZ$, write $ABCD \sim WXYZ$. When using this notation write corresponding vertices in the same order.

If $ABCD \sim WXYZ$, you know that

a. $\angle A \cong \angle W$, $\angle B \cong \angle X$, $\angle C \cong \angle Y$, $\angle D \cong \angle Z$, and

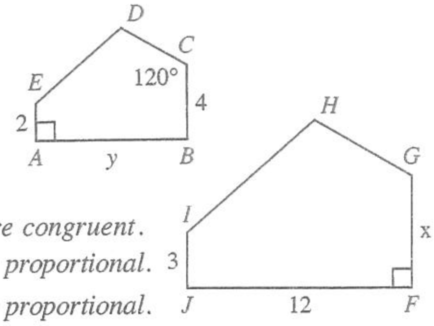
b. $\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{AD}{WZ}$

Conversely, if you know that all parts of *a* and *b* above are true, you can conclude that $ABCD \sim WXYZ$.

The ratio of lengths of corresponding sides of two similar polygons is called the **scale factor** between the similar polygons. Since $\frac{AB}{WX} = \frac{2}{4} = \frac{1}{2}$, the scale factor of $ABCD$ to $WXYZ$ is $\frac{1}{2}$. Since $\frac{WX}{AB} = \frac{4}{2} = 2$, the scale factor of $WXYZ$ to $ABCD$ is 2.

Example

ABCDE ~ *JFGHI* **Complete each statement.**



a. $m\angle J =$ ___ **b.** $m\angle G =$ ___ **c.** $\frac{ED}{IH} =$ ___

d. $x =$ ___ **e.** The scale factor of *JFGHI* to *ABCDE* is ___.

Solution

a. $m\angle J = 90$ **b.** $m\angle G = 120$ *Corresponding angles are congruent.*

c. $\frac{ED}{IH} = \frac{AE}{JI} = \frac{2}{3}$ *Corresponding sides are proportional.*

d. $\frac{AE}{JI} = \frac{BC}{FG}$ *Corresponding sides are proportional.*

$\frac{2}{3} = \frac{4}{x}$ *Substitute segment lengths and solve for x.*

$2x = 12$ or $x = 6$

e. The scale factor of *JFGHI* to *ABCDE* is $\frac{3}{2}$. $\frac{JI}{AE} = \frac{3}{2}$

Try This

In polygon *ABCDE*, find *y*.

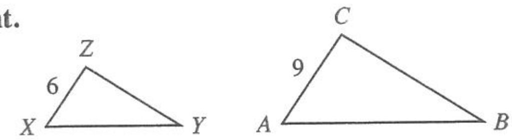
Class Exercises

✓ **Short Answer**

Complete each statement.

1. If two polygons are similar, then corresponding ___ are congruent.
2. If two polygons are similar, then corresponding ___ are proportional.
3. If for two polygons corresponding angles are ___ and corresponding sides are ___, then the polygons are similar.
4. If the scale factor between two similar triangles is one, then the triangles are ___.

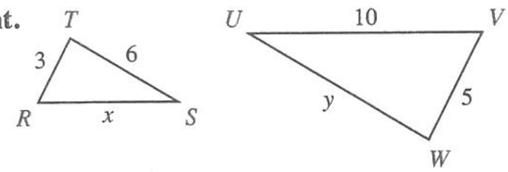
$\triangle ABC \sim \triangle XYZ$ **Complete each statement.**



5. $\angle A \cong$ ___ 6. $\angle C \cong$ ___
7. $\frac{AC}{XZ} =$ ___ 8. $\frac{XY}{AB} =$ ___

✓ **Sample Exercises**

$\triangle RST \sim \triangle VUW$ **Complete each statement.**



9. $\angle R \cong$ ___ 10. $\frac{RT}{VW} =$ ___
11. $x =$ ___ 12. $y =$ ___

✓ **Discussion**

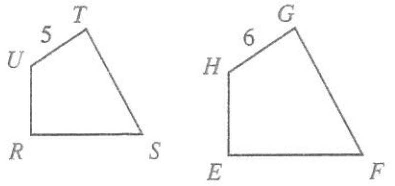
Determine whether each statement is always true, sometimes true, or never true. Explain your answer.

13. If two triangles are congruent, then they are similar.
14. If two triangles are similar, then they are congruent.
15. Squares *ABCD* and *EFGH* are similar.
16. Isosceles triangles *ABC* and *DEF* are similar.
17. If corresponding sides of two rectangles are proportional, then the rectangles are similar.

Exercises

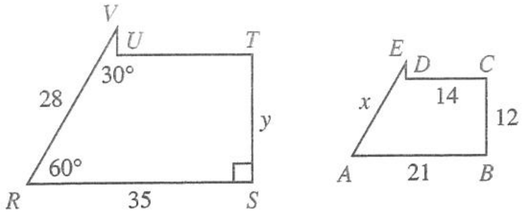
A

***RSTU* ~ *EFGH*   Complete each statement.**



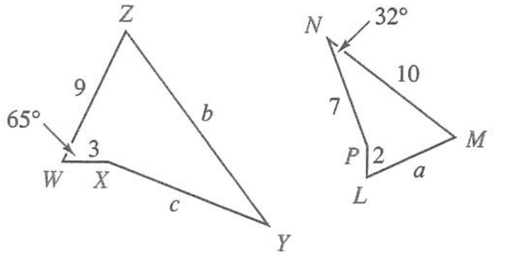
1. $\angle R \cong$ ___
2. $\angle S \cong$ ___
3. $\angle H \cong$ ___
4. $\angle G \cong$ ___
5. $\frac{HG}{UT} =$ ___
6. $\frac{RU}{EH} =$ ___
7. $\frac{ST}{FG} =$ ___
8. $\frac{RS}{EF} =$ ___

***ABCDE* ~ *RSTUV*   Complete each statement.**



9. $m\angle E =$ ___
10. $m\angle A =$ ___
11. $\angle T \cong$ ___
12. $m\angle B =$ ___
13. $x =$ ___
14. $y =$ ___
15. $UT =$ ___
16. $UV = 20$, $DE =$ ___
17. $\frac{RS}{AB} =$ ___
18. $\frac{UV}{DE} =$ ___
19. The scale factor of *ABCDE* to *RSTUV* is ___.

***WXYZ* ~ *LPNM*   Complete each statement.**

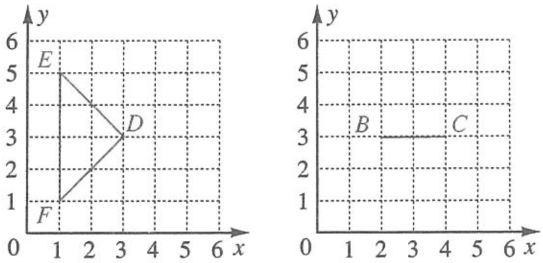


20. $m\angle L =$ ___
21. $m\angle Y =$ ___
22. $\angle M \cong$ ___
23. $\angle X \cong$ ___
24. $a =$ ___
25. $b =$ ___
26. $c =$ ___
27. If $WZ = 12$, $LM =$ ___.
28. The scale factor of *WXYZ* to *LPNM* is ___.

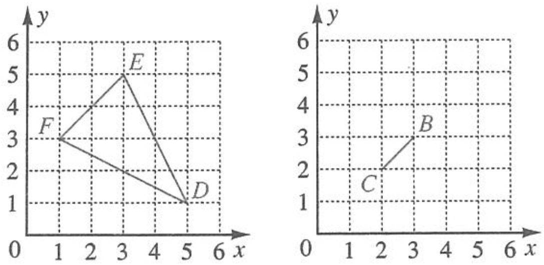
B

Find the coordinates of a point *A* so that $\triangle ABC \sim \triangle DEF$.

29.



30.



31. Plot points $A(1, 1)$, $B(7, 1)$, $C(6, 3)$, and $D(2, 3)$ and draw quadrilateral *ABCD*. Then plot points $A'(-3, 4)$ and $B'(-3, 1)$. Find points C' and D' so that $A'B'C'D'$ is similar to *ABCD*.

$\triangle ABC \sim \triangle DEF$ The scale factor of $\triangle ABC$ to $\triangle DEF$ is $\frac{3}{7}$. Complete each statement.

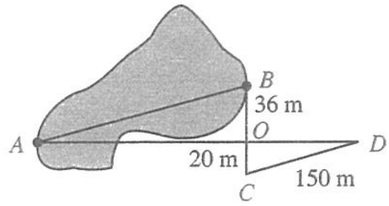
32. If $AB = 15$, then $DE = $ ___.

33. If $EF = 42$, then $BC = $ ___.

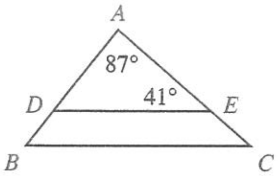
34. $\frac{AB}{DE} = $ ___

35. If $DF = 56$, then $AC = $ ___.

36. In order to find the distance AB across a lake, a surveyor constructed $\triangle OCD$ similar to $\triangle OBA$. He measured OB, OC, and CD directly to obtain the lengths shown. Find the length of AB.



$\triangle ABC \sim \triangle ADE$



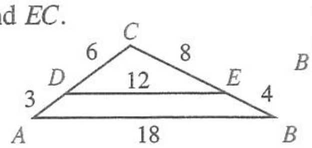
37. $m\angle A = 87$, $m\angle AED = 41$ Find $m\angle B$.

38. $AD = 8$, $AB = 12$, $AE = 14$ Find AC.

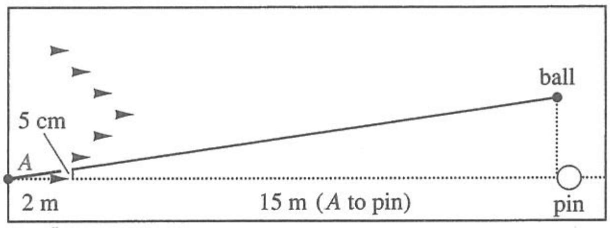
39. $AD = 9$, $AB = 13$, $AE = 14$ Find EC.

40. Prove that $\overline{DE} \parallel \overline{BC}$.

41. **Given:** $\overline{AB} \parallel \overline{DE}$
Prove: $\triangle ABC \sim \triangle DEC$

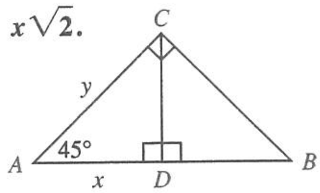


42. Keiko uses the sight marks to aim the ball. Suppose she misses the mark by 5 cm in a bowling alley that is 15 m long. If the mark is 2 m from the point A when the ball is released, by how much will she miss the pin?



C

$\triangle ACD$ and $\triangle ABC$ are right triangles and $y = x\sqrt{2}$. Complete each statement.



43. $CD = $ ___ **44.** $DB = $ ___

45. $BC = $ ___ **46.** $\frac{AB}{AC} = $ ___

47. Prove that $\triangle ACD \sim \triangle ABC$.

Determine whether each statement is always true, sometimes true, or never true.

48. An equilateral triangle and an equiangular triangle are similar.

49. If an angle of one rhombus is congruent to an angle of another rhombus, the two rhombi are similar.

50. Two isosceles trapezoids are similar.

51. Two parallelograms that each have a 132° angle are similar.

52. Two rectangles, each with the property that two sides are twice the length of the other two sides, are similar.

Critical Thinking

53. An arithmetic progression is a sequence of the form $x, x + a, x + 2a, x + 3a, \ldots$. Write the first five terms of each of these arithmetic progressions.
    a. $x = 3, a = 2$ b. $x = 4, a = 3$ c. $x = 2, a = 5$
54. Suppose the length of the sides of a triangle are the first three terms of an arithmetic progression. Show that the lengths of the sides of any similar triangle also form an arithmetic progression. (HINT: Assume that the lengths of the sides of one triangle are x, $x + a$, and $x + 2a$ and the lengths of the corresponding sides of the second triangle are y, $y + b$, and $y + c$ and show that $c = 2b$.)

Algebra Review

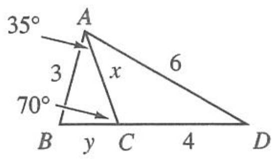
Write an equation and solve each problem.

1. In a triangle the second angle is 12° greater than the first angle. The third angle is 12° greater than the second angle. Find the measures of the three angles.
2. The measures of the angles of a triangle are in the ratio 4 : 5 : 6. Find the measure of each angle.
3. The perimeter of a triangle is 39. If the ratio of the lengths of the sides is 3 : 4 : 6, find the length of each side.
4. The vertex angle of an isosceles triangle has a measure 25 less than three times the measure of a base angle. Find the measures of the three angles.
5. The ratio of the measures of the angles of a quadrilateral is 2 : 4 : 7 : 11. Find the measure of each angle.

Quiz

Express each ratio in the simplest form.

1. $\frac{AB}{CD}$ 2. $m\angle ACB : m\angle BAC$
3. CD to AD 4. $BC + CD : AC$



5. The width and length of a rectangle are in the ratio 2 : 3. If the perimeter of the rectangle is 20, find the width and length.

Given the proportion $\frac{x}{2} = \frac{7}{y}$, indicate whether the following proportions are true or false.

6. $\frac{2}{x} = \frac{y}{7}$ 7. $\frac{y}{2} = \frac{x}{7}$ 8. $7 : x = y : 2$ 9. $\frac{x + 2}{2} = \frac{y + 7}{y}$

quadrilateral *ABCD* ~ quadrilateral *KLMN* **Complete each statement.**

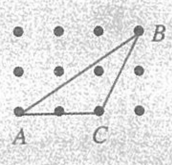
10. $\angle C \cong \angle$ ___
11. $AD = 20$, $BC = 32$, $LM = 24$ $KN =$ ___
12. The scale factor of $ABCD$ to $KLMN$ is ___.

OBJECTIVE: *Use the AA Similarity Postulate to draw conclusions about triangles.*

6-4 AA Similarity Postulate

EXPLORE

Trace angles A, B, and C in this triangle. Then draw a large triangle ABC on dot paper so that the corresponding angles of the two triangles are congruent. Use a protractor if necessary.



Compare the ratios of corresponding sides of the two triangles.

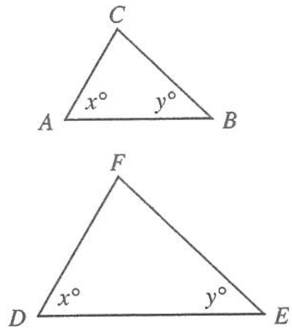
State a generalization.

In order to prove that two triangles are similar using the definition of similarity, you must verify six relationships. You must establish that all three pairs of corresponding angles are congruent and that the three ratios of corresponding sides are equal.

In the Explore you may have discovered that if all three pairs of corresponding angles are congruent, then the two triangles are similar. In fact, knowing that two pairs of angles are congruent is sufficient to conclude similarity. Consider $\triangle ABC$ and $\triangle DEF$ in which two pairs of angles are congruent. You can use the Triangle Angle Sum Theorem to conclude that

$m\angle C = 180 - (x + y)$ and $m\angle F = 180 - (x + y)$

Consequently, if two pairs of angles are congruent, all three pairs of angles are congruent.



$\angle A \cong \angle D$
$\angle B \cong \angle E$

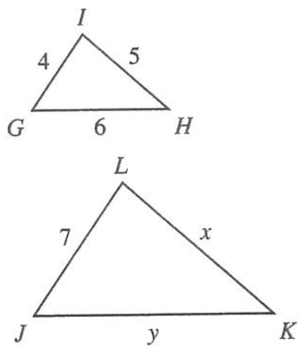
● **POSTULATE 19** AA Similarity

If two angles of one triangle are congruent respectively to two angles of another triangle, then the two triangles are similar.

Example 1

$\angle G \cong \angle J$, $\angle H \cong \angle K$

Find x and y.



Solution

$\triangle GHI \sim \triangle JKL$ — *AA Similarity Postulate*

$\frac{x}{5} = \frac{7}{4}$ or $x = \frac{35}{4}$ — *Ratios of corresponding sides are equal.*

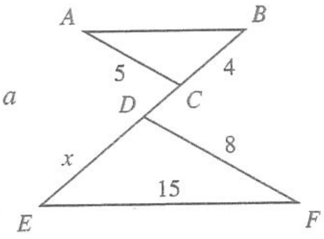
$\frac{y}{6} = \frac{7}{4}$ or $y = \frac{21}{2}$

Try This

In the figure to the right, $JL = 12$. Find JK and KL.

Example 2

$\angle A \cong \angle F$, $\overline{AB} \parallel \overline{EF}$, $BC = 4$, $AC = 5$, $DF = 8$ Find ED.



Solution

$\frac{BC}{ED} = \frac{AC}{FD}$

$\frac{4}{x} = \frac{5}{8}$

$5x = 32$

$x = \frac{32}{5}$

$\angle A \cong \angle F$ When parallel lines are cut by a transversal, alternate interior angles are congruent so $\angle B \cong \angle E$. Therefore $\triangle ABC \sim \triangle FED$ by the AA Postulate.

Try This

If $EF = 15$, find AB.

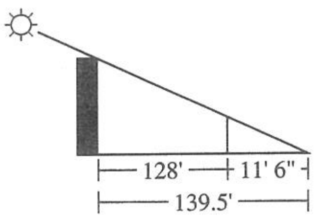
THEOREM 6.3 Right Triangle Similarity

If an acute angle of one right triangle is congruent to an acute angle of another right triangle, then the triangles are similar.

Example 3

Application A man 6 ft tall casts a shadow that is 11 ft 6 in. long. The end of his shadow coincides with the end of the shadow cast by a building 128 ft from the man. Find the height of the building.

Solution



Let x = height of the building.

$\frac{6}{11.5} = \frac{x}{139.5}$

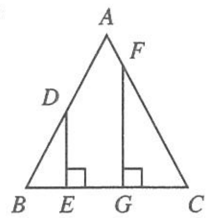
$x = \frac{139.5(6)}{11.5}$

$x \approx 72.78$ ft The building is about 73 ft high.

Try This

How tall would the building be if the man's shadow were 10 ft long?

Example 4



Given: $\triangle ABC$ is isosceles with base $\overline{BC}$. $\overline{DE} \perp \overline{BC}$, $\overline{FG} \perp \overline{BC}$

Prove: $\frac{DE}{FG} = \frac{BE}{CG}$

Proof

Statements	Reasons
1. $\triangle ABC$ is isosceles.	1. Given
2. $\angle B \cong \angle C$	2. Base $\angle$s of an isos. $\triangle$ are $\cong$.
3. $\overline{DE} \perp \overline{BC}$, $\overline{FG} \perp \overline{BC}$	3. Given
4. $\angle BED$ and $\angle CGF$ are rt. $\angle$s.	4. ?
5. $\triangle BED$ and $\triangle CGF$ are rt. $\triangle$s.	5. Definition of rt. $\triangle$
6. $\triangle DEB \sim \triangle FGC$	6. Rt. $\triangle$ Similarity Theorem
7. $\frac{DE}{FG} = \frac{BE}{CG}$	7. Ratios of corr. sides of $\sim\triangle$s are $=$.

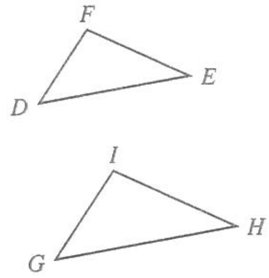
Solution

4. Definition of $\perp$ line segments

Class Exercises

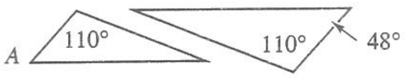
Short Answer

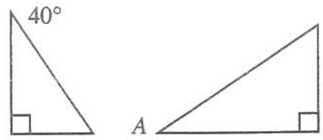
Which of these statements about $\triangle DEF$ and $\triangle GHI$ are true?

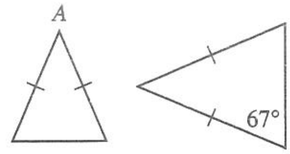


1. If $m\angle E = 32$, $m\angle H = 32$, $m\angle D = 68$, and $m\angle G = 68$, then $\triangle DEF \sim \triangle GHI$.
2. If $m\angle F = m\angle I = 90$, $m\angle D = 35$, and $m\angle H = 50$, then $\triangle DEF \sim \triangle GHI$.
3. If $m\angle F = m\angle I = 115$, $m\angle E = 30$, and $m\angle H = 40$, then $\triangle DEF \sim \triangle GHI$.

State the measure of $\angle A$ that would make the given triangles similar.

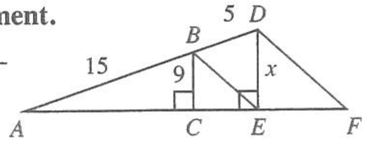
4. 

5. 

6. 

Sample Exercises

$\overline{BE} \parallel \overline{DF}$, $\overline{BC} \perp \overline{AC}$, $\overline{DE} \perp \overline{AE}$ **Complete each statement.**



7. $\triangle ABC \sim$ ___ 8. $\triangle ADF \sim$ ___ 9. $\frac{AE}{AC} =$ ___
10. Write a proportion that can be used to find x.
11. Find x.

Discussion

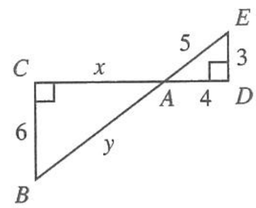
Determine whether each statement is always true, sometimes true, or never true. Draw pictures as needed and explain your answer.

12. Two right triangles are similar. 13. Two obtuse triangles are similar.
14. Two isosceles triangles with congruent vertex angles are similar.
15. Two isosceles right triangles are similar.

Exercises

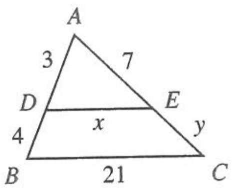
A

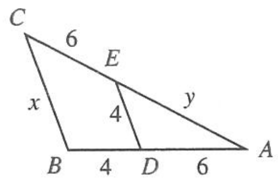
Complete each statement.

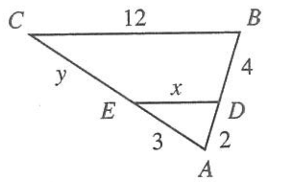


1. $\triangle ABC \sim$ ___ 2. The proportion ___ can be used to find x.
3. $x =$ ___ 4. The proportion ___ can be used to find y.
5. $y =$ ___

$\overline{BC} \parallel \overline{DE}$ **Find x and y.**

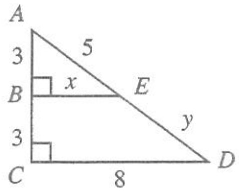
6. 

7. 

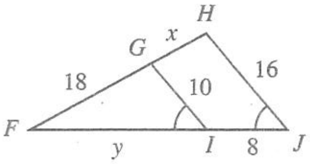
8. 

Identify a pair of similar triangles and find x and y.

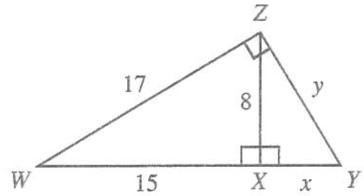
9.



10.

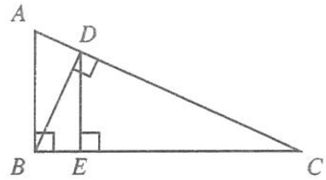


11.



Prove that each pair of triangles is similar.

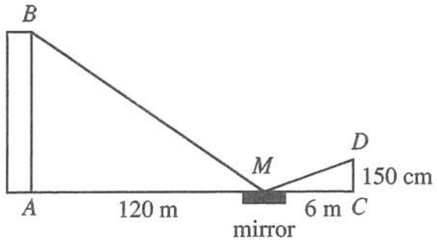
12. $\triangle ABC \sim \triangle DEC$ **13.** $\triangle ABC \sim \triangle BDC$ **14.** $\triangle BED \sim \triangle ADB$ **15.** $\triangle ADB \sim \triangle DEC$



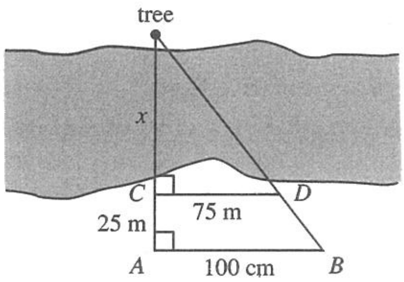
16. Prove Theorem 6.3.
Given: right triangles ABC and XYZ with $\angle A$ and $\angle X$ right angles, $\angle B \cong \angle Y$
Prove: $\triangle ABC \sim \triangle XYZ$

17. A person 5 ft 6 in. tall casts a shadow that is 14 ft long, the end of which lies at the end of the shadow cast by a building that is 328 ft away from the person. How tall is the building?

✓**18.** When a mirror is placed on the ground so that the top of a building can be seen beside a person standing by the mirror, $m\angle BMA = m\angle DMC$. A person 150 cm tall who is 6 m from the mirror observes the top of the tower when the mirror is 120 m from the tower. Find the height of the building.

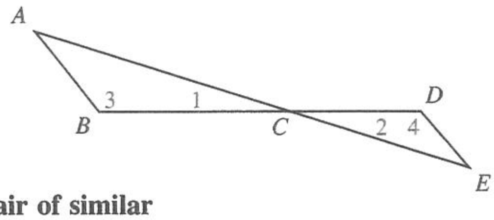


✓**19.** To find the distance x across a river, a surveyor located points A, B, C, and D through direct measurement. Find the distance x.



20. Given: $\angle 3 \cong \angle 4$
Prove: $\triangle ABC \sim \triangle EDC$

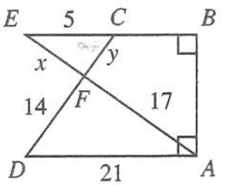
21. Given: $\overline{AB} \parallel \overline{ED}$
Prove: $\frac{AB}{ED} = \frac{BC}{DC}$



B

***ABCD* is a trapezoid with bases $\overline{AD}$ and $\overline{BC}$. Identify a pair of similar triangles and find x and y.**

22.



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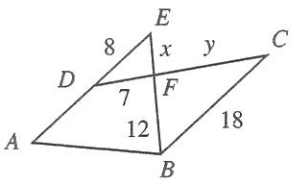

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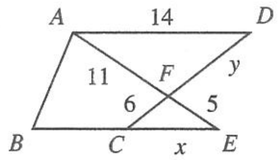
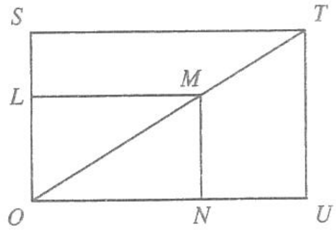
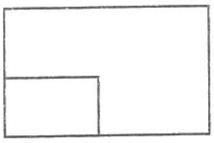


25. Rectangle $LMNO$ coincides with rectangle $STUO$ in such a way that $\overline{OL}$ lies on $\overline{OS}$ and $\overline{ON}$ lies on $\overline{OU}$. Show that when the diagonal from point O to point T passes through point M, the rectangles are similar.



Use a straightedge and the diagonal test to determine whether the rectangles are similar.

26.



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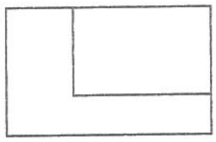

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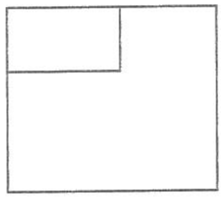
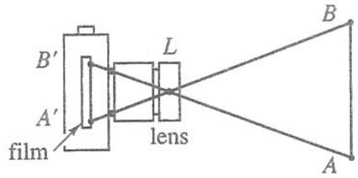
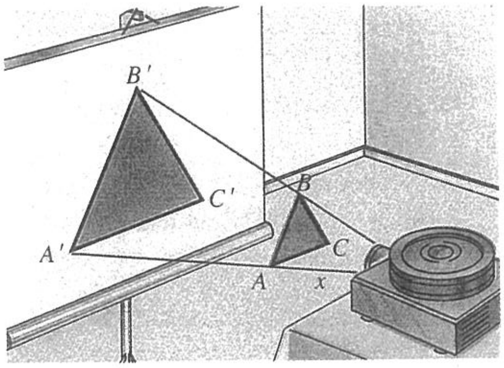


29. When a photograph is taken, the image formed on the film is similar to the object being photographed. Similar triangles help to explain this. If $\overline{AB}$ and $\overline{A'B'}$ are parallel, prove $\triangle LAB$ and $\triangle LB'A'$ are similar.

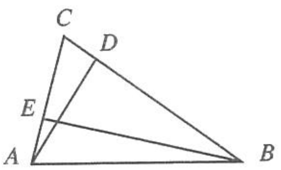


Suppose a slide projector and screen are set up as shown with the screen 20 ft from the projector. Assume ABC is similar to $A'B'C'$ and these planes are parallel.

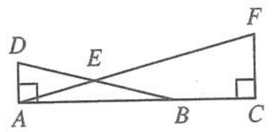


30. If the triangular cutout ABC is placed 2 ft in front of the projector, find the length of $A'B'$ in terms of AB.
31. If the triangular cutout ABC is placed x ft in front of the projector, calculate the length $A'B'$ in terms of the length AB and the distance x.
32. If x is halved, what happens to $A'B'$?
33. If x is doubled, what happens to $A'B'$?

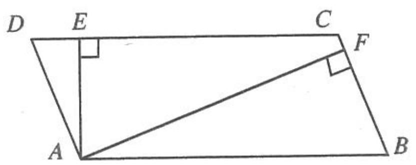
34. **Given:** $\overline{BE}$ and $\overline{AD}$ are altitudes.
Prove: $\frac{AD}{AC} = \frac{BE}{BC}$



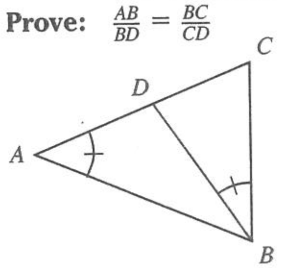
35. **Given:** $\triangle AEB$ is isosceles.
Prove: $AD \cdot AF = CF \cdot BD$



36. **Given:** $ABCD$ is a parallelogram. $\overline{AE} \perp \overline{CD}$, $\overline{AF} \perp \overline{BC}$
Prove: $AE \cdot CD = AF \cdot BC$

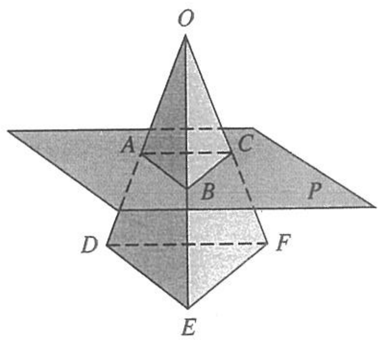


37. **Given:** $\overline{AB} \cong \overline{AC}$, $\angle A \cong \angle CBD$
Prove: $\frac{AB}{BD} = \frac{BC}{CD}$

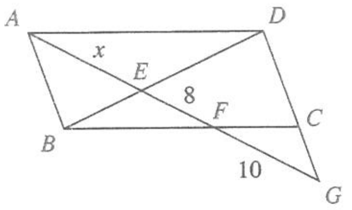


C

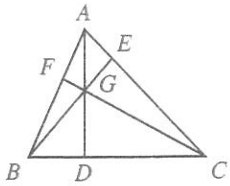
38. **Given:** Plane P is parallel to $\triangle DEF$.
Prove: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.



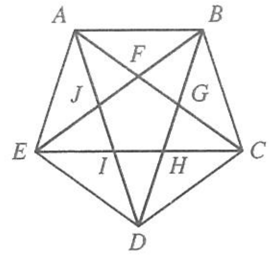
39. *ABCD* is a parallelogram. Find *x*. (HINT: Set up two proportions by using $\triangle AED$ and $\triangle AEB$ and triangles similar to them.)



40. **Given:** $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are altitudes.
Prove: $\frac{AF}{BF} \cdot \frac{BD}{CD} \cdot \frac{CE}{AE} = 1$
(HINT: Begin by writing three proportions using three pairs of similar right triangles.)



Critical Thinking



41. Trace pentagon *ABCDE* and all its diagonals as shown in the figure to the right. Assume that all sides of *ABCDE* are equal in length and all vertex angles are equal in measure. Label the angle measures of as many angles in your tracing as possible.

42. Show that $\triangle JAE \sim \triangle ABE$ and conclude that $\frac{EJ}{AE} = \frac{AE}{EB}$. Show that $AE = EF$ and write a proportion using *EJ*, *EF*, and *EB*.

43. Suppose that $AE = EF = 1$. Let $x = EJ$. Use the proportion from Exercise 42 to show that $\frac{x}{1} = \frac{1}{1 + x}$. Show that $x = \frac{-1 + \sqrt{5}}{2}$.

Mixed Review

✓ **Any two equilateral triangles are similar.**

1. Write the statement as a conditional.
2. Write the inverse of the statement.
3. Write the contrapositive of the statement

✓ **If two triangles are congruent, then they are similar.**

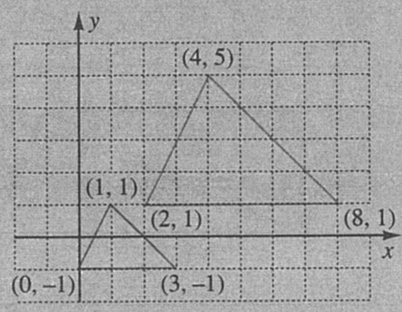
4. Write the converse of the statement.
5. Show that the converse is false.
6. If the inverse is false, give a counterexample.

CONNECTIONS

Transformations

Similarity as discussed in this lesson is related to concepts studied in algebra. Consider the transformation that associates each point (x, y) in the coordinate plane with the point $(2x + 2, 2y + 3)$—the red triangle with the blue triangle.

1. Calculate the lengths of the sides of each triangle to show that the three ratios of corresponding sides are proportional.
2. Draw on graph paper the triangle whose vertices are $(-2, -1)$, $(4, -1)$, and $(2, 3)$. Then draw the triangle associated with it by the transformation above.
3. Show that the ratios of corresponding sides are proportional.



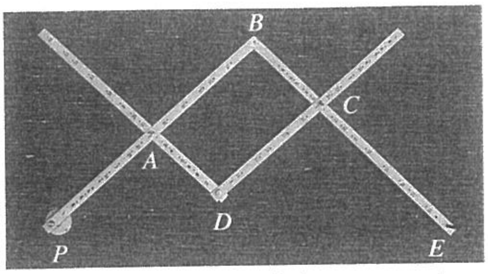
$(x, y) \rightarrow (2x + 2, 2y + 3)$

OBJECTIVE: *Use the SAS and SSS Similarity Theorems to draw conclusions about triangles.*

6-5 SAS and SSS Similarity Theorems

The AA Similarity Postulate provides what is probably the most often used method of showing that two triangles are similar, since it requires proving only two pairs of angles congruent.

However, there are other methods for proving triangles similar. The SAS Theorem below describes a comparison between two pairs of sides and one pair of included angles.



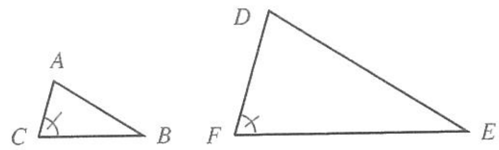
A mechanical linkage, called a pantograph, is used by draftsmen to enlarge or reduce drawings.

THEOREM 6.4 SAS Similarity Theorem

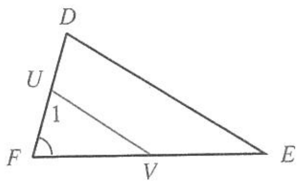
If an angle of one triangle is congruent to an angle of another triangle and if the lengths of the sides including these angles are proportional, then the triangles are similar.

Given: $\angle C \cong \angle F$, $\frac{AC}{DF} = \frac{BC}{EF}$

Prove: $\triangle ABC \sim \triangle DEF$



Plan Suppose that $CA < FD$. Choose point U on $\overline{FD}$ so that $UF = AC$ and point V on $\overline{FE}$ so that $\angle 1 \cong \angle FDE$. Then $\triangle UVF \sim \triangle DEF$ by AA Similarity. Use this similarity to conclude that $\frac{UF}{DF} = \frac{VF}{EF}$. It is given that $\frac{AC}{DF} = \frac{BC}{EF}$ from which it can be concluded that $VF = BC$ (recall, $AC = UF$). Conclude that $\triangle UFV \cong \triangle ACB$ by SAS and that $\triangle ACB \sim \triangle UFV \sim \triangle DEF$.



Example 1

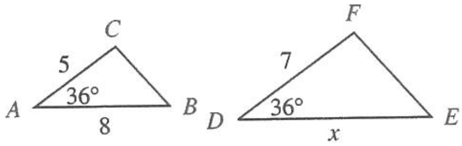
Find the value of x for which $\triangle ABC \sim \triangle DEF$. State the theorem that justifies your answer.

Solution

$\frac{5}{7} = \frac{8}{x}$

$5x = 56$

$x = \frac{56}{5}$



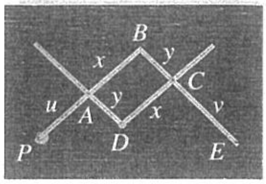
The triangles are similar by the SAS Similarity Theorem.

Try This

Given: $\triangle ABC$ and $\triangle XYZ$ If $\angle A \cong \angle X$, state the proportion that must be true to conclude that $\triangle ABC \sim \triangle XYZ$ by SAS Similarity.

Example 2

Application: In the pantograph, P, D, and E are collinear, $AD = BC$, $AB = CD$, and $\frac{AB}{AP} = \frac{CE}{BC}$. How does PD compare to DE?



Solution

$ABCD$ is a parallelogram.	*By construction opposite sides of ABCD are congruent.*
$\angle PAD \cong \angle B \cong \angle DCE$	*If parallel lines are cut by a transversal, corresponding angles are congruent.*
$\frac{x}{u} = \frac{v}{y}$	*This is true by the pantograph construction.*
$\triangle APD \sim \triangle CDE$	*SAS Similarity Theorem*
$\frac{PD}{DE} = \frac{u}{x}$ or $PD = \frac{u}{x} \cdot DE$	*Corresponding sides of similar triangles are proportional.*

Try This

In the pantograph shown, if $PA = 5$ cm and $AB = 10$ cm, find $\frac{DE}{PD}$.

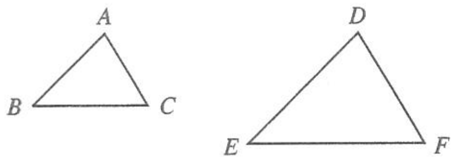
The next theorem describes a situation where three pairs of sides of two triangles allow you to conclude that the triangles are similar.

◆ **THEOREM 6.5** SSS Similarity Theorem

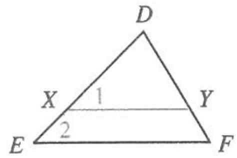
If corresponding sides of two triangles are proportional, then the two triangles are similar.

Given: $\frac{AC}{DF} = \frac{AB}{DE} = \frac{AB}{DE}$

Prove: $\triangle ABC \sim \triangle DEF$

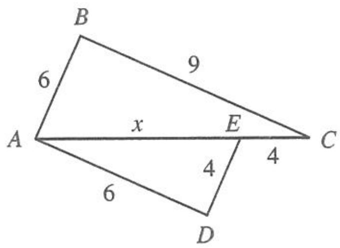


Plan Suppose that $AB < DE$. Choose point X on $\overline{DE}$ so that $DX = AB$ and choose point Y on $\overline{DF}$ so that $\overline{XY}$ is parallel to $\overline{EF}$. It follows that $\angle 1 \cong \angle 2$ and $\triangle DXY \sim \triangle DEF$ by AA Similarity. Consequently, $\frac{DY}{DF} = \frac{XY}{EF}$. Use this proportion together with the given proportion to conclude that $AC = DY$, $AB = DX$, and $XY = BC$. It follows that $\triangle ABC \cong \triangle DXY$. Therefore, $\triangle ABC \sim \triangle DXY \sim \triangle DEF$.



✓ **Example 3**

Find the value of x for which $\triangle ABC \sim \triangle EDA$.



Solution

$\frac{6}{4} = \frac{9}{6} = \frac{x+4}{x}$ *If* $\frac{AB}{ED} = \frac{BC}{DA} = \frac{AC}{EA}$, *then the triangles are similar.*

$\frac{3}{2} = \frac{x+4}{x}$

$3x = 2x + 8$

$x = 8$

Try This

If $EC = 3$ and the triangles are similar, show that the two triangles are isosceles.

Class Exercises

Short Answer

Determine whether the two triangles shown are similar. If so, state the postulate or theorem that justifies your answer.

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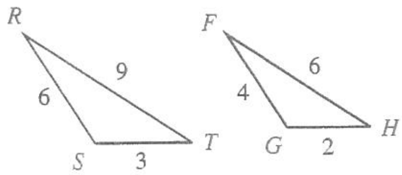

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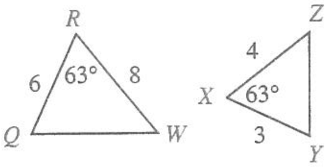

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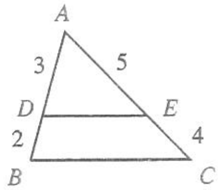

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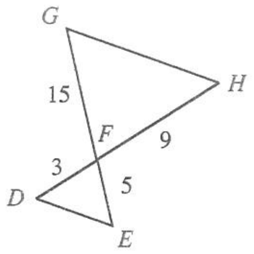

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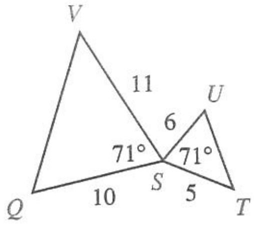

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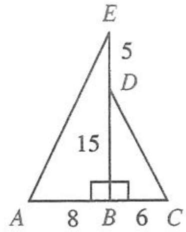

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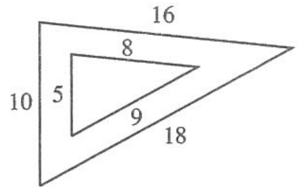

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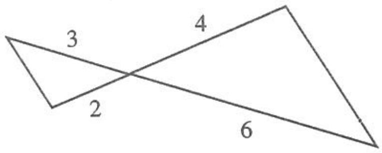

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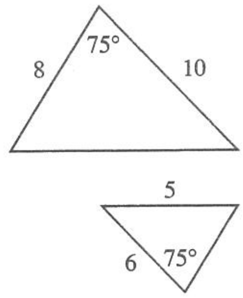
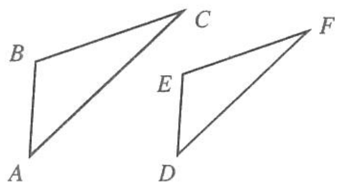


Sample Exercises

State whether you can conclude that $\triangle ABC \sim \triangle DEF$ from the given information.



10. $\angle A \cong \angle D$, $\frac{AC}{DF} = \frac{AB}{DE}$

11. $\frac{DF}{AC} = \frac{DE}{AB} = \frac{EF}{BC}$

12. $\angle B \cong \angle E$, $\frac{EF}{BC} = \frac{AC}{FD}$

13. $\angle C \cong \angle F$, $\frac{AC}{BC} = \frac{DF}{EF}$

14. Given: $\triangle ABC$ and $\triangle DEF$, $\angle B \cong \angle E$, $AB = 6$, $DE = 2$, $BC = 4$
Find the length of $\overline{EF}$ for which $\triangle ABC \sim \triangle DEF$.

15. Given: $\triangle RST$ and $\triangle UVW$, $RS = 6$, $UV = 8$, $ST = 9$, $RT = 12$
Find lengths of $\overline{VW}$ and $\overline{UW}$ for which $\triangle RST \sim \triangle UVW$.

16. Given: $\triangle ABC$ and $\triangle DEF$, $DE = 6$, $EF = 3$, $DF = 9$, $AB = 4$, $BC = 2$
Find the length of $\overline{AC}$ for which $\triangle ABC \sim \triangle DEF$.

Discussion

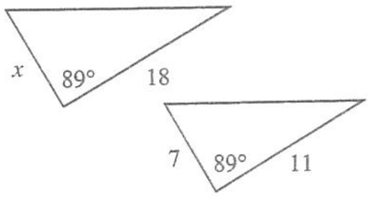
17. Given: quadrilaterals $ABCD$ and $WXYZ$, $\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{AD}{WZ}$
Are the quadrilaterals similar? If so, give a convincing argument to support your answer. If not, give a counterexample.

Exercises

A

Find a value of x for which the triangles are similar. State the theorem that justifies your answer.

1.



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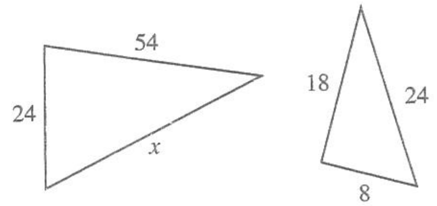

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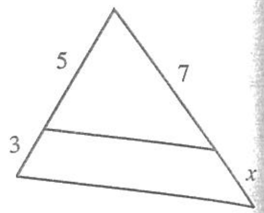

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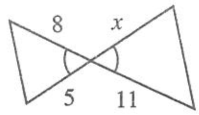

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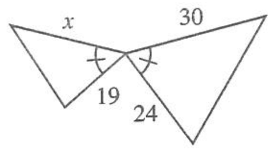

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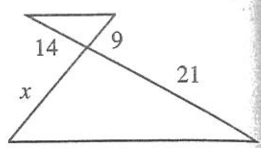

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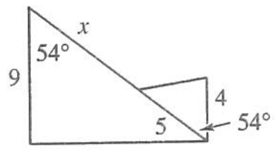

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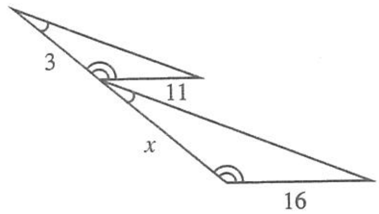
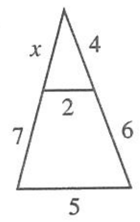
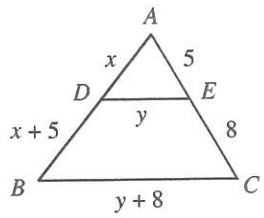


9.

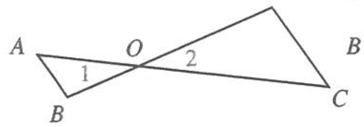


Draw and label figures for each exercise.

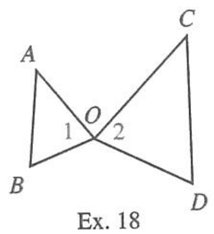
10. Given: $\triangle ABC$ and $\triangle DEF$ If $\angle B \cong \angle E$, state the proportion that must be true if $\triangle ABC \sim \triangle DEF$ by SAS Similarity.
11. Given: $\triangle UAZ$ and $\triangle RBN$ If $\angle U \cong \angle R$, state the proportion that must be true if $\triangle UAZ \sim \triangle RBN$ by SAS Similarity.
12. Given: $\triangle GHI$ and $\triangle KLM$ If $\angle I \cong \angle M$, state the proportion that must be true if $\triangle GHI \sim \triangle KLM$ by SAS Similarity.
13. Given: $\triangle XYZ$ and $\triangle UVW$ State the proportions that must be true if $\triangle XYZ \sim \triangle UVW$ by SSS Similarity.



14. $\triangle ADE \sim \triangle ABC$ Find AD and BD.
15. $y = 7$ Is $\triangle ADE \sim \triangle ABC$?
16. Find the value of y that makes $\overline{DE} \parallel \overline{BC}$.
17. **Given:** $\frac{AO}{CO} = \frac{BO}{DO}$
    Prove: $\angle B \cong \angle D$



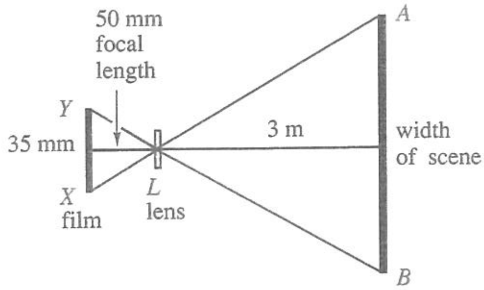
Ex. 17



Ex. 18

18. **Given:** $\frac{AB}{CD} = \frac{AO}{OC} = \frac{BO}{OD}$
    Prove: $\angle 1 \cong \angle 2$
19. In the pantograph in Example 2, the ratio $\frac{PE}{PD}$ is called the magnification factor. If $PA = 5$ in. and $AB = 7$ in., what is the magnification factor of the pantograph?

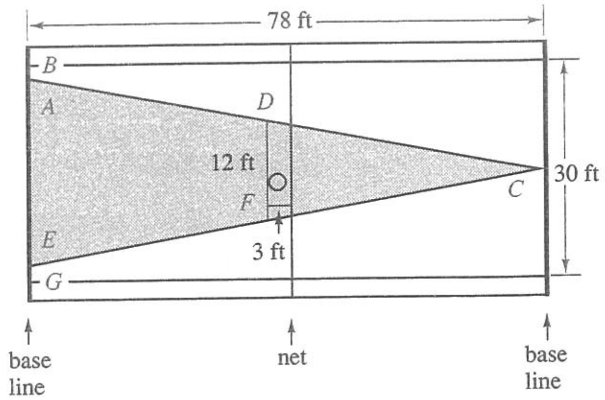
The figure describes a camera with film width *XY* that is 35 mm and with focal length 50 mm. The scene has width *AB*.



20. What assumptions do you make about this figure that allow the conclusion $\triangle ALB \sim \triangle XLY$?
21. What is the width of the scene *AB*?
22. If the lens of the camera has a focal length of 100 mm, what is the width of the scene?

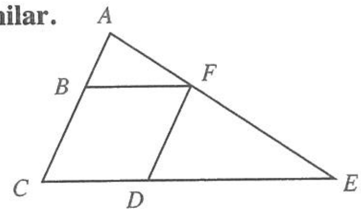
B

Use this figure for Exercises 23–25.



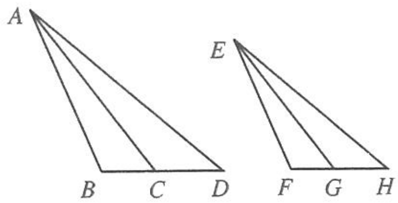
23. Suppose that a tennis player standing 3 ft from the net at the center of the court can return all balls hit within 6 ft on each side of him. Then his opponent standing at the center of the base line would have to hit the ball outside the shaded region in order to hit a "winner." What assumptions do you make in order to ensure that $\triangle ACE \sim \triangle DCF$?
24. What is the distance *AB* from the corner of the shaded triangle to the corner of the singles court?
25. If the distance from the player to the net is 6 ft, will the distance *AB* increase or decrease? by how much?

Use the figure and the table to identify which triangles, if any, are similar.

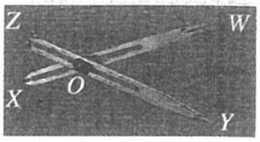


	AB	*BC*	*CD*	*DE*	*FE*	*AF*	*BF*	*FD*
26.	1	1	1	1	1	1	1	1
27.	3	6	6	3	6	3	3	5.2
28.	1	2	2	4	4	2	2	2
29.	8	16	24	3	20	4	8	18
30.	1	2	2	1	1	2	1.7	1
31.	4	10	10	2	4	8	6.5	4

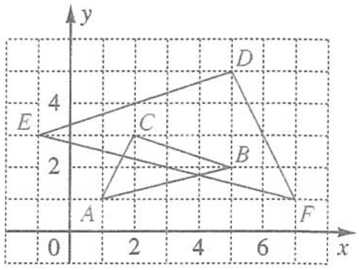
32. **Given:** $\triangle ABD \sim \triangle EFH$
C is the midpoint of $\overline{BD}$.
G is the midpoint of $\overline{FH}$.
Prove: $\triangle ABC \sim \triangle EFG$



33. Proportional dividers are used to construct segments of proportional length. Two pieces of the same length are connected with a set screw. Suppose the set screw O is set so that $OW = OY = 3OX = 3OZ$. How does the length WY compare to the length XZ? Explain.



34. Find the lengths of the sides of these triangles.
35. Use the SSS Similarity Theorem to show that $\angle DFE \sim \angle CAB$.
36. Name the coordinates of points X, Y, Z so that $\triangle XYZ \sim \triangle ABC$.



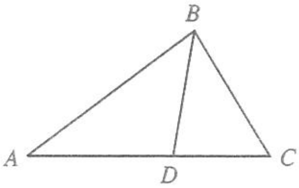
Ex. 34–36

37. **Given:** $\angle A \cong \angle CBD$

Prove: $AC \cdot DC = BC \cdot BC$

38. **Given:** $AC \cdot DC = BC \cdot BC$

Prove: $\angle A \cong \angle CBD$

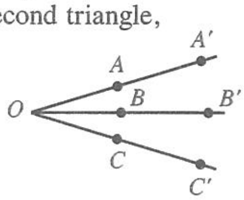


C

39. Give a counterexample to show that this statement is false. If two sides of one triangle are proportional to two sides of another triangle and an angle of the first triangle is congruent to an angle of the second triangle, the triangles are similar.

40. **Given:** point O on lines $\overline{AA}$, $\overline{BB}$, $\overline{CC}$, $\frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC}$

Prove: $\overline{AB} \parallel \overline{A'B'}$, $\overline{BC} \parallel \overline{B'C'}$, $\overline{AC} \parallel \overline{A'C'}$



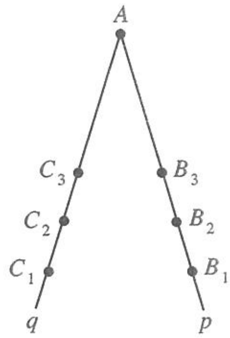
Critical Thinking

41. Suppose that lines p and q intersect at A and points $B_1, B_2, B_3, \ldots$ are on p and $C_1, C_2, C_3, \ldots$ are on q so that the following pattern of equations is true. State the next two equations in each of these patterns.

a. $AC_1 \cdot AC_2 = (AB_1)^2$
$AC_2 \cdot AC_3 = (AB_2)^2$

b. $AB_1 \cdot AB_2 = (AC_2)^2$
$AB_2 \cdot AB_3 = (AC_3)^2$

42. Use the equations in Exercise 41 to show that $\triangle AC_1B_1 \sim \triangle AB_1C_2 \sim \triangle AC_2B_2 \sim \triangle AB_2C_3 \sim \triangle AC_3B_3$. (HINT: Rewrite each equation as a proportion.)



Algebra Review

Solve.

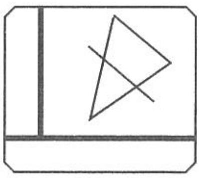
1. $n^2 - 9 = 0$ **2.** $n^2 - 6 = 0$ **3.** $n^2 - 4 = 0$ **4.** $n^2 - 25 = 0$

5. $2n^2 - 8 = 0$ **6.** $(3n)^2 = 81$ **7.** $3n^2 + 9 = 2n^2 + 25$ **8.** $n^2 + 6n = 18$

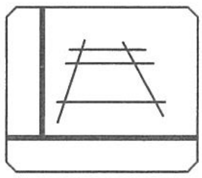
Computer Activity

Computer Construction

Use a computer software program to complete the following constructions. Then measure the ratios of the segment lengths. Suggest theorems that you think could be proven.



a line parallel to a side of a triangle



three parallel lines cut by transversals