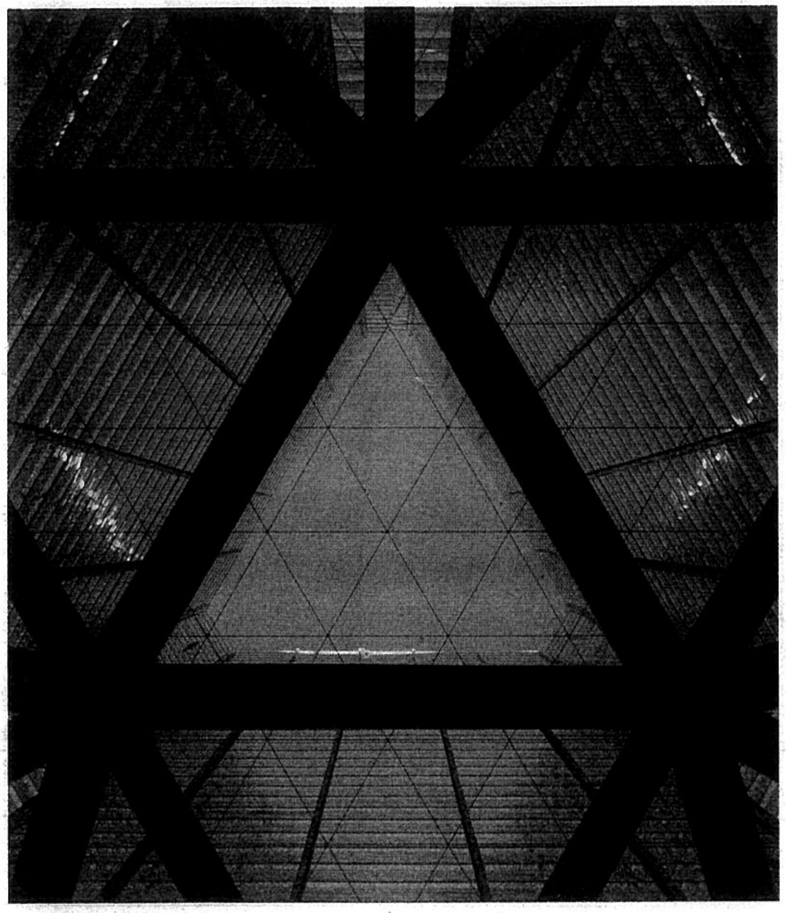
4

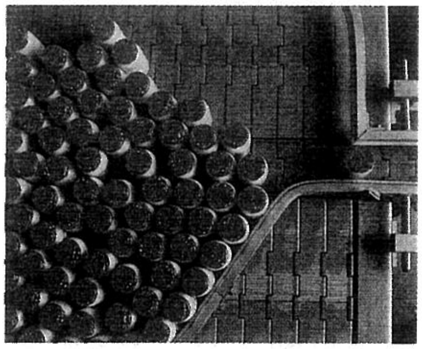
Congruent Triangles



Give a convincing argument that the equilateral triangles of this structure are congruent.

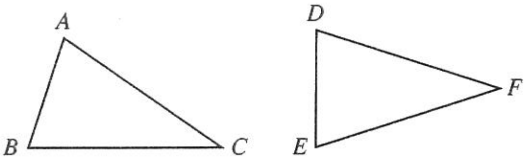
OBJECTIVE: *Given two congruent triangles, identify congruent parts.*

4-1 Congruent Triangles


Individual parts must be made identical in size and shape so that they fit any one of the items on the assembly line.

You learned in Chapter 1 that congruent segments have the same length and that congruent angles have the same measure. Use these ideas to define congruent triangles. Two figures are called congruent if they have the same size and shape.

If you were to trace triangle ABC shown below and position the tracing over triangle DEF, you would see that triangles ABC and DEF are identical in size and shape.



When the tracing of triangle ABC matches up with triangle DEF, the vertices of the two triangles match up as follows:

$A \leftrightarrow D \qquad B \leftrightarrow E \qquad C \leftrightarrow F$

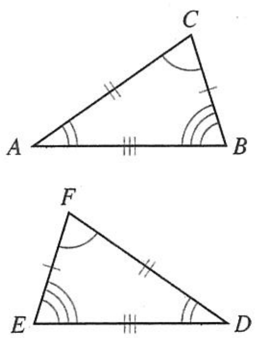
You can also see a **correspondence** between the angles and the sides of the two triangles.

Corresponding angles: $\angle A \leftrightarrow \angle D$, $\angle B \leftrightarrow \angle E$, $\angle C \leftrightarrow \angle F$
Corresponding sides: $\overline{AB} \leftrightarrow \overline{DE}$, $\overline{BC} \leftrightarrow \overline{EF}$, $\overline{AC} \leftrightarrow \overline{DF}$

DEFINITION

Two triangles are congruent if and only if there is a correspondence between the vertices such that each pair of corresponding sides and each pair of corresponding angles are congruent.

You write $\triangle ABC \cong \triangle DEF$.



In a diagram congruent sides can be indicated with "hatch" marks and congruent angles by "arc" marks as shown in this figure. When using the notation $\triangle ABC \cong \triangle DEF$, list corresponding vertices in the same order. In this pair of congruent triangles, the corresponding vertices are A and D, B and E, and C and F. These six statements are correct as written.

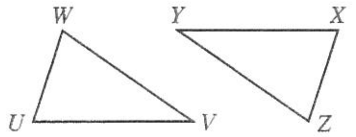
$\triangle ABC \cong \triangle DEF \qquad \triangle ACB \cong \triangle DFE \qquad \triangle BCA \cong \triangle EFD$
$\triangle BAC \cong \triangle EDF \qquad \triangle CAB \cong \triangle FDE \qquad \triangle CBA \cong \triangle FED$

However, it would not be correct to write $\triangle ABC \cong \triangle EDF$ since vertices A and E, and B and D are not corresponding vertices.

Example 1

$\triangle UVW \cong \triangle XYZ$ **Complete each statement.**

a. $\overline{VW} \cong$ ___ b. $\angle V \cong$ ___



Solution

a. $\overline{VW} \cong \overline{YZ}$ b. $\angle V \cong \angle Y$

Try This

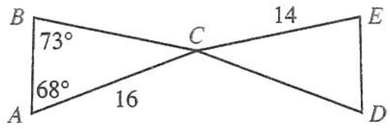
Complete each statement.

a. $\overline{YX} \cong$ ___ b. $\angle Z \cong$ ___

Example 2

$\triangle ABC \cong \triangle DEC$ **Complete each statement.**

a. $m\angle D =$ ___ b. $CD =$ ___



Solution

a. $m\angle D = 68$ b. $CD = 16$

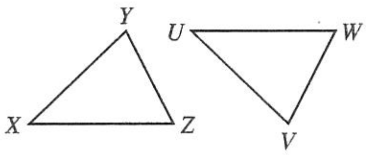
Try This

a. $m\angle E =$ ___ b. $BC =$ ___

Class Exercises

Short Answer

$\triangle XYZ \cong \triangle UVW$



1. Name the three pairs of corresponding angles.
2. Name the three pairs of corresponding sides.

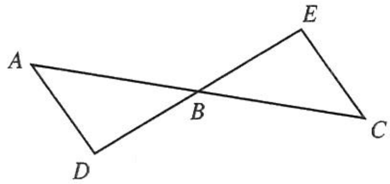
Determine whether each statement is true or false for the above figure.

3. $\triangle YZX \cong \triangle VWU$ 4. $\triangle XZY \cong \triangle UVW$ 5. $\overline{XY} \cong \overline{VY}$ 6. $\angle Y \cong \angle V$

7. Which statement is correct?
   $\triangle YZX \cong \triangle UVW$, $\triangle YZX \cong \triangle VWU$, $\triangle ZXY \cong \triangle WVU$
8. Which statement is not correct? $\overline{XZ} \cong \overline{UW}$, $\angle Y \cong \angle V$, $\overline{XZ} \cong \overline{UV}$

Sample Exercises

$\triangle ABD \cong \triangle CBE$ **Complete each statement.**



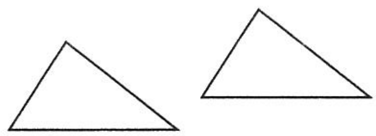
9. $\overline{AD} \cong$ ___ 10. $\overline{BE} \cong$ ___ 11. $\angle ABD \cong$ ___
12. $\overline{BC} \cong$ ___ 13. $\triangle BDA \cong$ ___ 14. $\triangle CEB \cong$ ___

$\triangle MOP \cong \triangle HAT$ **Complete each statement.**

15. $\angle M \cong$ ___ 16. $\overline{OP} \cong$ ___ 17. $m\angle P =$ ___
18. $\overline{MP} \cong$ ___ 19. $\angle A \cong$ ___ 20. $\overline{HA} \cong$ ___

Discussion

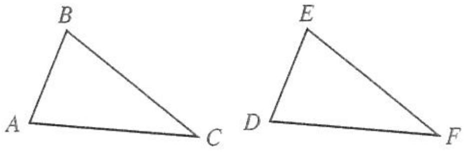
21. Given that the two triangles to the right are congruent, explain how the figure could be labeled and what notation should be used to show that these triangles are congruent.



Exercises

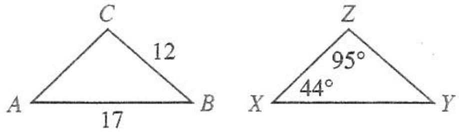
A

$\triangle ABC \cong \triangle DEF$ **Complete each statement.**



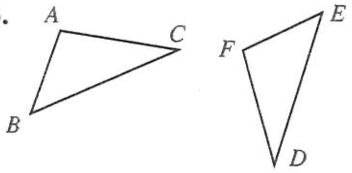
1. $\overline{AB} \cong$ ___ 2. $\overline{EF} \cong$ ___
3. $\angle B \cong$ ___ 4. $\angle D \cong$ ___
5. $\angle F \cong$ ___ 6. $\overline{AC} \cong$ ___
7. $\triangle FED \cong$ ___ 8. $\triangle BCA \cong$ ___

$\triangle ABC \cong \triangle XYZ$ **Complete each statement.**

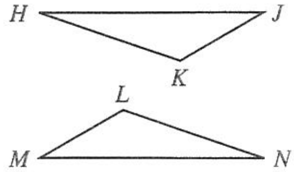


9. $AB =$ ___ 10. $m\angle A =$ ___
11. $m\angle C =$ ___ 12. $YZ =$ ___

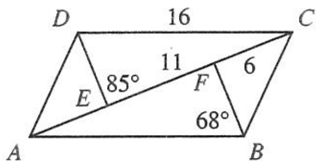
The two triangles shown are congruent. Name three pairs of angles and three pairs of sides that appear to be congruent for each pair of triangles.

13. 

14. 

15. 

$\triangle ABF \cong \triangle CDE$, $EF = 11$, $FC = 6$ **Complete each statement.**



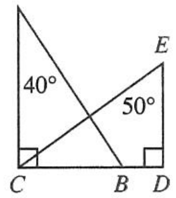
16. $AB =$ ___ 17. $AF =$ ___
18. $m\angle AFB =$ ___ 19. $m\angle CDE =$ ___
20. $m\angle CFB =$ ___ 21. $m\angle FAB =$ ___

$\triangle JIP \cong \triangle MAT$ **Complete each statement.**

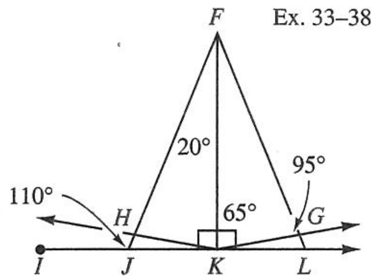
22. $\angle J \cong$ ___ 23. $\angle P \cong$ ___ 24. $\angle I \cong$ ___
25. $\overline{IP} \cong$ ___ 26. $\overline{JP} \cong$ ___ 27. $\overline{JI} \cong$ ___

28. Draw $\triangle ABC$. Use a compass and a straightedge to construct a $\triangle XYZ$, such that $\triangle XYZ \cong \triangle ABC$. Use hatch marks and arc marks to label all congruent parts of these figures.

Ex. 29–32



Ex. 33–38



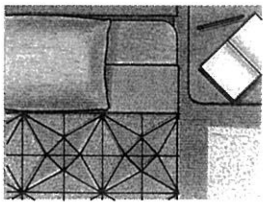
B

$\triangle ABC \cong \triangle CED$ **Complete each statement.**

29. $m\angle ABC =$ ___ 30. $m\angle ECD =$ ___
31. $m\angle ABD =$ ___ 32. $m\angle ACE =$ ___

$\triangle FKH \cong \triangle FKG$ **Complete each statement.**

33. $m\angle KFL =$ ___ 36. $m\angle FHK =$ ___ 37. $m\angle FJI =$ ___
34. $m\angle FKH =$ ___ 35. $m\angle HKJ =$ ___ 38. $m\angle FJK =$ ___



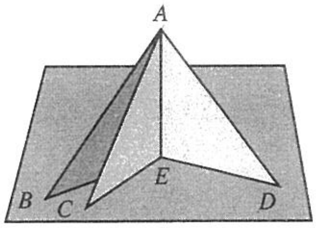
39. This quilt pattern is made by sewing together triangular shapes. How many different (non-congruent) shapes are used? (Consider the shapes of the pieces only, not the fabric design in the shapes.)

40. Explain why these two triangles , though congruent, must be treated as different if you are cutting out the pieces for this quilt.

Plot each point on a rectangular coordinate system. Choose the unit of length large enough that the resulting triangles are easy to measure. Measure the lengths and angles of these triangles. Is $\triangle ABC$ congruent to $\triangle DEF$?

41. $A(-4, 1)$, $B(-4, 3)$, $C(-1, 3)$, $D(1, 1)$, $E(1, 3)$, $F(4, 3)$
42. $A(-1, -2)$, $B(-3, 1)$, $C(-2, 5)$, $D(1, -2)$, $E(3, 1)$, $F(2, 5)$
43. $A(-3, 1)$, $B(-1, -1)$, $C(-4, 4)$, $D(1, 2)$, $E(1, 4)$, $F(3, 3)$

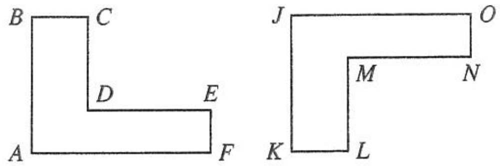
C



44. Explain why, if one of two congruent triangles is isosceles, then the other triangle is also isosceles.

The three shaded triangles are congruent.

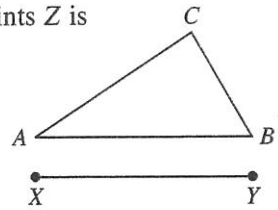
45. List a collection of three sides that are congruent to one another.
46. List a collection of three angles that are congruent to one another.



These figures are congruent, and the congruence symbol in the statement $ABCDEF \cong JKLMNO$ is interpreted just as it is for triangles. Complete each statement.

47. $\angle C \cong$ ___ **48.** $\angle M \cong$ ___
49. $\overline{AB} \cong$ ___ **50.** $\overline{ON} \cong$ ___

Critical Thinking

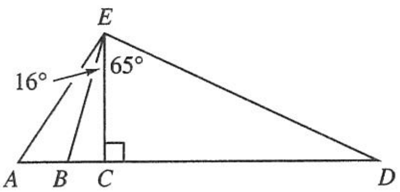


51. In the figure at the right, $XY = AB$. For how many different points Z is it true that

a. triangles ABC and XYZ are congruent?
b. $\triangle ABC \cong \triangle XYZ$?

Explain why the answers to these two questions are different.

Mixed Review



$\overrightarrow{EB}$ **bisects** $\angle AEC$. **Complete each statement.**

1. $m\angle AEB =$ ___ **2.** $m\angle BED =$ ___
3. $m\angle AED =$ ___

4. Name six triangles.
5. What type of triangle is $\triangle AED$?

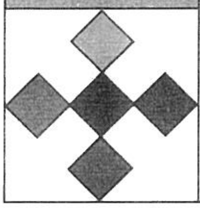
Study Skills

1. Find a word in this lesson that means "same size and shape."
2. Suppose $\triangle ABC \cong \triangle DEF$. Explain why it is wrong to write $\triangle ABC \cong \triangle FED$.
3. Suppose triangle ABC is congruent to triangle DEF with angles A and F congruent, angles B and D congruent, and angles C and E congruent. Describe the congruence between these triangles using the appropriate symbols.

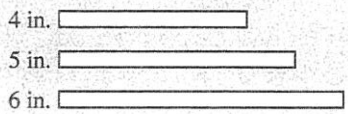
OBJECTIVE: *State and apply the SSS, ASA, and SAS Postulates.*

4-2 Congruence Postulates

EXPLORE



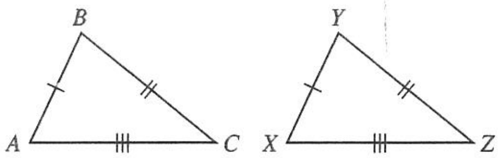
Cut a 4-in., a 5-in., and a 6-in. strip of paper and arrange them to form a triangle. Repeat this experiment. How do the two triangles compare in size and shape? State any generalizations that you discover.



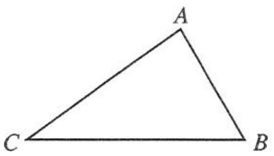
You learned in the last section that when two triangles are congruent, there are three pairs of congruent sides and three pairs of congruent angles. In the Explore above, you may have discovered that you do not need to show all six of these congruence relationships true to conclude that two triangles are congruent as state in the following postulate.

POSTULATE 16 SSS Postulate

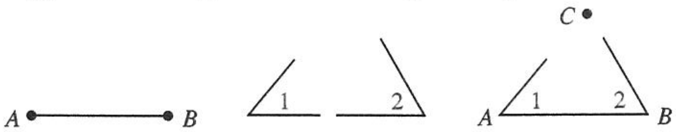
If each of the three sides of one triangle are congruent respectively to corresponding sides of another triangle, then the two triangles are congruent.



In $\triangle ABC$, $\angle B$ is *opposite* $\overline{AC}$ and $\overline{AB}$ is *opposite* $\angle C$. $\angle C$ is *included* between $\overline{AC}$ and $\overline{BC}$, and $\overline{BC}$ is *included* between $\angle C$ and $\angle B$.



Suppose that a segment and two angles are given as shown here.

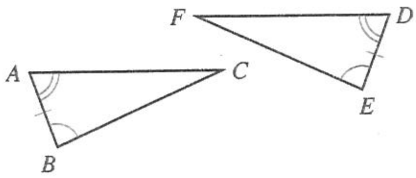


If an angle congruent to angle 1 is constructed at point A and an angle congruent to angle 2 is constructed at point B, then the sides of these angles can be extended to intersect at a point C, forming one and only one triangle.

This reasoning can be used as a convincing argument to describe how you might reach the conclusion given in Postulate 17.

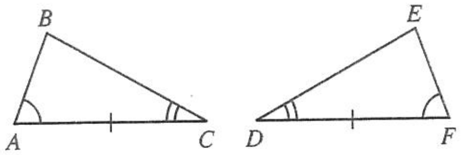
● **POSTULATE 17** ASA Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.



Example 1

Write a congruence between these triangles and state the postulate by which the triangles are congruent.



Solution

$\triangle ABC \cong \triangle FED$ *ASA Postulate*

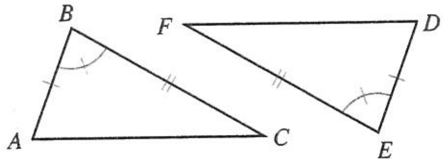
Try This

$\angle V \cong \angle Y$, $\angle W \cong \angle Z$, $\overline{VW} \cong \overline{YZ}$

Which postulate would you use to show that $\triangle UVW \cong \triangle XYZ$?

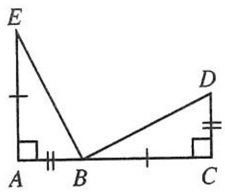
● **POSTULATE 18** SAS Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.



Example 2

Apply the SAS Postulate to show that a pair of triangles are congruent.



Solution

$\triangle BAE \cong \triangle DCB$ *SAS Postulate*

Try This

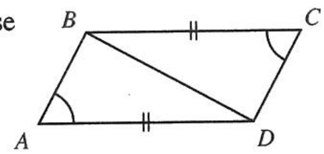
$\angle V \cong \angle Y$, $\overline{UV} \cong \overline{XY}$, $\overline{VW} \cong \overline{YZ}$

Which postulate would you use to show that $\triangle UVW \cong \triangle XYZ$?

Example 3

Name the additional congruent sides or angles needed to show these triangles are congruent by the given postulate.

a. SAS **b.** ASA



Solution

a. $\overline{AB} \cong \overline{CD}$ **b.** $\angle ADB \cong \angle CBD$

Try This

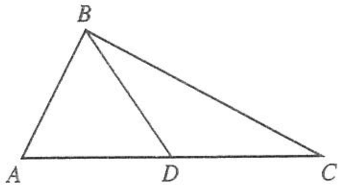
$\overline{UV} \cong \overline{XY}$, $\overline{VW} \cong \overline{YZ}$, $\overline{UW} \cong \overline{XZ}$

Which postulate would you use to show $\triangle UVW \cong \triangle XYZ$?

Class Exercises

Short Answer

State the angle or side opposite the given side or angle.



1. $\overline{AD}$ 2. $\angle A$ 3. $\overline{AC}$ 4. $\angle BDC$

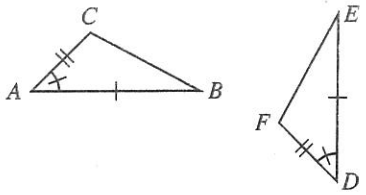
For each pair of sides or angles state the included angle or side.

5. $\overline{AB}, \overline{AD}$ 6. $\angle A, \angle C$ 7. $\angle ABD, \angle ADB$ 8. $\overline{AD}, \overline{BD}$

Sample Exercises

Write a congruence between each pair of triangles and state the postulate applied.

9.



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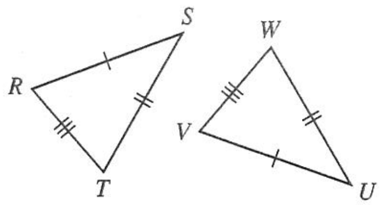
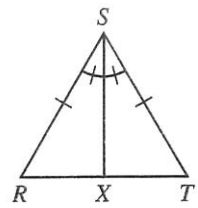


11.



Discussion

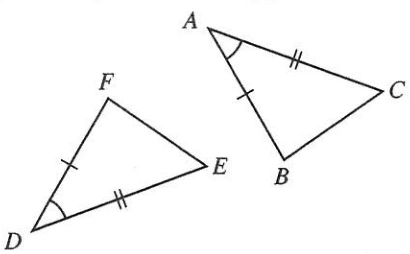
12. Give a convincing argument to describe how you might reach the conclusion in Postulate 17.
13. Give a convincing argument to describe how you might reach the conclusion in Postulate 18.
14. Give a convincing argument in favor of or against an AAA method of proving two triangles congruent: If three angles of one triangle are congruent to three angles of another triangle, the triangles are congruent.

Exercises

A

Write a congruence between each pair of triangles and state the postulate applied. If you cannot apply a postulate, write *no conclusion can be made.*

1.



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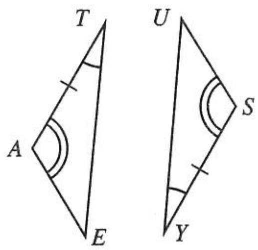

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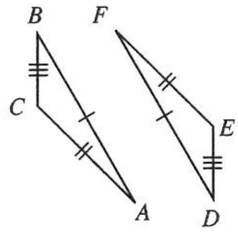

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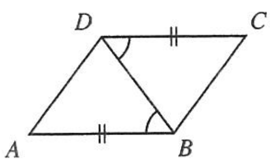

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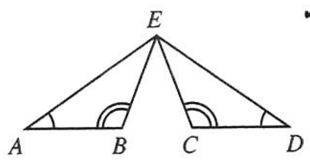

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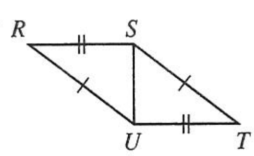
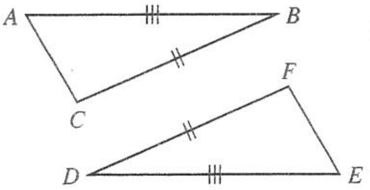


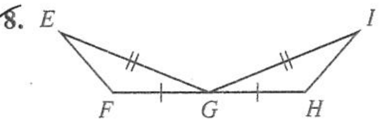
Name the additional congruent sides or angles needed to show that the pair of triangles are congruent by the given postulate.

7.



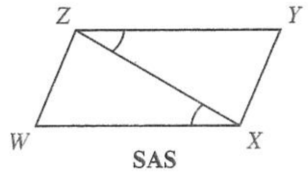
SSS

8.



SAS

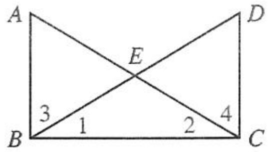
9.



SAS

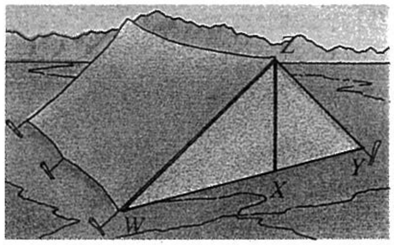
Complete each statement. Give the angle or side that allows you to show that each triangle is congruent to another triangle by the given postulate.

10. SAS for $\triangle ABC$: $\overline{AC}$, ___, $\overline{BC}$
11. ASA for $\triangle BCE$: $\angle 1$, $\overline{BC}$, ___
12. SSS for $\triangle CDE$: ___, $\overline{EC}$, $\overline{CD}$
13. SAS for $\triangle BCD$: $\overline{BC}$, ___, ___



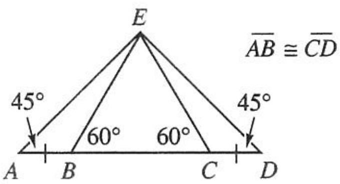
B

Tent pole $\overline{ZX}$ is placed perpendicular to the ground in such a way that X is the midpoint of the opening $\overline{WY}$ at the front of the tent.

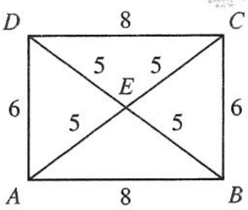


14. Name a pair of congruent triangles.
15. What congruence postulate justifies your choice in Exercise 14?
16. Explain why $\triangle WYZ$ is an isosceles triangle.

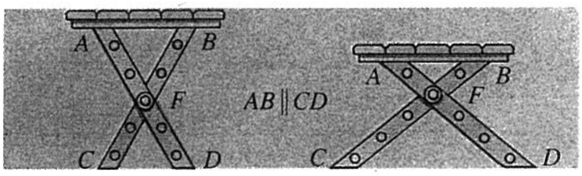
17. Name a triangle that is congruent to $\triangle ABE$.
18. Name a congruence postulate that can be used to show that the triangles in Exercise 17 are congruent.
19. Explain how you can conclude $\triangle BCE$ is an isosceles triangle.



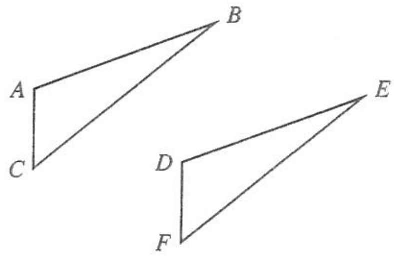
20. Name three triangles that are congruent to $\triangle ABD$.



21. The legs of a table are hinged at points A and B. A bolt is placed at F in one of several predrilled holes. The height of the table is determined by the placement of the bolt F. Where should the bolt F be placed if the base CD is equal in length to the top AB? Explain your answer.



22. $\angle A \cong \angle D$, $m\angle A = 3x + 15$, $m\angle D = x + 75$
$AB = 2x + 15$, $DE = 3x - 15$, $AC = x + 4$, $DF = 2x - 26$
Prove that $\triangle ABC$ and $\triangle DEF$ are congruent.



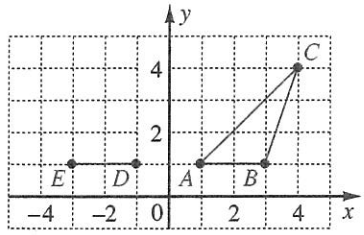
23. $\angle A \cong \angle D$, $m\angle A = 9x - 48$, $m\angle D = 6x - 15$
$AB = x + 5$, $DE = 2x - 6$, $AC = x - 1$, $DF = 3x - 23$
Prove that $\triangle ABC$ and $\triangle DEF$ are congruent.

24. In $\triangle RST$ and $\triangle XYZ$, $\angle R \cong \angle X$, $RS = XY$
$m\angle R = 11x - 1$, $m\angle X = 9x + 5$, and $RT = 7x + 5$
Find XZ if $\triangle RST \cong \triangle XYZ$.

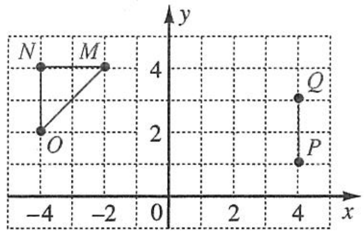
25. $\triangle KLM \cong \triangle NOP$, $m\angle K = 3x + 7$, $m\angle N = 2x + 24$, $m\angle L = 5x - 42$,
$m\angle O = 4x - 25$ Find the measure of $\angle P$.

26. $\triangle CDE \cong \triangle FGH$, $m\angle D = x^2 - 39$, $m\angle G = x + 17$, $m\angle E = 19 - x$,
$m\angle H = 27 - 2x$, $GH = 39 - 3x$ Find DE.

27. Name the coordinates of point F so that $\triangle ABC \cong \triangle DEF$.
28. Calculate the lengths of the sides and use the SSS Congruence Postulate to verify the congruence stated in Exercise 27.

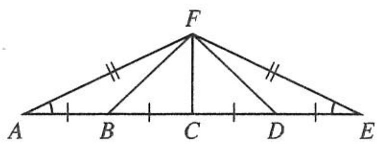


29. Name the coordinates of point R so that $\triangle MNO \cong \triangle PQR$.
30. Calculate the lengths of the sides and use the SSS Congruence Postulate to verify the congruence stated in Exercise 29.

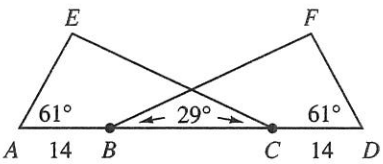


C

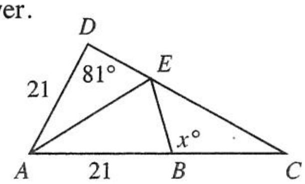
31. $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE}$, $\overline{AF} \cong \overline{EF}$, $\angle A \cong \angle E$
List all the pairs of triangles that you think could be shown to be congruent in the figure to the right.



32. For each pair of triangles listed in Exercise 31 explain why you think your choice is correct.
33. Which postulate can be used to show that $\triangle ACE \cong \triangle DBF$? Explain your reasoning.

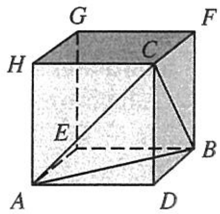


34. Given that $\overline{AE}$ bisects $\angle BAD$, $AD = AB = 21$, find x. Explain how you found the answer.

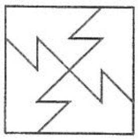


Critical Thinking

35. *ADBEHCFG* is a cube. Write a paragraph that would convince someone that $\triangle ADC$, $\triangle BDC$, and $\triangle ADB$ are all congruent to one another.



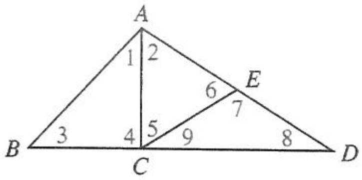
36. The figure to the right shows a square divided into four congruent eight-sided polygons. Draw a second way of doing this.



37. Draw a third way of dividing a square into four congruent eight-sided polygons.

38. Draw a triangle. Label it *RST*. Using a compass and straightedge, construct $\overline{XY} \cong \overline{RS}$, $\angle X \cong \angle R$, and $\angle Y \cong \angle S$. Extend the exterior sides of angles *X* and *Y* so that they meet in a point *Z*. State a congruence between the triangles. What postulate supports your statement?

Mixed Review

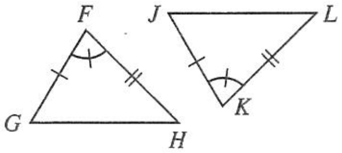


$\triangle ACE$ is equilateral. $\angle ACB$ is a right angle. $\angle 1 \cong \angle 3$
Complete each statement.

1. $m\angle 1 =$ ___ **2.** $m\angle 2 =$ ___ **3.** $m\angle 3 =$ ___
4. $m\angle 4 =$ ___ **5.** $m\angle 5 =$ ___ **6.** $m\angle 6 =$ ___

Write a congruence between each pair of triangles and state the postulate applied.

7.



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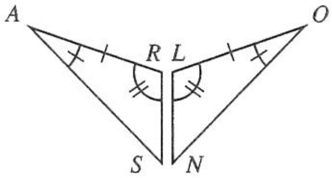

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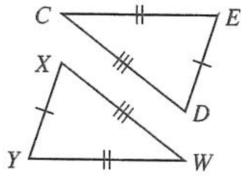
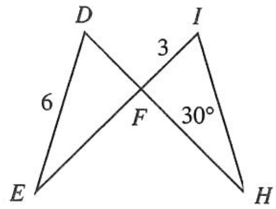


10. $\triangle ABC \cong \triangle LMN$, $m\angle B = x^2$, $m\angle L = 9x$, $BC = x - 2$, $MN = 16 - x$, $AB = 2x - 14$ How long is $\overline{LM}$?

Quiz

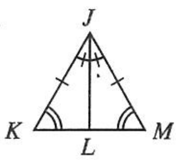


$\triangle DEF \cong \triangle IHF$ **Complete each statement.**

1. $\overline{EF} \cong$ ___ **2.** $m\angle E =$ ___ **3.** $IH =$ ___
4. $\angle I \cong$ ___ **5.** $DF =$ ___ **6.** $\triangle FDE \cong$ ___

Write a congruence between each pair of triangles and state the postulate applied.

7.



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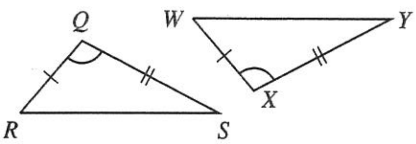

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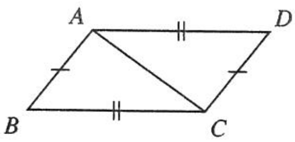
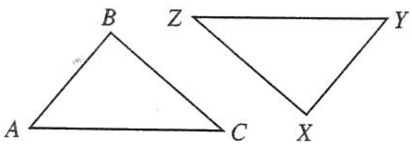


State the additional sides or angles that are needed to prove $\triangle ABC \cong \triangle YXZ$ by the indicated postulate.



10. SAS: $\overline{AB} \cong \overline{YX}$, $\overline{AC} \cong \overline{YZ}$
11. ASA: $\angle C \cong \angle Z$, $\overline{BC} \cong \overline{XZ}$
12. SSS: $\overline{AB} \cong \overline{YX}$, $\overline{BC} \cong \overline{XZ}$

PROVING TRIANGLES CONGRUENT

OBJECTIVE: *Use definitions and the SSS, SAS, and ASA congruence postulates to prove triangles congruent.*

4-3 Proofs: Using Congruence Postulates

In this lesson you will use the SSS, SAS, and ASA congruence postulates to write proofs in the two-column format you learned to use in Chapter 2. The hypothesis and the conclusion of the SAS Postulate are highlighted.

If	two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle,	then	the two triangles are congruent.
	hypothesis		*conclusion*

Many professionals must reason carefully and logically. Geometry proofs provide practice in logical thinking.

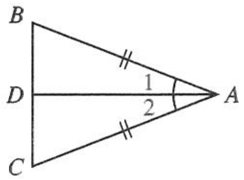
When you use a congruence postulate in a two-column proof, you must include statements to show that the hypothesis is true. The final statement is the conclusion.

Example 1

Given: $\overline{AB} \cong \overline{AC}$

$\angle 1 \cong \angle 2$

Prove: $\triangle ABD \cong \triangle ACD$



Proof

Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Given
3. $\overline{AD} \cong \overline{AD}$	3. Reflexive Property
4. $\triangle ABD \cong \triangle ACD$	4. SAS Postulate

In the above proof statements 1, 2, and 3 show that the hypothesis of the SAS Postulate is satisfied. Statement 4 is drawn from the conclusion of the SAS Postulate. Other congruence postulates are used similarly in proofs.

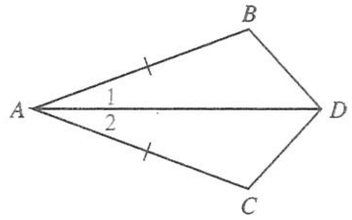
A statement and its reasons often refer to a previously learned definition. Use the sentences listed below as reasons in your proof when you are using the definitions listed. Note the third reason in Example 2.

An angle *bisector* forms two congruent angles.
A segment *bisector* forms two congruent segments.
A segment *midpoint* forms two congruent segments.
Perpendicular segments form right angles.

Example 2

Given: $\overline{AD}$ bisects $\angle BAC$.
$\overline{AB} \cong \overline{AC}$

Prove: $\triangle ABD \cong \triangle ACD$



Proof

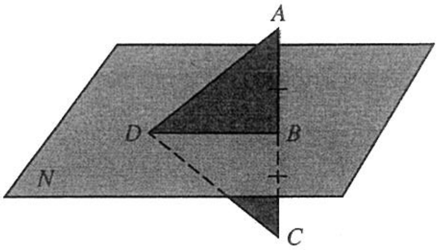
Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	1. Given
2. $\overline{AB}$ bisects $\angle BAC$.	2. Given
3. $\angle 1 \cong \angle 2$	3. An angle bisector forms two congruent angles.
4. $\overline{AD} \cong \overline{AD}$	4. Reflexive Property
5. $\triangle ABD \cong \triangle ACD$	5. SAS Postulate

The proof in Example 3 uses the definition of a line perpendicular to a plane introduced in Chapter 1.

Example 3

Given: $\overline{AB} \perp N$
B in plane N
$\overline{AB} \cong \overline{CB}$

Prove: $\triangle ABD \cong \triangle CBD$



Proof

Statements	Reasons
1. $\overline{AB} \perp N$, B in N	1. Given
2. $\overline{AB} \cong \overline{CB}$	2. Given
3. $\overline{AD} \perp \overline{BD}$	3. If a line is $\perp$ to plane N at B then it is $\perp$ to each line in N through B.
4. $\angle ABD$ and $\angle CBD$ are rt. $\angle$s.	4. $\perp$ lines form rt. $\angle$s.
5. $\angle ABD \cong \angle CBD$	5. All rt. $\angle$s are $\cong$.
6. $\overline{BD} \cong \overline{BD}$	6. Reflexive Property
7. $\triangle ABD \cong \triangle CBD$	7. SAS Postulate

Class Exercises

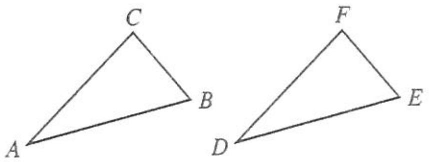
Short Answer

State the hypothesis and conclusion of each postulate.

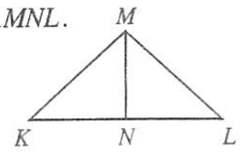
1. SSS Congruence Postulate
2. SAS Congruence Postulate
3. ASA Congruence Postulate
4. State the hypothesis and conclusion of this statement.
   Two triangles are congruent if corresponding pairs of angles and sides are congruent.

A conclusion can be drawn from the information given in each of Exercises 5–9. State your conclusion and the postulate or definition on which it is based.

5. $\overline{DF} \cong \overline{AC}$, $\overline{DE} \cong \overline{AB}$, $\angle D \cong \angle A$
6. $\overline{EF} \cong \overline{BC}$, $\angle F \cong \angle C$, $\angle B \cong \angle E$



7. $\overline{MN}$ bisects $\angle KML$ and $\angle KNM \cong \angle MNL$.
8. N is the midpoint of $\overline{KL}$. $\overline{KM} \cong \overline{LM}$
9. $\overline{MN}$ bisects $\angle KML$. $\overline{KM} \cong \overline{LM}$

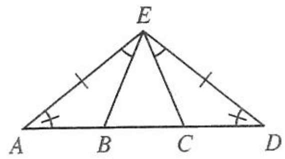


Sample Exercises

Complete each proof.

10. **Given:** $\overline{AE} \cong \overline{DE}$, $\angle A \cong \angle D$
$\angle AEB \cong \angle DEC$

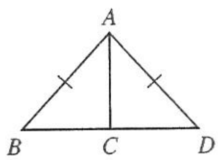
Prove: $\triangle AEB \cong \triangle DEC$



Statements	Reasons
1. $\angle AEB \cong \angle DEC$	1. ___
2. $\overline{AE} \cong \overline{DE}$	2. ___
3. $\angle A \cong \angle D$	3. ___
4. $\triangle AEB \cong \triangle DEC$	4. ___

11. **Given:** $\overline{AB} \cong \overline{AD}$
C is the midpoint of $\overline{BD}$.

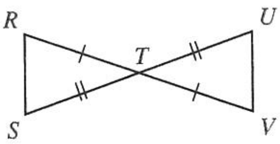
Prove: $\triangle ABC \cong \triangle ADC$



Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$	1. ___
2. C is the midpoint of $\overline{BD}$.	2. ___
3. $\overline{BC} \cong \overline{DC}$	3. ___
4. ___	4. Reflexive Property
5. $\triangle ABC \cong \triangle ADC$	5. ___

12. **Given:** $\overline{RT} \cong \overline{VT}$, $\overline{ST} \cong \overline{UT}$

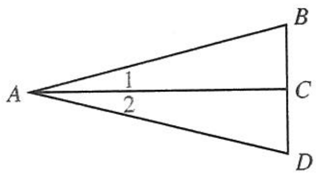
Prove: $\triangle RST \cong \triangle VUT$



Statements	Reasons
1. $\overline{RT} \cong \overline{VT}$	1. ___
2. $\angle RTS \cong \angle VTU$	2. ___
3. $\overline{ST} \cong \overline{UT}$	3. ___
4. ___	4. ___

Discussion

13. Suppose that $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$. Give a convincing argument that $\triangle ABC \cong \triangle ADC$.



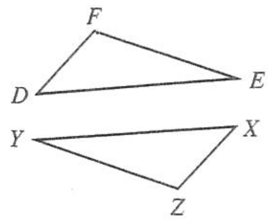
Exercises

A

Complete each proof.

1. Given: $\angle D \cong \angle X$, $\angle F \cong \angle Z$, $\overline{DF} \cong \overline{XZ}$

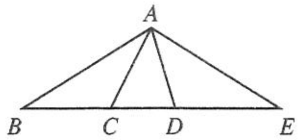
Prove: $\triangle DEF \cong \triangle XYZ$



Statements	Reasons
1. $\angle D \cong \angle X$	1. ___
2. $\overline{DF} \cong \overline{XZ}$	2. ___
3. $\angle F \cong \angle Z$	3. ___
4. $\triangle DEF \cong \triangle XYZ$	4. ___

2. Given: $\overline{AC} \cong \overline{AD}$, $\overline{BC} \cong \overline{DE}$, $\overline{AB} \cong \overline{AE}$

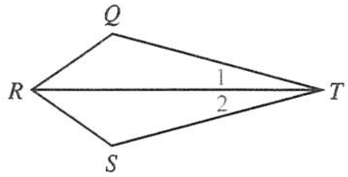
Prove: $\triangle ABC \cong \triangle AED$



Statements	Reasons
1. $\overline{AC} \cong \overline{AD}$	1. ___
2. $\overline{BC} \cong \overline{DE}$	2. ___
3. ___	3. Given
4. $\triangle ABC \cong \triangle AED$	4. ___

3. Given: $\overline{RT}$ bisects $\angle QRS$. $\angle 1 \cong \angle 2$

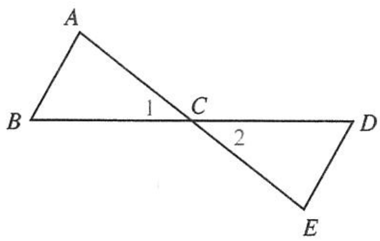
Prove: $\triangle RTQ \cong \triangle RTS$



Statements	Reasons
1. $\overline{RT}$ bisects $\angle QRS$.	1. ___
2. $\angle QRT \cong \angle SRT$	2. ___
3. $\overline{RT} \cong \overline{RT}$	3. ___
4. $\angle 1 \cong \angle 2$	4. ___
5. ___	5. ASA

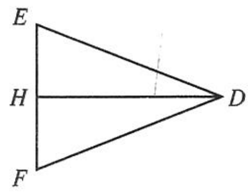
4. Given: $\overline{AE}$ bisects $\overline{BD}$. $\overline{AC} \cong \overline{EC}$

Prove: $\triangle ABC \cong \triangle EDC$



Statements	Reasons
1. $\overline{AE}$ bisects $\overline{BD}$.	1. ___
2. $\overline{BC} \cong \overline{DC}$	2. ___
3. $\angle 1 \cong \angle 2$	3. ___
4. ___	4. ___
5. $\triangle ABC \cong \triangle EDC$	5. ___

Write a two-column proof for each exercise.



5. Given: $\overline{DE} \cong \overline{DF}$, $\overline{DH}$ bisects $\overline{EF}$.

Prove: $\triangle DHE \cong \triangle DHF$

6. Given: $\overline{DE} \cong \overline{DF}$, $\overline{DH}$ bisects $\angle EDF$.

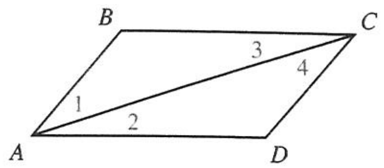
Prove: $\triangle DHE \cong \triangle DHF$

7. Given: $\overline{AB} \cong \overline{CD}$, $\angle 1 \cong \angle 4$

Prove: $\triangle ABC \cong \triangle CDA$

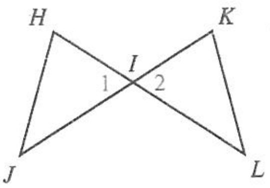
8. Given: $\angle 2 \cong \angle 3$, $\angle 1 \cong \angle 4$

Prove: $\triangle ABC \cong \triangle CDA$



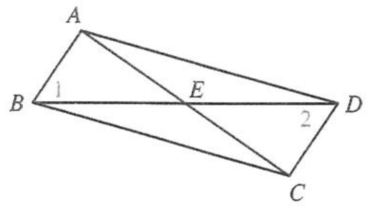
9. **Given:** $\overline{HI} \cong \overline{KI}$, $\angle 1 \cong \angle 2$
$\overline{JI} \cong \overline{IL}$

Prove: $\triangle HIJ \cong \triangle KIL$



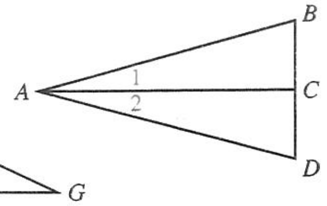
10. **Given:** $\overline{AB} \cong \overline{CD}$, $\angle 1 \cong \angle 2$
$\overline{AC}$ bisects $\overline{BD}$.

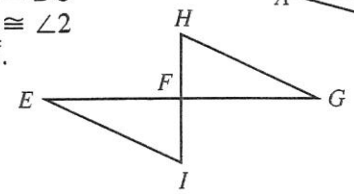
Prove: $\triangle ABE \cong \triangle CDE$



Write a conclusion that follows from the given information. State the congruence postulate and/or definition on which your conclusion is based.

11. $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{DC}$
12. $\overline{AB} \cong \overline{AD}$, $\angle 1 \cong \angle 2$
13. $\overline{AC}$ is perpendicular to $\overline{BD}$. $\overline{BC} \cong \overline{DC}$
14. $\overline{AC}$ is perpendicular to $\overline{BD}$. $\angle 1 \cong \angle 2$
15. F is the midpoint of $\overline{EG}$ and $\overline{HI}$.
16. $\overline{EG}$ and $\overline{HI}$ bisect each other.

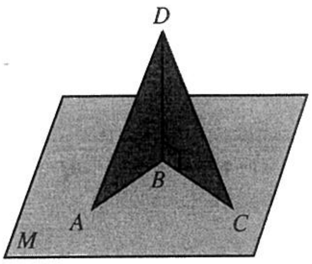




Write a two-column proof for each exercise.

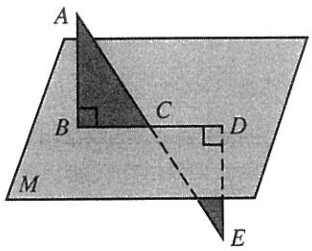
17. **Given:** $\overline{BD} \perp$ to plane M
$\overline{AB} \cong \overline{BC}$

Prove: $\triangle ABD \cong \triangle CBD$



18. **Given:** $\overline{AB} \perp$ plane M, $\overline{DE} \perp$ plane M
C bisects $\overline{BD}$.

Prove: $\triangle ACB \cong \triangle ECD$

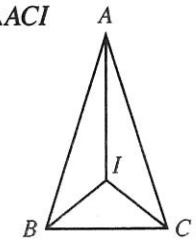


B

Write a two-column proof for each exercise.

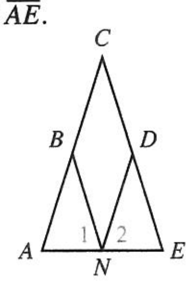
19. **Given:** $\overline{AI}$ bisects $\angle BAC$.
$\overline{AB} \cong \overline{AC}$

Prove: $\triangle ABI \cong \triangle ACI$

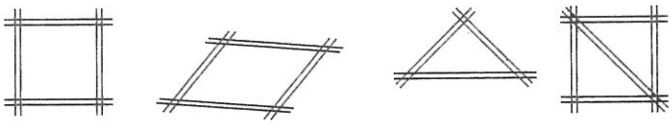


20. **Given:** $\angle 1 \cong \angle 2$, $\angle A \cong \angle E$
N is the midpoint of $\overline{AE}$.

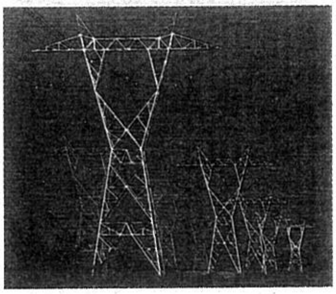
Prove: $\triangle ABN \cong \triangle EDN$



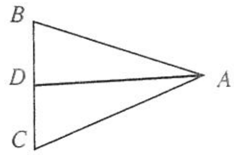
21. Supporting struts are often added to a structure to form triangles so that the structure will become rigid. Explain how the congruence postulates relate to the fact that triangles form rigid figures.



A nonrigid figure collapses. Triangles form rigid figures.

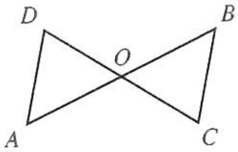


22. Give a convincing argument that quadrilaterals do not satisfy an SSSS congruence postulate.



23. D is the midpoint of $\overline{BC}$. $AB = 9x + 3$, $BD = 9x - 2$, $AC = 15x - 3$, $CD = 4x + 3$ Show that $\triangle ABD \cong \triangle ACD$.

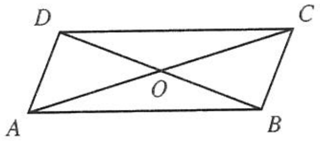
24. $\overline{AD}$ bisects $\angle BAC$. $m\angle BAD = 31 - x$, $m\angle CAD = 15 + 3x$, $AB = 3x + 4$, $AC = 7x - 12$ Show that $\triangle BDA \cong \triangle CDA$.



25. $m\angle AOD = 2x + 9$, $m\angle BOC = 3x + 5$, $AO = x^2 - 2$, $OB = 3x + 2$, $OC = 2x^2 - 17$, $DO = 5x - 5$ Show that $\overline{AB}$ and $\overline{CD}$ bisect each other.

C

26. **Given:** In figure $ABCD$, O bisects $\overline{AC}$ and $\overline{BD}$.
Prove: $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{AD}$



Critical Thinking

Decide whether each statement is always, sometimes, or never true. Draw a figure to explain your answer. Give a convincing argument when the statement is always true.

27. A ray that bisects one angle of a triangle divides the triangle into two congruent triangles.
28. The segment from a vertex of a triangle to the midpoint of the opposite side divides the triangle into a pair of congruent triangles.
29. If two sides of a triangle are congruent, the segment from the common vertex to the midpoint of the third side bisects the angle of the vertex.

Mixed Review

Complete each statement.

1. Two lines in different planes that do not intersect are ___.
2. Four lines intersect in, at most, ___ points.
3. Adjacent angles have no ___ points in common.
4. Two lines that form congruent adjacent angles are ___.
5. If two sides of a triangle lie in a plane, the third side ___.
6. A triangle divides a plane into three sets of points, those on the exterior, those on the interior, and ___.

Computer Activity

Suppose you have two boxes and each box contains eight dowel rods with lengths 8 cm, 9 cm, 10 cm, 11 cm, 12 cm, 13 cm, 14 cm, and 15 cm. Suppose you reach into one box with your right hand and select three rods at random, and then you reach into the other box with your left hand and select three rods at random.

What are the chances that the triangle made with the right-hand rods will be congruent to the triangle made with the left-hand rods? Are your chances very good?

The program provided here simulates this experiment. When you run the program, it asks how many times you want to simulate the experiment. Then it records the number of times the two randomly selected triangles are congruent. It also records the experimental probability of selecting a pair of congruent triangles.

```
10 INPUT "HOW MANY EXPERIMENTS DO YOU WANT TO RUN ";N
15 FOR COUNT = 1 TO N
20 INDEX = 0
25 FOR TRIANGLE 1 TO 2
30 GOSUB 1000
35 A(INDEX + 1) = NUMBER
40 GOSUB 1000
45 A(INDEX + 2) = NUMBER
50 IF A(INDEX + 1) = A(INDEX + 2) THEN GOTO 40
55 IF A(INDEX + 1) > A(INDEX + 2), THEN SWITCH = A(INDEX + 1):
     A(INDEX + 1) = A(INDEX + 2): A(INDEX + 2) = SWITCH
60 GOSUB 1000
65 A(INDEX + 3) = NUMBER
70 IF A(INDEX + 3) = A(INDEX + 2) OR A(INDEX + 3) = A(INDEX + 1)
     THEN GOTO 60
75 IF A(INDEX + 1) > A(INDEX + 3), THEN SWITCH = A(INDEX + 1):
     A(INDEX + 1) = A(INDEX + 3): A(INDEX + 3) = SWITCH
80 IF A(INDEX + 2) > A(INDEX + 3), THEN SWITCH = A(INDEX + 2);
     A(INDEX + 2) = A(INDEX + 3): A(INDEX + 3) = SWITCH
85 INDEX = 3
90 NEXT TRIANGLE
95 IF A(1) = A(4) AND A(2) = A(5) AND A(3) = A(6) THEN PAIRS = PAIRS + 1
100 NEXT COUNT
110 PRINT "THE NUMBER OF CONGRUENT PAIRS IS "; PAIRS
115 PRINT "THE EXPERIMENTAL PROBABILITY IS "; PAIRS/N
120 END
1000 NUMBER = INT(8*RND(1)) + 8: RETURN
```

Exercises

1. Run the program to simulate the experiment 100 times.
2. Run the program to simulate the experiment 500 times.
3. Which result do you think is closer to the actual probability? Why?

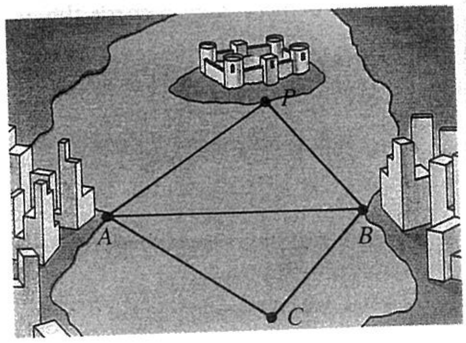
OBJECTIVE: *Deduce that segments and angles are congruent by first proving that triangles are congruent.*

4-4 Proving Segments and Angles Congruent

In this lesson you will prove two triangles are congruent in order to conclude that a pair of angles or sides are congruent. In this first example you will learn how a surveyor can use congruent triangles to find a distance that he or she is unable to measure directly.

Example 1

Application A surveyor is unable to measure the distances from docks A and B to point P on an island. Explain how to find distances AP and BP.

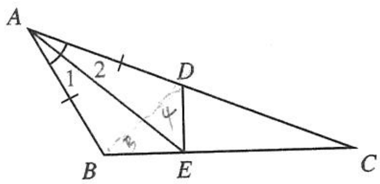


Solution

1. Measure $\angle BAP$ and $\angle ABP$.
2. Locate point C so that $m\angle BAP = m\angle BAC$ and $m\angle ABP = m\angle ABC$.
3. Use the ASA postulate to conclude that $\triangle ABP \cong \triangle ABC$.
4. Since corresponding sides of congruent triangles are the same length, we can conclude that $AP = AC$ and $BP = BC$. Measure $\overline{AC}$ and $\overline{BC}$.

Step 4 in Example 1 uses the fact that corresponding sides of congruent triangles are congruent. It is also true that corresponding angles of congruent triangles are congruent.

Corresponding parts of congruent triangles are congruent.
Abbreviation: Corr. parts of $\cong$ $\triangle$s are $\cong$.



Example 2

Complete the proof.

Given: $\overline{AB} \cong \overline{AD}$, $\angle 1 \cong \angle 2$

Prove: $\overline{BE} \cong \overline{DE}$

Plan To prove that $\overline{BE} \cong \overline{DE}$, find a pair of congruent triangles that contain these segments. $\triangle ABE$ and $\triangle ADE$ include $\overline{BE}$ and $\overline{DE}$. Angles 1 and 2 are included angles that lead to the use of the SAS postulate.

Proof

Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Given
3. $\overline{AE} \cong \overline{AE}$	3. Reflexive Property
4. $\triangle ABE \cong \triangle ADE$	4. SAS Postulates
5. $\overline{AB} \cong \overline{CD}$	5. —

Solution

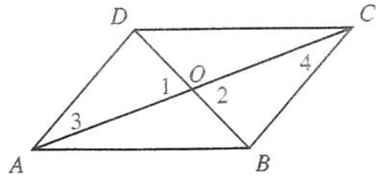
Corr. parts of $\cong$ $\triangle$s are $\cong$.

A summary of the general procedure that can be used to prove two segments or two angles congruent by using congruent triangles is described as follows.

Proving Two Segments or Two Angles Congruent

1. Identify two triangles that contain the two segments or two angles as corresponding parts.
2. Prove that the two triangles are congruent.
3. State that the two segments or angles are congruent by using "Corr. parts of ≅ △s are ≅."

Example 3



Write a plan for proof and a two-column proof.

Given: $\overline{AC}$ and $\overline{BD}$ bisect each other.

Prove: $\angle 3 \cong \angle 4$

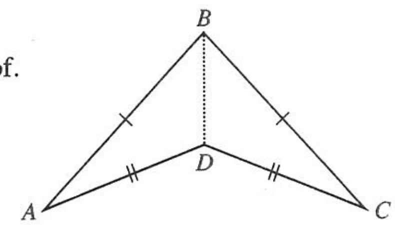
Solution

Plan To prove that $\angle 3 \cong \angle 4$, first prove that $\triangle ADC \cong \triangle CBA$ or that $\triangle AOD \cong \triangle COB$. Since $\angle 1$ and $\angle 2$ are vertical angles, they are congruent. I can use this fact together with the given information to prove $\triangle AOD \cong \triangle COB$ by SAS.

Proof

Statements	Reasons
1. $\overline{AC}$ and $\overline{BD}$ bisect each other.	1. Given
2. $\overline{AO} \cong \overline{CO}$, $\overline{BO} \cong \overline{DO}$	2. A segment bisector forms two ≅ segments.
3. $\angle 1 \cong \angle 2$	3. Vertical angles are ≅.
4. $\triangle AOD \cong \triangle COB$	4. SAS Postulates
5. $\angle 3 \cong \angle 4$	5. Corr. parts of ≅ △s are ≅.

Example 4



Write a plan for proof and a two-column proof. Use an auxiliary line.

Given: $\overline{AB} \cong \overline{CB}$, and $\overline{AD} \cong \overline{CD}$

Proof: $\angle A \cong \angle C$

Solution

Plan I can prove $\angle A \cong \angle C$ if these two angles are corresponding parts of a pair of congruent triangles. If I draw the auxiliary segment $\overline{BD}$, I can use the SSS congruence postulate to prove that $\triangle ABD \cong \triangle CBD$.

Proof

Statements	Reasons
1. Draw $\overline{BD}$.	1. For any two points, there is exactly one line containing them.
2. $\overline{BD} \cong \overline{BD}$	2. Reflexive Property
3. $\overline{AB} \cong \overline{CB}$	3. Given
4. $\overline{AD} \cong \overline{CD}$	4. Given
5. $\triangle ABD \cong \triangle CBD$	5. SSS Postulate
6. $\angle A \cong \angle C$	6. Corr. parts of ≅ △s are ≅.

Class Exercises

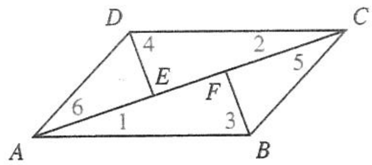
Short Answer

Select a pair of triangles that you might try to prove congruent if you wanted to conclude each of the following.

1. $\overline{AD} \cong \overline{CB}$ 2. $\angle 1 \cong \angle 2$
3. Select a second pair of triangles that you could use for Exercise 1.
4. Select a second pair of triangles that you could use for Exercise 2.

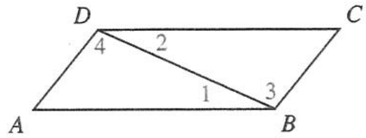
Sample Exercises

Give a plan for each proof.



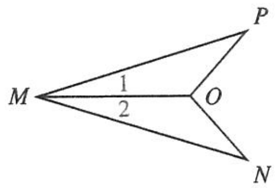
5. **Given:** $\overline{AB} \cong \overline{CD}$, $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$
   Prove: $\overline{BF} \cong \overline{DE}$
6. **Given:** $\angle 5 \cong \angle 6$, $\overline{BC} \cong \overline{DA}$, $\overline{CF} \cong \overline{AE}$
   Prove: $\angle DEA \cong \angle BFC$

Complete each proof.



7. **Given:** $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$
   Prove: $\angle A \cong \angle C$

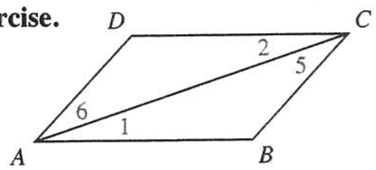
Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. ___
2. $\overline{DB} \cong \overline{DB}$	2. ___
3. ___	3. ___
4. $\triangle ADB \cong \triangle CBD$	4. ___
5. $\angle A \cong \angle C$	5. ___



8. **Given:** $\overline{MN} \cong \overline{MP}$, $\overline{NO} \cong \overline{PO}$
   Prove: $\overline{MO}$ bisects $\angle PMN$.

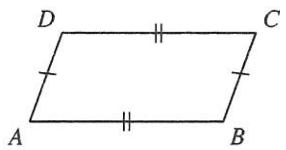
Statements	Reasons
1. $\overline{MN} \cong \overline{MP}$	1. ___
2. ___	2. Given
3. $\overline{MO} \cong \overline{MO}$	3. ___
4. $\triangle PMO \cong \triangle NMO$	4. ___
5. $\angle 1 \cong \angle 2$	5. ___
6. $\overline{MO}$ bisects $\angle PMN$.	6. ___

Write a two-column proof for each exercise.



9. **Given:** $\overline{AB} \cong \overline{CD}$, $\angle 1 \cong \angle 2$
   Prove: $\angle 5 \cong \angle 6$
10. **Given:** $\angle 1 \cong \angle 2$, $\angle 5 \cong \angle 6$
    Prove: $\angle B \cong \angle D$

Discussion



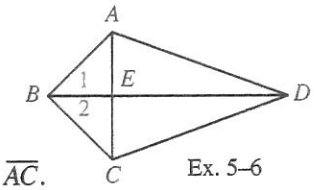
11. Suppose that $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$. Explain how you would use an auxiliary line to prove that $\angle A \cong \angle C$. How many ways can this be done?

Exercises

A

Suppose that $\triangle ABC \cong \triangle XYZ$. Complete each statement.

1. $XY = 14$ cm, ___ $= 14$ cm
2. $m\angle B = 63$, then ___ $= 63$
3. $m\angle Z = 131$, ___ $= 131$
4. $AC = 21$ cm, ___ $= 21$ cm



Ex. 5–6

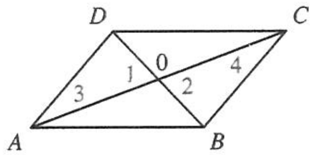
5. **Given:** $\overline{BD}$ bisects $\angle ABC$.
$\overline{AB} \cong \overline{CB}$
Prove: $\overline{AD} \cong \overline{CD}$

Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$	1. ___
2. $\overline{BD}$ bisects $\angle ABC$.	2. ___
3. $\angle 1 \cong \angle 2$	3. ___
4. ___	4. Reflexive Property
5. $\triangle ABD \cong \triangle CBD$	5. ___
6. $\overline{AD} \cong \overline{CD}$	6. ___

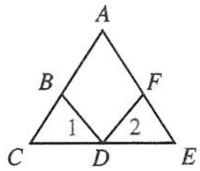
6. **Given:** $\overline{AC} \perp \overline{BD}$
$\overline{BD}$ bisects $\overline{AC}$.
Prove: $\angle ABD \cong \angle CBD$

Statements	Reasons
1. $\overline{AC} \perp \overline{BD}$	1. ___
2. ___	2. $\perp$ lines form $\cong$ adjacent angles.
3. $\overline{BD}$ bisects $\overline{AC}$.	3. ___
4. $\overline{AE} \cong \overline{CE}$	4. ___
5. ___	5. Reflexive Property
6. $\triangle ABE \cong \triangle CBE$	6. ___
7. $\angle ADB \cong \angle CBD$	7. ___

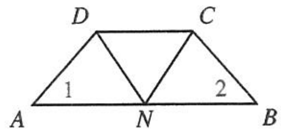
Write a two-column proof for each exercise.



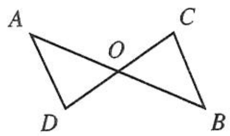
7. **Given:** $\overline{AC}$ and $\overline{BD}$ bisect each other.
Prove: $\overline{AD} \cong \overline{CB}$
8. **Given:** O bisects $\overline{AC}$. $\angle 3 \cong \angle 4$
Prove: $\overline{AC}$ and $\overline{BD}$ bisect each other.

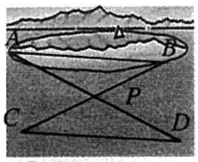


9. **Given:** D bisects $\overline{CE}$. $\angle 1 \cong \angle 2$, $\angle C \cong \angle E$
Prove: $\overline{BD} \cong \overline{FD}$
10. **Given:** N is the midpoint of $\overline{AB}$. $\overline{AD} \cong \overline{BC}$, $\angle 1 \cong \angle 2$
Prove: $\overline{CN} \cong \overline{DN}$

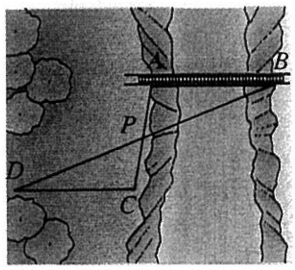


11. **Given:** O is the midpoint of $\overline{AB}$ and $\overline{CD}$.
Prove: $\overline{AD} \parallel \overline{BC}$
12. **Given:** $\overline{AD} \parallel \overline{BC}$, $\overline{AD} \cong \overline{BC}$
Prove: O is the midpoint of $\overline{AB}$ and $\overline{CD}$.



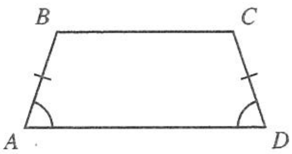


13. To find the distance AB across this lake, a surveyor located points C and D with A, P, and D collinear and B, P, and C collinear. $AP = DP$ and $BP = CP$. Explain why distance CD is equal to distance AB.



14. An engineer designing a bridge to be built across a deep canyon needs to find the distance AB across the canyon. He locates points P and C so that $AP = CP$. Next he uses a transit to measure $\angle PAB$. He then locates point D so that $m\angle PAB = m\angle PCD$. How can the engineer find the distance AB?

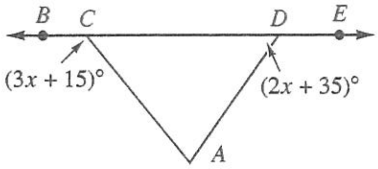
15. Write a two-column proof.
Add auxiliary lines as needed.
Given: $\overline{AB} \cong \overline{DC}$, $\angle A \cong \angle D$
Prove: $\angle B \cong \angle C$



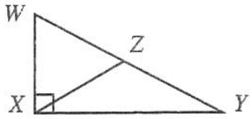
B

16. $\angle ADE \cong \angle ACB$, $m\angle ACB = 3x + 15$,
$m\angle ADC = 2x + 35$
Find $m\angle ADC$.

17. $\angle XWZ \cong \angle WXZ$, $m\angle XWZ = 3x + 40$,
$m\angle ZXY = 2x + 10$
Find $m\angle ZXY$.



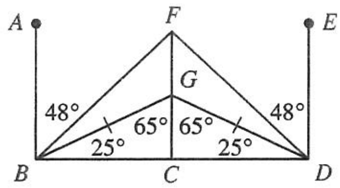
Ex. 16



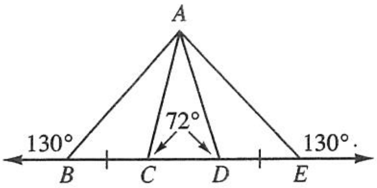
Ex. 17

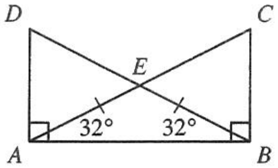
List sufficient given information (taken from each figure) and then write a two-column proof.

18. Prove that C is the midpoint of $\overline{BD}$.
19. Prove that $\overline{BF} \cong \overline{DF}$.
20. Prove that $\overline{AB} \cong \overline{AE}$.
21. Prove that $\angle D \cong \angle C$.



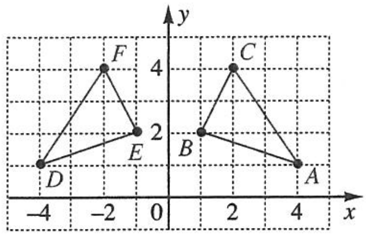
Ex. 18–19





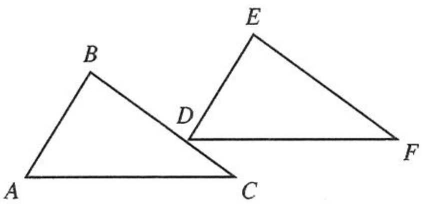
Use the distance formula to calculate each length.

22. AB, DE
23. AC, DF
24. BC, EF
25. Use Exercises 22–24 to show that $\angle A \cong \angle D$.



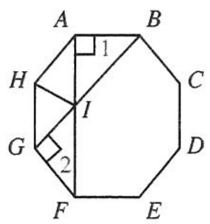
C

26. **Given:** $\overline{AC} \parallel \overline{DF}$, $\overline{AB} \parallel \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{DE}$
Prove: $\angle B \cong \angle E$
(HINT: Draw auxiliary lines.)



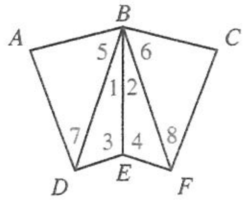
In Exercises 27–30 you may need to prove one pair of triangles congruent in order to gain information needed to show a second pair of triangles congruent. Write a two-column proof for each exercise.

27. **Given:** In figure $ABCDEFGH$, all sides are congruent to each other.
$\overline{AF} \perp \overline{AB}$, $\overline{BG} \perp \overline{GF}$, $\angle 1 \cong \angle 2$
Prove: $\triangle AHI \cong \triangle GHI$

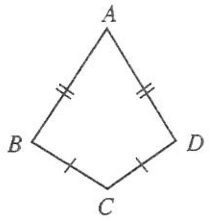


28. **Given:** $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $\angle 5 \cong \angle 6$, $\angle 7 \cong \angle 8$

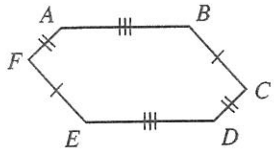
Prove: $\angle A \cong \angle C$



29. Prove that if $\overline{AD} \cong \overline{AB}$ and $\overline{CD} \cong \overline{CB}$, then $\angle D \cong \angle B$.

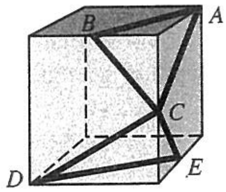


30. Prove that if $\overline{AB} \cong \overline{DE}$, $\overline{AF} \cong \overline{CD}$, $\overline{EF} \cong \overline{BC}$, and $\angle F \cong \angle C$, then $\overline{AB} \parallel \overline{ED}$.



Critical Thinking

31. In the cube, all faces are squares and all edges are equal in length. Suppose that points B, C, and E are midpoints of edges. Write a convincing argument that $\triangle ABC$ is congruent to $\triangle DCE$.



Algebra Review

1. In $\triangle DEF$, $m\angle E$ is six more than $m\angle D$. The measure of $\angle F$ is ten less than twice $m\angle D$. Find the measure of each angle.

2. isosceles $\triangle LOR$ with vertex L, $LO = x^2 + 5$, $LR = 4x + 2$ Find LO and LR.

3. $\triangle CAW$ has $m\angle C = x^2 + 10x + 2$, $m\angle A = x^2 - 9$, and $m\angle W = 8x + 7$. Find the measure of each angle.

Enrichment

Factorials

If three dowel rods are selected from a box of eight rods of different lengths, how many different ways can this selection be completed? (See the Computer Activity on page 165.)

If n is a positive whole number, n factorial ($n!$) is defined as $n! = n(n-1)(n-2)\ldots 3\cdot 2\cdot 1$.

Example

How many ways are there to select

a. three things from nine different things?

b. r things from n different things, where $r < n$?

Solution

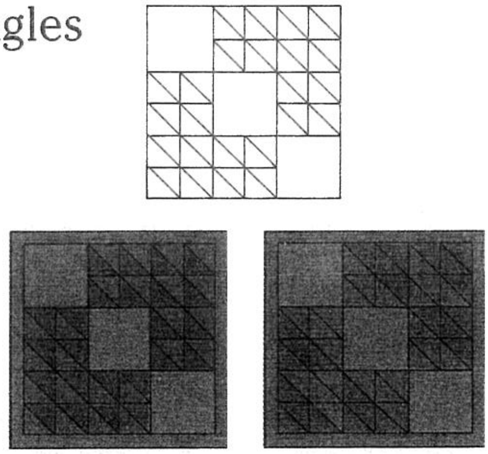
a. $\frac{9!}{3!6!} = 84$ **b.** $\frac{n!}{r!(n-r)!}$

Part b of this example gives a general formula to count the number of ways to select r things from n different things.

Select seven points on a circle and use the general formula.

1. How many different segments can be drawn between pairs of these points?
2. How many different triangles can be drawn with vertices selected among these points?
3. How many different segments and triangles can be drawn with vertices selected from among nine points distributed around a circle?

4-5 Proofs: Overlapping Triangles



In the last lesson, you learned that to prove two segments or two angles congruent, it is often necessary to first prove that a pair of triangles are congruent. Sometimes it is difficult to visualize the triangles that you should prove congruent because they overlap.

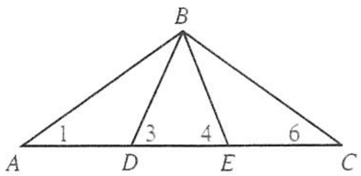
So it is helpful to separate the triangles mentally, or to redraw the two triangles and then separate them—to help analyze the proof. The first example focuses on formulating a plan for a proof involving overlapping triangles.

Example 1

Write a plan for the proof.

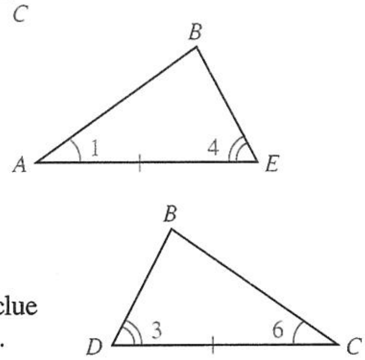
Given: $\angle 1 \cong \angle 6$, $\angle 3 \cong \angle 4$
$\overline{AE} \cong \overline{CD}$

Prove: $\angle ABE \cong \angle CBD$



Solution

Plan I can prove that the overlapping angles $\angle ABE$ and $\angle CBD$ are congruent if I can show that they are corresponding angles in a pair of congruent triangles. Since the angles are overlapping, I can use the ASA Postulate to prove that the overlapping triangles $\triangle ABE$ and $\triangle CBD$ are congruent.



Sometimes the angles or segments that you want to prove congruent are not overlapping so there is no clue that you should be looking for overlapping triangles.

Try This

Write a plan for the proof.

Given: $\angle 1 \cong \angle 6$, $\overline{AE} \cong \overline{CD}$, $\overline{AB} \cong \overline{CB}$

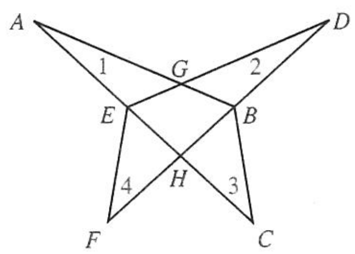
Prove: $\angle 3 \cong \angle 4$

Example 2

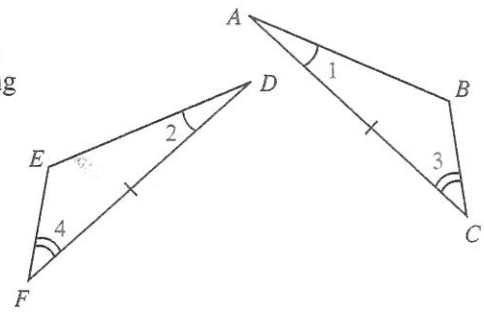
Complete the proof.

Given: $\angle 1 \cong \angle 2$, $\overline{AC} \cong \overline{DF}$
$\angle 3 \cong \angle 4$

Prove: $\overline{EF} \cong \overline{BC}$



Plan To prove that $\overline{EF} \cong \overline{BC}$, I need to choose a pair of triangles that contain $\overline{EF}$ and $\overline{BC}$ as corresponding sides. I cannot prove $\triangle EFH \cong \triangle BCH$. I can prove overlapping triangles $\triangle EFD \cong \triangle BCA$ by using the ASA postulate.



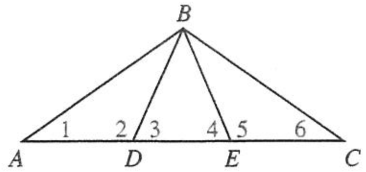
Solution

Proof

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\overline{AC} \cong \overline{DF}$	2. Given
3. $\angle 3 \cong \angle 4$	3. Given
4. $\triangle ABC \cong \triangle DEF$	4. ASA Postulate
5. $\overline{EF} \cong \overline{BC}$	5. Corr. parts of $\cong$ $\triangle$s are $\cong$.

Example 3

Complete the proof.



Given: $\angle 1 \cong \angle 6$, $\angle 3 \cong \angle 4$, $\overline{AD} \cong \overline{CE}$

Prove: $\angle ABE \cong \angle CBD$

Plan To prove that $\angle ABE \cong \angle CBD$, I need to prove that a pair of congruent triangles contain these corresponding angles. I can prove (overlapping triangles) $\triangle AEB \cong \triangle CDB$ by ASA if I can show that $\overline{AE} \cong \overline{CD}$. I can do that by using the Common Segment Theorem and the fact that $\overline{AD} \cong \overline{DF}$.

Solution

Proof

Statements	Reasons
1. $\angle 1 \cong \angle 6$	1. Given
2. $\overline{AD} \cong \overline{CE}$	2. Given
3. $\overline{AE} \cong \overline{CD}$	3. Common Segment Theorem
4. $\angle 4 \cong \angle 3$	4. Given
5. $\triangle AEB \cong \triangle CDB$	5. ASA Postulate
6. $\angle ABE \cong \angle CBD$	6. Corr. parts. of $\cong$ $\triangle$s are $\cong$.

Try This

Complete the proof.

Given: $\overline{AB} \cong \overline{CB}$, $\angle 1 \cong \angle 6$, $\overline{AE} \cong \overline{CD}$

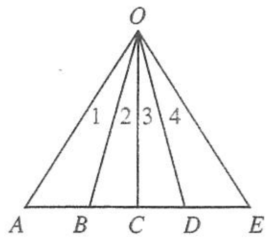
Prove: $\angle 2 \cong \angle 5$

Plan First show that $\triangle AEB \cong \triangle CDB$ by SAS. Then $\angle 3 \cong \angle 4$ by corresponding parts of congruent triangles and $\angle 2 \cong \angle 5$ by the Congruent Supplements Theorem.

Proof

Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$, $\angle 1 \cong \angle 6$, $\overline{AE} \cong \overline{CD}$	1. ___
2. ___	2. ___
3. $\angle 3 \cong \angle 4$	3. ___
4. ___	4. ___

Class Exercises



Short Answer

1. Name two different triangles that overlap $\triangle AOC$.
2. Name two different triangles that overlap $\triangle BOD$.
3. Name two different triangles that overlap $\triangle BOE$.
4. Name two different triangles that overlap $\triangle AOD$.

Sample Exercises

Use the Common Segment Theorem or the Common Angle Theorem to complete each statement.

5. If $\overline{AB} \cong \overline{DE}$, then $\overline{AD} \cong$ ___.
6. If $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong$ ___.
7. If $\angle 1 \cong \angle 3$, then $\angle AOC \cong$ ___.
8. If $\angle 2 \cong \angle 4$, then $\angle EOC \cong$ ___.
9. If $\overline{AC} \cong \overline{BD}$, then $\overline{AB} \cong$ ___.
10. If $\overline{AD} \cong \overline{BE}$, then $\overline{AB} \cong$ ___.

11. Write a two-column proof for Example 1 on page 172.

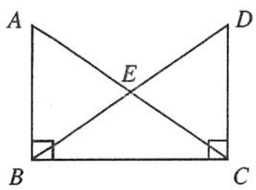
Discussion

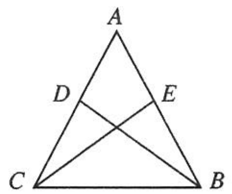
12. Suppose that in the above figure $\overline{AO} \cong \overline{EO}$, $\angle 1 \cong \angle 4$, and $\overline{OB} \cong \overline{OD}$. Use this information to give a convincing argument that a pair of nonoverlapping triangles are congruent and a pair of overlapping triangles are congruent.

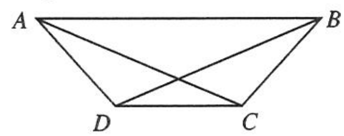
Exercises

A

Name a pair of overlapping triangles that appear to be congruent.

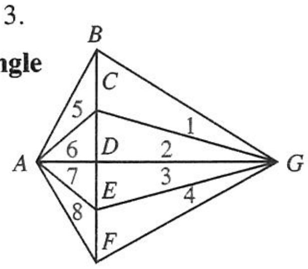
1.


2.


3.


4. Name a second pair of overlapping triangles in Exercise 2.
5. Name a second pair of overlapping triangles in Exercise 3.

Use the Common Segment Theorem and the Common Angle Theorem to complete each statement.



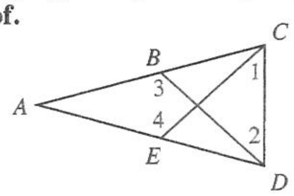
6. If $\angle 1 \cong \angle 4$, then $\angle BGE \cong$ ___.
7. If $\overline{BE} \cong \overline{FC}$, then $\overline{FE} \cong$ ___.
8. If $\angle BAE \cong \angle FAC$, then $\angle FAE \cong$ ___.
9. If $\angle 5 \cong \angle 8$, then $\angle BAE \cong$ ___.

Redraw each figure, separating the overlapping triangles that you plan to use in the proof. Write a two-column proof.

10. Given: $\overline{BC} \cong \overline{ED}$, $\angle ADC \cong \angle ACD$
Prove: $\overline{BD} \cong \overline{CE}$

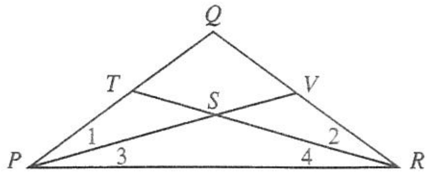
11. Given: $\angle ADC \cong \angle ACD$, $\angle 1 \cong \angle 2$
Prove: $\overline{BC} \cong \overline{ED}$

12. Given: $\overline{BC} \cong \overline{ED}$, $\overline{BD} \cong \overline{EC}$
Prove: $\angle 3 \cong \angle 4$



13. Given: $\angle 1 \cong \angle 2$, $\overline{PQ} \cong \overline{RQ}$
Prove: $\overline{QT} \cong \overline{QV}$

14. Given: $\overline{PT} \cong \overline{RV}$, $\overline{TR} \cong \overline{VP}$
Prove: $\angle 3 \cong \angle 4$

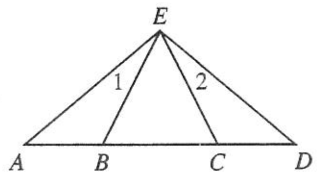


B

15. $AB = CD$, $AC = 3x + 5$, $BD = 4x - 3$, $BC = x + 2$
Find AB and CD.

16. $\angle 1 \cong \angle 2$, $m\angle AEC = 14x - 3$, $m\angle DEB = 17x - 15$,
$AE = 4x + 7$, $DE = 3x + 11$, $BE = 3x + 7$, $CE = 5x - 1$
Show that $\triangle AEC \cong \triangle DEB$.

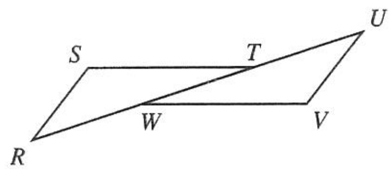
17. $\angle AEC \cong \angle DEB$, $\triangle AEB \cong \triangle DEC$, $m\angle 1 = 5x - 3$,
$m\angle 2 = 3x + 5$, $AC = 3x - 1$
Find DB.



Write a two-column proof for each exercise.

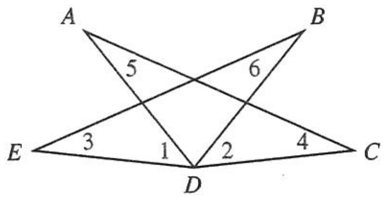
18. Given: $\overline{RS} \cong \overline{UV}$
$\overline{ST} \cong \overline{VW}$
$\angle S \cong \angle V$
Prove: $\overline{RW} \cong \overline{UT}$

19. Given: $\overline{ST} \cong \overline{VW}$
$\angle S \cong \angle V$
$\angle STR \cong \angle VWU$
Prove: $\overline{RS} \parallel \overline{UV}$



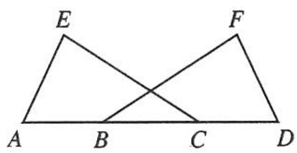
20. Given: $\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$
$\overline{ED} \cong \overline{CD}$
Prove: $\overline{EB} \cong \overline{CA}$

21. Given: $\angle 1 \cong \angle 2$
$\overline{AD} \cong \overline{BD}$
$\overline{ED} \cong \overline{CD}$
Prove: $\angle 5 \cong \angle 6$

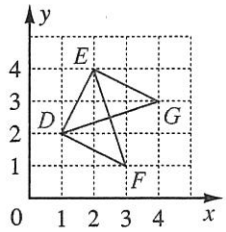


22. Given: $\overline{AB} \cong \overline{DC}$
$\angle A \cong \angle D$
$\overline{AE} \cong \overline{DF}$
Prove: $\overline{EC} \cong \overline{FB}$

23. Given: $\overline{AE} \cong \overline{DF}$
$\angle E \cong \angle F$
$\overline{EC} \cong \overline{FB}$
Prove: $\overline{AB} \cong \overline{CD}$



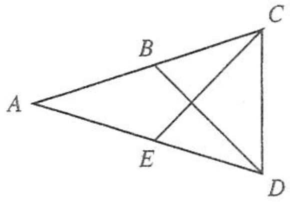
24. Show that $\angle F \cong \angle G$.



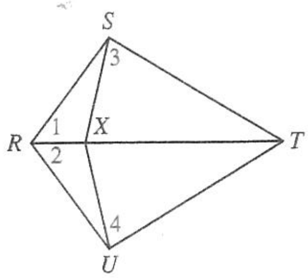
C

Write a two-column proof for each exercise.

25. Given: $\overline{BC} \cong \overline{ED}$
$\angle ADC \cong \angle ACD$
Prove: $\overline{AB} \cong \overline{AE}$

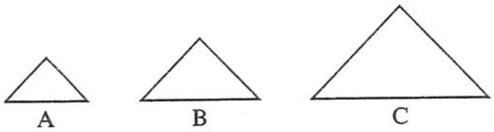


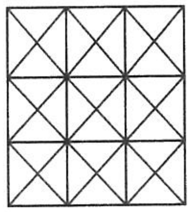
26. Given: $\overline{RS} \cong \overline{RU}$, $\angle 1 \cong \angle 2$
Prove: $\angle 3 \cong \angle 4$



Critical Thinking

27. How many triangles in the figure to the right are congruent to triangle A? triangle B? triangle C?





Algebra Review

1. $\triangle KAW$ is equiangular. $m\angle K = x^2 + 10x + 4$, $m\angle A = x^2 - x + 48$
$m\angle W = 2x^2 + 5x + 8$ Find x.

2. $\triangle JOR$, $\overline{JO} \cong \overline{JR}$, $JO = 2x + 1$, $JR = 4x - 15$, $OR = 3x - 4$
Find OR.

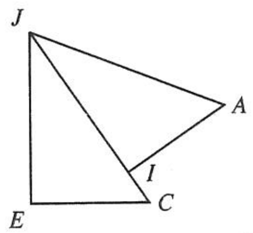
3. $\triangle PSN$, $\angle S \cong \angle N$, $m\angle P = 3x^2 - 4x - 11$, $m\angle S = x^2 - 2x + 1$
$m\angle N = 2x^2 - 6x - 20$ Find $m\angle P$.

Quiz

Write a two-column proof for each exercise.

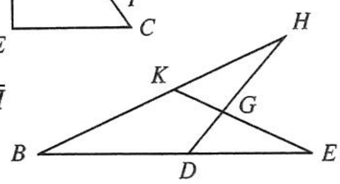
1. **Given:** $\overline{JC}$ bisects $\angle EJA$.
$\overline{JE} \cong \overline{JI}$, $\angle E \cong \angle JIA$
Prove: $\overline{EC} \cong \overline{IA}$

2. **Given:** $\overline{JE} \cong \overline{JI}$, $\overline{JC} \cong \overline{JA}$
$\overline{EC} \cong \overline{IA}$
Prove: $\overline{JC}$ bisects $\angle EJA$.



3. **Given:** $\overline{BH} \cong \overline{BE}$, $\overline{BK} \cong \overline{BD}$
Prove: $\angle BDH \cong \angle BKE$

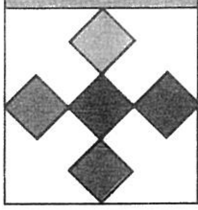
4. **Given:** $\overline{BK} \cong \overline{KE} \cong \overline{BD} \cong \overline{DH}$
$\angle BDH \cong \angle BKE$
Prove: $\angle E \cong \angle H$



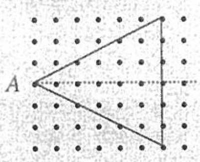
OBJECTIVE: *Prove and apply the theorems about isosceles triangles.*

4-6 Isosceles Triangles

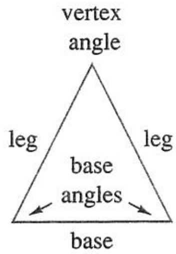
EXPLORE



Use square dot paper or graph paper to draw a large triangle with two sides equal in length. Fold the dot paper with the fold line through the vertex A and perpendicular to the side opposite A. (This fold line is the reflection line for a reflection transformation.) What do you discover about the angles of this triangle? Repeat for a variety of triangle shapes and state any generalizations that you discover.



You have learned that an isosceles triangle has a pair of congruent sides. The congruent sides are called **legs** and the third side is called the **base.** The angles opposite the legs are called **base angles** and the third angle is called the **vertex angle.**



The following theorem describes a relationship between the legs and the base angles of an isosceles triangle. Note the use of the five steps in proving a theorem as discussed in Chapter 2.

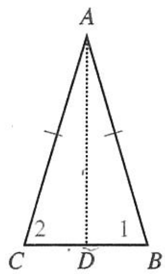
THEOREM 4.1 Isosceles Triangle Theorem

If two sides of a triangle are congruent then the angles opposite those sides are congruent.

Given: $\overline{AB} \cong \overline{AC}$

Prove: $\angle 1 \cong \angle 2$

Plan Let D be the midpoint of $\overline{BC}$ and draw the auxiliary line $\overline{AD}$. Then prove that $\triangle ABD \cong \triangle ACD$ using the SSS Postulate and conclude that $\angle 1 \cong \angle 2$.



Proof

Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	**1.** Given
2. D is the midpoint of $\overline{BC}$.	**2.** A segment has exactly one midpoint.
3. $\overline{BD} \cong \overline{CD}$	**3.** A midpoint forms two $\cong$ segments.
4. $\overline{AD} \cong \overline{AD}$	**4.** Reflexive Property
5. $\triangle ABD \cong \triangle ACD$	**5.** SSS Postulate
6. $\angle 1 \cong \angle 2$	**6.** Corr. parts of $\cong$ $\triangle$s are $\cong$.

Sometimes a theorem can be stated concisely in a form that is not if-then form. Theorem 4.1 could have been stated as follows.
The base angles of an isosceles triangle are congruent.

Remember that a theorem that follows directly from another theorem is called a corollary. Below are several corollaries of Theorem 3.1. You will be asked to prove these corollaries in Exercises 30–31.

COROLLARY 4.1a

If a triangle is equilateral then it is equiangular.

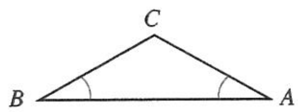
COROLLARY 4.1b

The segment from the vertex of an isosceles triangle to the midpoint of the base bisects the vertex angle.

Recall that the converse of a conditional statement "if p, then q" is the statement "if q, then p." The next theorem is the converse of Theorem 4.1.

THEOREM 4.2

If two angles of a triangle are congruent, then the sides opposite them are congruent.



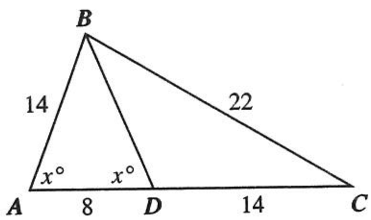
You will be asked to prove Theorem 4.2 in Exercise 37.

COROLLARY 4.2a

If a triangle is equiangular, then the triangle is equilateral.

Example

Find BD and $m\angle ABC$.



Solution

$BD = 14$ — *Since $\angle BAD \cong \angle BDA$, then $AB = BD$ by Theorem 4.2.*

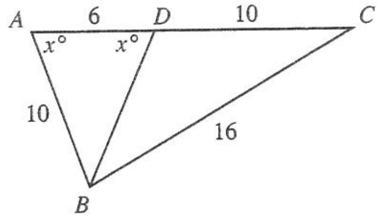
$m\angle ABC = x$ — *Since $AC = 8 + 14 = 22$ and $BC = 22$, then $\angle A \cong \angle ABC$ by Theorem 4.1.*

Try This

Explain why $\angle DBC \cong \angle DCB$.

Class Exercises

Short Answer

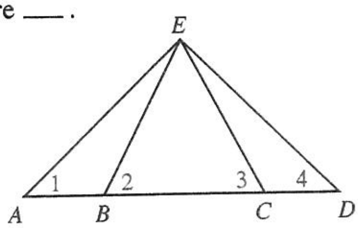


1. Name the legs of $\triangle ABD$.
2. Name the base angles of $\triangle ABC$.
3. Name the base angles of $\triangle ABD$.
4. Name the legs of $\triangle ABC$.
5. Name the vertex angle of $\triangle ABC$.
6. Name the vertex angle of $\triangle ABD$.

Sample Exercises

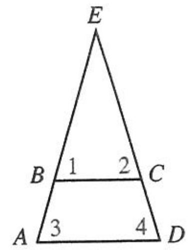
Complete each statement based on the theorems in this lesson.

7. The base angles of an isosceles triangle are ___.
8. If two angles of a triangle are congruent, then ___.
9. If all three sides of a triangle are congruent then ___.
10. If all three angles of a triangle are congruent then ___.
11. The legs of an isosceles triangle are ___.



12. If $\overline{AE} \cong \overline{DE}$, then ___ $\cong$ ___.
13. If $\angle 2 \cong \angle 3$, then ___ $\cong$ ___.
14. If $\overline{BE} \cong \overline{CE}$, then ___ $\cong$ ___.
15. If $\angle 1 \cong \angle 4$, then ___ $\cong$ ___.

Discussion

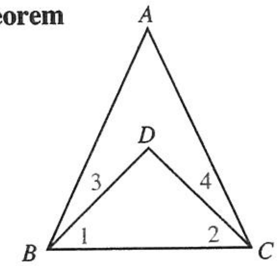


16. In the figure to the right $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. Give a convincing argument that $AB = CD$.

Exercises

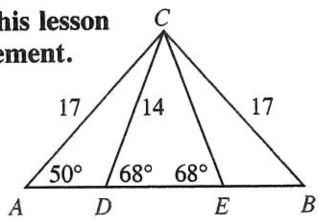
A

Complete each of the following statements. State the theorem that you used.



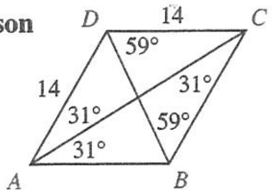
1. If $\angle 1 \cong \angle 2$, then ___ $\cong$ ___.
2. If $\angle ABC \cong \angle ACB$, then ___ $\cong$ ___.
3. If $\overline{DB} \cong \overline{DC}$, then ___ $\cong$ ___.
4. If $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$, then ___ $\cong$ ___.
5. If $\overline{AB} \cong \overline{AC}$, then ___ $\cong$ ___.

Use the theorems in this lesson to complete each statement.



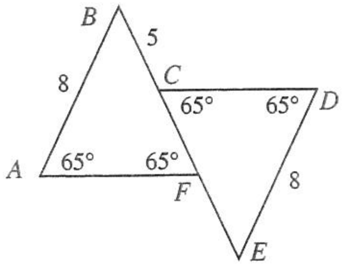
6. $m\angle ABC =$ ___
7. $CE =$ ___

Use the theorems in this lesson to complete each statement.



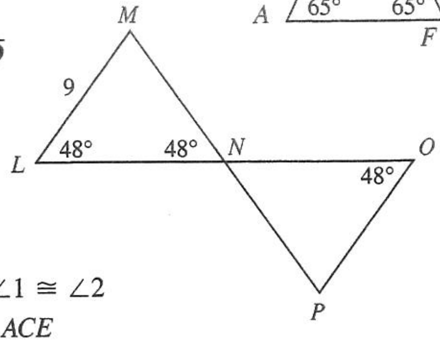
8. $BC =$ __
9. $AB =$ __
10. $m\angle ACD =$ __
11. $m\angle ADB =$ __
12. $m\angle ABC =$ __

Use $\triangle ABF$ and $\triangle CED$ in the figure.



13. $CF =$ __
14. $FE =$ __

***N* is the midpoint of $\overline{LO}$ in the figure.**

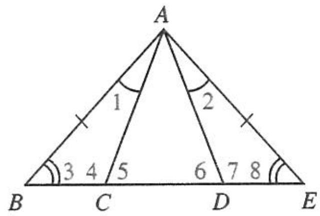


15. Find MN.
16. Find PN.
17. Find PO.

Complete each proof.

18. **Given:** $\overline{AC} \cong \overline{AD}$, $\angle 1 \cong \angle 2$

Prove: $\triangle ADB \cong \triangle ACE$



Statements	Reasons
1. $\overline{AC} \cong \overline{AD}$	1. __
2. $\angle 5 \cong \angle 6$	2. __
3. $\angle 1 \cong \angle 2$	3. __
4. $\angle BAD \cong \angle EAC$	4. Common Angle Theorem
5. $\triangle ADB \cong \triangle ACE$	5. __

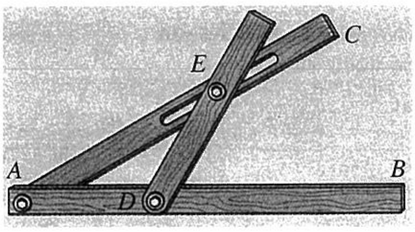
19. Use the figure in Exercise 18.

Given: $\overline{AB} \cong \overline{AE}$, $\angle 1 \cong \angle 2$

Prove: $\triangle ACD$ is isosceles.

Statements	Reasons
1. $\overline{AB} \cong \overline{AE}$	1. __
2. $\angle 3 \cong \angle 8$	2. __
3. $\angle 1 \cong \angle 2$	3. __
4. $\triangle ABC \cong \triangle AED$	4. __
5. $\angle 4 \cong \angle 7$	5. __
6. $\angle 5 \cong \angle 6$	6. Supplements of $\cong \angle$s are $\cong$.
7. $\overline{AD} \cong \overline{AC}$	7. __
8. $\triangle ACD$ is isosceles.	8. __

20. The device to the right can be used to determine one half of any angle. Point E is allowed to move along the slot so that $AD = DE$. Explain why the measure of $\angle EAD$ is always one half the measure of $\angle BDE$.

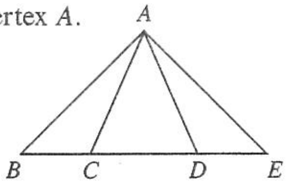


B

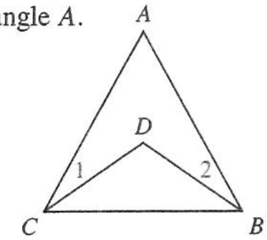
21. $\triangle ABC$ is isosceles with $AB = BC$. If $AB = 4x$ and $BC = 6x - 15$, find AB and BC.
22. $\triangle DEF$ is isosceles with base $\overline{DF}$. If $DE = 4x + 15$, $EF = 2x + 45$, and $DF = 3x + 15$, find the lengths of the sides of the triangle.
23. In $\triangle XYZ$, $XY = YZ$. If $m\angle X = 4x + 60$, $m\angle Y = 2x + 30$, and $m\angle Z = 14x + 30$, find $m\angle X$, $m\angle Y$, and $m\angle Z$.
24. In $\triangle ABC$, $\angle A \cong \angle C$. If $AB = 4x + 25$, $BC = 2x + 45$, and $AC = 3x - 15$, find the lengths of the three sides.

Write a two-column proof for each exercise.

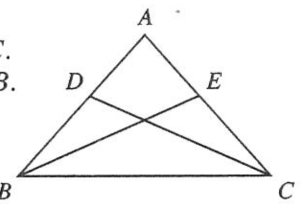
25. **Given:** $\triangle ADC$ is isosceles with vertex A.
$\overline{BC} \cong \overline{ED}$
Prove: $\triangle ABE$ is isosceles.



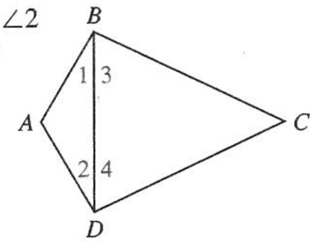
26. **Given:** $\triangle ABC$ is isosceles with vertex angle A.
$\angle 1 \cong \angle 2$
Prove: $\triangle BCD$ is isosceles.



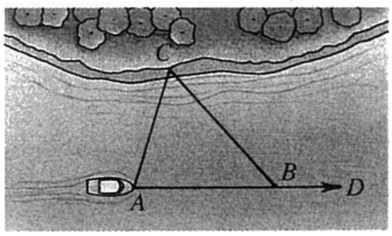
27. **Given:** $\overline{AB} \cong \overline{AC}$
$\overline{BE}$ bisects $\angle ABC$.
$\overline{CD}$ bisects $\angle ACB$.
Prove: $\overline{AD} \cong \overline{AE}$



28. **Given:** $\angle ABC \cong \angle ADC$, $\angle 1 \cong \angle 2$
Prove: $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$



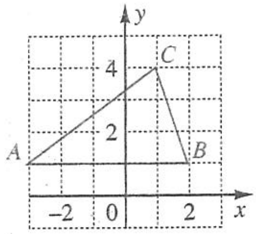
29. If a ship is moving in the direction $\overrightarrow{AD}$, its distance from a point on shore can be determined in the following way. At point A, $m\angle DAC$ can be determined. At some point B along $\overrightarrow{AD}$, $m\angle DBC$ is twice $m\angle DAC$. The distance AB, which can be determined by the ship's log, is the same as the distance BC. Explain why $m\angle DBC = 2m\angle DAC$ means that $AB = BC$.



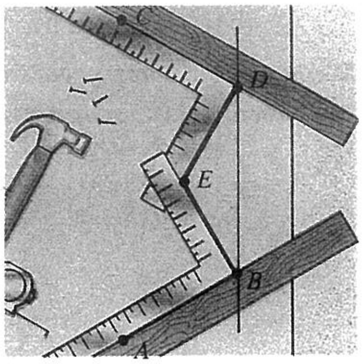
30. Prove Corollary 4.1a.

31. Prove Corollary 4.1b.

32. Use the distance formula and conclude that $m\angle ABC = m\angle ACB$.



33. Two converging timbers are to be cut along a line as shown so that $m\angle CDB = m\angle ABD$. Two carpenter's squares are positioned so that $BE = DE$. (A carpenter's square has a right-angle corner.)
Prove that $\angle CDB \cong \angle ABD$.

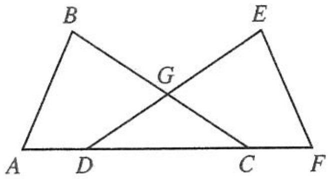


C

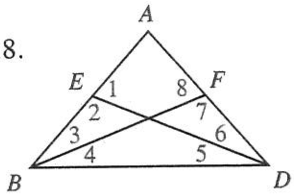
34. Suppose that $\triangle ABC$ is an isosceles triangle with vertex angle A and that D is a point on side $\overline{BC}$. Prove that $\overline{AD}$ bisects $\angle A$ if D is the midpoint of $\overline{BC}$.

35. **Given:** $\overline{AD} \cong \overline{FC}$, $\overline{AB} \cong \overline{FE}$
$\angle A \cong \angle F$

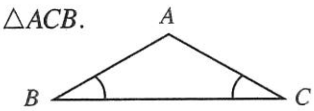
Prove: $\triangle CDG$ is isosceles.



36. $\angle ABD \cong \angle ADB$, $\overline{BF}$ and $\overline{DE}$ bisect $\angle ABD$ and $\angle ADB$ respectively. Prove that $\angle 1 \cong \angle 8$.

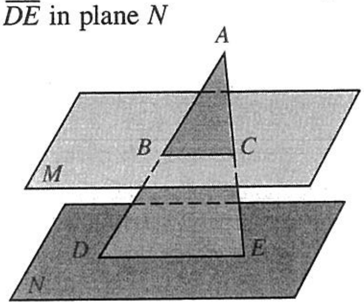


37. Given that, in $\triangle ABC$, $\angle B \cong \angle C$. Prove that $\triangle ABC \cong \triangle ACB$. Explain why you have just proven Theorem 4.2.



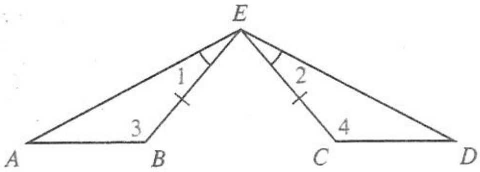
38. **Given:** $M \parallel N$, $\overline{BC}$ in plane M, $\overline{DE}$ in plane N
$m\angle ABC = m\angle ACB$

Prove: $\triangle ADE$ is isosceles.

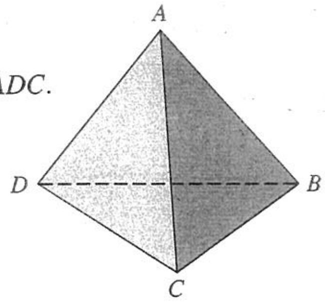


Critical Thinking

39. Suppose that $\overline{EB} \cong \overline{EC}$ and $\angle 1 \cong \angle 2$. Consider the following argument. Since $\overline{EB} \cong \overline{EC}$, $\triangle EBC$ is an isosceles triangle with the base angles at B and C congruent. This means that $\angle 3 \cong \angle 4$ since they are supplements of congruent angles and $\triangle ABE \cong \triangle DCE$. Therefore, $\overline{AE} \cong \overline{DE}$ since they are corresponding segments in congruent triangles. Consequently $\triangle AED$ is an isosceles triangle. Is this a convincing argument? Why or why not?

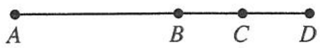


40. For pyramid $ABCD$, $m\angle ABC \cong \angle ACB \cong \angle ACD \cong \angle ADC$. What can you deduce about $\triangle$ABD?
41. Under what conditions is $\triangle ABD$ an equilateral triangle?



Mixed Review

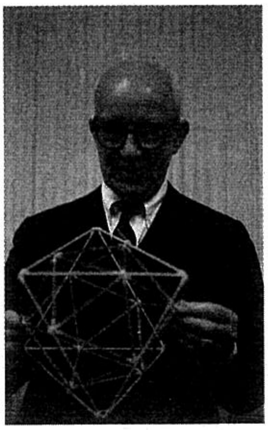
1. The ratio of the measures of the angles of a triangle is $13:15:17$. Find the measures of the angles and classify the triangle as acute, right, or obtuse.
2. The ratio of the measures of the acute angles of a right triangle is $7:8$. Find the measures of the two acute angles.
3. $BC = CD$, $BD = AB - 1$, $AD = 9$ Find AB.



Biographical Note

R. Buckminster Fuller (1895–1983)

R. Buckminster Fuller believed in using knowledge and technology to solve world problems. As an engineer, he designed fuel-efficient, omnidirectional automobiles. As a cartographer, he developed a map that projects the world onto a flat surface with almost no distortions. As an educator, he taught at Southern Illinois University. A poet and philosopher, Fuller described his views on the optimum use of energy resources in several books.



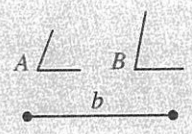
Fuller's goal of doing more with less is apparent in his architectural designs. His Dymaxion house was factory-assembled and easily transported. Thousands of examples of his geodesic domes exist throughout the world. Based on his system of geometry, these domes join networks of triangles into frameworks for lightweight, but extremely strong, spheres—the most efficient way to enclose space. Fuller envisioned domes over entire cities, allowing their inhabitants total control over the environment.

OBJECTIVE: *Use the AAS Congruence Postulate in proofs. Use the HL and HA Congruence Postulates in proofs.*

4-7 AAS Congruence and Right Triangle Congruence

EXPLORE

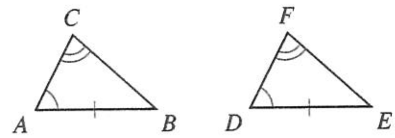
Given angles A and B and a segment with length b. Use a protractor and ruler to draw a triangle DEF with $\angle D$ congruent to the given $\angle A$, $\angle E$ congruent to the given $\angle B$, and whose side opposite $\angle E$ has length b.



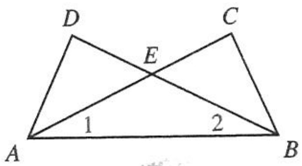
You have studied the SSS, SAS, and ASA methods of proving triangles congruent. The above Explore suggests the AAS method.

THEOREM 4.3 AAS Congruence

If two angles and a side opposite one of them in one triangle are congruent to two angles and a side opposite one of them of another triangle, then the two triangles are congruent.



You will be asked to prove the AAS Theorem in Exercise 27. It is used to complete the proof of this example.



Example 1

Given: $\angle 1 \cong \angle 2$, $\angle C \cong \angle D$

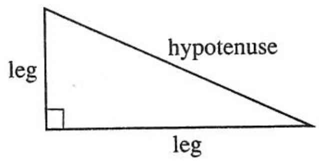
Prove: $\overline{AD} \cong \overline{BC}$

Solution

Statements	Reasons
1. $\angle 1 \cong \angle 2$, $\angle C \cong \angle D$	**1.** Given
2. $\overline{AB} \cong \overline{AB}$	**2.** Reflexive
3. $\triangle ABC \cong \triangle BAD$	**3.** AAS Congruence Theorem
4. $\overline{AD} \cong \overline{BC}$	**4.** Corr. parts of $\cong$ $\triangle$s are $\cong$.

In addition to the SSS, SAS, ASA, and AAS methods for proving triangles congruent, there are two methods that apply to right triangles only. The HA Congruence Theorem is a consequence of the AAS Congruence Theorem and will be proven in Exercise 28.

Recall that in a right triangle, the side opposite the right angle is called the *hypotenuse* and the other two sides are called *legs*.



◆ THEOREM 4.4 HA Congruence

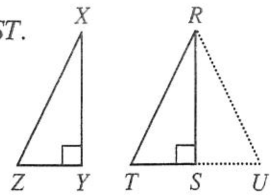
If the hypotenuse and one acute angle of a right triangle are congruent to the hypotenuse and one acute angle of another right triangle, then the triangles are congruent.

◆ THEOREM 4.5 HL Congruence

If the hypotenuse and a leg of one right triangle are congruent to the corresponding parts of a second right triangle, then the triangles are congruent.

Given: $\angle Y$ and $\angle RST$ are right $\angle$s in $\triangle XYZ$ and $\triangle RST$.
$\overline{XZ} \cong \overline{RT}$, $\overline{XY} \cong \overline{RS}$

Prove: $\triangle XYZ \cong \triangle RST$



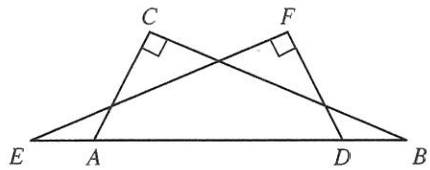
Plan I will construct $\triangle RUS$, with U selected on $\overline{TS}$ so that $\overline{SU} \cong \overline{YZ}$. If I can show $\triangle XYZ \cong \triangle RSU$, then I can conclude that $\overline{ZX} \cong \overline{UR}$ and that $\triangle TRU$ is isosceles. Consequently $\angle T \cong \angle U$ by Theorem 4.1 and $\angle U \cong \angle Z$ by corr. parts of $\cong$ $\triangle$s are $\cong$. Therefore $\angle T \cong \angle Z$ and I can conclude that $\triangle XYZ \cong \triangle RST$ by HA Congruence.

Example 2

Complete the proof.

Given: $\angle C$ and $\angle F$ are right angles in $\triangle ACB$ and $\triangle DFE$.
$\overline{AE} \cong \overline{DB}$, $\overline{AC} \cong \overline{DF}$

Prove: $\overline{EF} \cong \overline{BC}$



Proof

Statements	Reasons
1. $\angle C$ and $\angle F$ are rt. $\angle$s.	1. Given
2. $\triangle DEF$ and $\triangle ABC$ are rt. $\triangle$s.	2. ___
3. $\overline{AE} \cong \overline{DB}$, $\overline{AC} \cong \overline{DF}$	3. ___
4. $\overline{ED} \cong \overline{BA}$	4. ___
5. $\triangle DEF \cong \triangle ABC$	5. ___
6. $\overline{EF} \cong \overline{BC}$	6. Corr. parts of $\cong$ $\triangle$s are $\cong$.

Solution

2. Definition of a right triangle
3. Given
4. Common Segment Theorem
5. HL Theorem

Six ways to prove triangles congruent have been presented in this chapter. Four ways apply to all triangles and two ways apply to right triangles only.

Summary of Ways To Prove Triangles Congruent

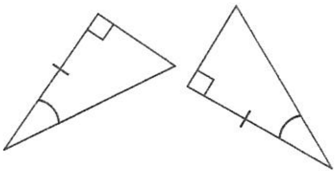
All triangles	SSS	SAS	ASA	AAS
Right triangles	HL	HA		

Class Exercises

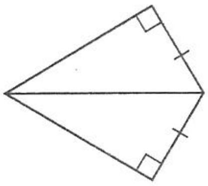
Short Answer

Indicate whether the given pair of triangles is congruent by SSS, SAS, ASA, AAS, HA, HL, or none of these.

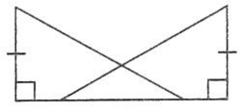
1.



2.



3.



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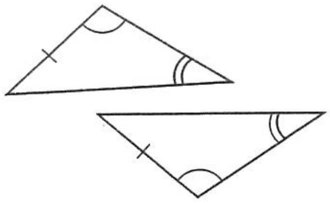

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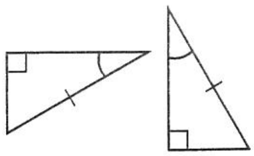

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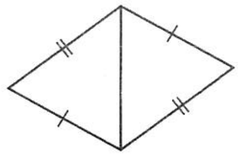
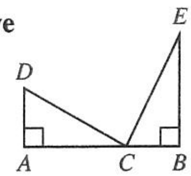


Sample Exercises

$\overline{CD} \cong \overline{CE}$ State the additional information you need to prove $\triangle ACD \cong \triangle BEC$ by the given method.



7. HA 8. HL 9. SSS
10. AAS 11. ASA 12. SAS

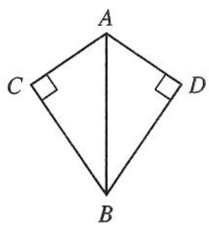
Discussion

13. Formulate a theorem that you would call the LL Congruence Theorem for right triangles. Do you think it is true? Why?
14. Formulate a theorem that you would call the LA Congruence Theorem for right triangles. Do you think it is true? Why?

Exercises

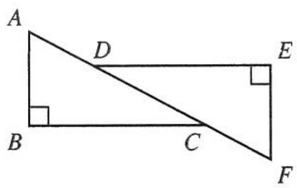
A

State the additional information you need to prove $\triangle ABC \cong \triangle ABD$ by the given method.



1. HA 2. HL 3. AAS
4. SAS 5. ASA 6. SSS

State the additional information you need for the given method to prove that $\triangle ABC \cong \triangle FED$ if $\overline{AD} \cong \overline{CF}$.

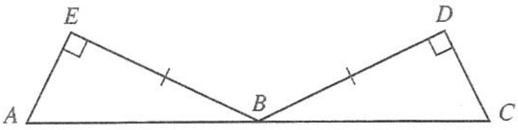


7. HA 8. ASA 9. HL

10. Complete the proof.

Given: B is the midpoint of $\overline{AC}$.
$\angle E$ and $\angle D$ are right $\angle$s. $\overline{EB} \cong \overline{DB}$

Prove: $\angle A \cong \angle C$



Statements	Reasons
1. $\angle E$, $\angle D$ are rt. $\angle$s.	1. ___
2. $\triangle ABC$ and $\triangle CBD$ are rt. $\triangle$s.	2. ___
3. B is the midpoint of $\overline{AC}$.	3. ___
4. $\overline{AB} \cong \overline{CB}$	4. ___
5. $\overline{EB} \cong \overline{DB}$	5. ___
6. $\triangle ABE \cong \triangle CBD$	6. ___
7. $\angle A \cong \angle C$	7. ___

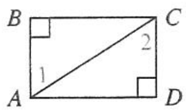
Write a two-column proof for each exercise.

11. **Given:** $\angle B$ and $\angle D$ are rt. $\angle$s.
$\overline{AB} \cong \overline{CD}$

Prove: $\triangle ABC \cong \triangle CDA$

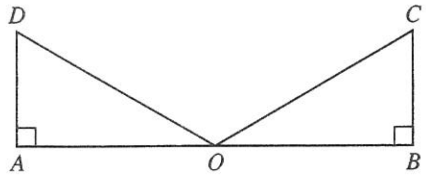
12. **Given:** $\angle B$ and $\angle D$ are rt. $\angle$s.
$\angle 1 \cong \angle 2$

Prove: $\triangle ABC \cong \triangle CDA$



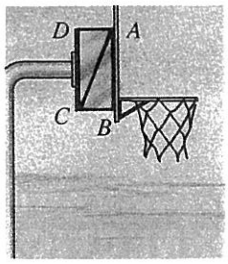
Use each given to prove the right triangles $\triangle AOD$ and $\triangle BOC$ congruent.

13. O is the midpoint of $\overline{AB}$.
$\angle A$ and $\angle B$ are rt. $\angle$s. $\overline{OD} \cong \overline{OC}$
14. $\angle A$ and $\angle B$ are rt. $\angle$s. $\overline{OC} \cong \overline{OD}$, $\angle D \cong \angle C$
15. O is the midpoint of $\overline{AB}$.
$\angle A$ and $\angle B$ are rt. $\angle$s. $\angle AOD \cong \angle BOC$



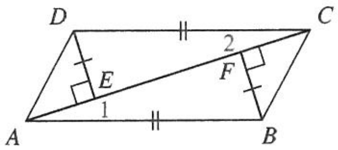
B

16. A basketball backboard is mounted on a wall with a metal bracket $ABCD$ so that $\angle B$ and $\angle D$ are right angles and $AB = CD$. Show that the two corners A and B of the backboard are equidistant from the wall.



$\overline{DE} \perp \overline{AC}$, $\overline{BF} \perp \overline{AC}$, $\overline{AB} \cong \overline{CD}$, $\overline{DE} \cong \overline{BF}$

17. Prove that $\angle 1 \cong \angle 2$.
18. Prove that $\overline{AD} \cong \overline{CB}$.
19. Prove that $\overline{AE} \cong \overline{CF}$.

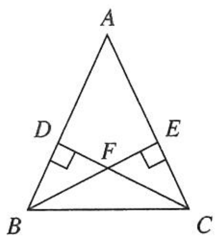


20. **Given:** $\overline{CD} \perp \overline{AB}$, $\overline{BE} \perp \overline{AC}$, $\triangle ABC$ is isosceles.

Prove: $\triangle BFC$ is isosceles.

21. **Given:** $\overline{CD} \perp \overline{AB}$, $\overline{BE} \perp \overline{AC}$, $\overline{BE} \cong \overline{CD}$

Prove: $\triangle ABC$ is isosceles.



22. $\overline{CD} \perp \overline{AB}$, $\overline{BE} \perp \overline{AC}$, $\overline{CD} \cong \overline{BE}$, $BD = 5x - 7$, $CE = 2x + 14$, $DF = 2x + 5$, $EF = 3x - 2$ Prove that $AEFD$ has two pairs of congruent sides.
23. $\overline{CD} \perp \overline{AB}$, $\overline{BE} \perp \overline{AC}$, $\overline{AC} \cong \overline{AB}$, $AD = 4x - 5$, $AE = 2x + 7$
$m\angle A = 3x + 8$ Find $m\angle A$.
24. A milling cutter with seven teeth is made by cutting seven right triangles out of a seven-sided regular polygon. If $\overline{AB}$ is cut the same length for each tooth, why are the sharp points of the cutter all the same size angle?