CHAPTER 22

Antiderivatives

If $F'(x) = f(x)$, then F is called an *antiderivative* of f.

EXAMPLE 22.1: x^3 is an antiderivative of $3x^2$, since $D_x(x^3) = 3x^2$. But $x^3 + 5$ is also an antiderivative of $3x^2$, since $D_x(5) = 0$.

(I) In general, if $F(x)$ is an antiderivative of $f(x)$, then $F(x) + C$ is also an antiderivative of $f(x)$, where C is any constant.

(II) On the other hand, if $F(x)$ is an antiderivative of $f(x)$, and if $G(x)$ is any other antiderivative of $f(x)$, then $G(x) = F(x) + C$, for some constant C.

Property (II) follows from Problem 13 of Chapter 18, since $F'(x) = f(x) = G'(x)$.

From Properties (I) and (II) we see that, if $F(x)$ is an antiderivative of $f(x)$, then the antiderivatives of $f(x)$ are precisely those functions of the form $F(x) + C$, for an arbitrary constant C.

Notation: $\int f(x)\,dx$ will denote any antiderivative of $f(x)$. In this notation, $f(x)$ is called the *integrand*.

Terminology: An antiderivative $\int f(x)\,dx$ is also called an *indefinite integral*.

An explanation of the peculiar notation $\int f(x)\,dx$ (including the presence of the differential dx) will be given later.

EXAMPLE 22.2: (a) $\int x\,dx = \frac{1}{2}x^2 + C$; (b) $\int -\sin x\,dx = \cos x + C$.

Laws for Antiderivatives

Law 1. $\int 0\,dx = C$.

Law 2. $\int 1\,dx = x + C$.

Law 3. $\int a\,dx = ax + C$.

Law 4. $\int x^r\,dx = \dfrac{x^{r+1}}{r+1} + C$ for any rational number $r \neq -1$.

(4) follows from the fact that $D_x\left(\dfrac{x^{r+1}}{r+1}\right) = x^r$ for $r \neq -1$.

Law 5. $\int af(x)\,dx = a\int f(x)\,dx$.

Note that $D_x\left(a\int f(x)\,dx\right) = aD_x\left(\int f(x)\,dx\right) = af(x)$.

Law 6. $\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$.

Note that $D_x\left(\int f(x)\,dx + \int g(x)\,dx\right) = D_x\left(\int f(x)\,dx\right) + D_x\left(\int g(x)\,dx\right) = f(x) + g(x)$.

Law 7. $\int (f(x) - g(x))\,dx = \int f(x)\,dx - \int g(x)\,dx$.

Note that $D_x\left(\int f(x)\,dx - \int g(x)\,dx\right) = D_x\left(\int f(x)\,dx\right) - D_x\left(\int g(x)\,dx\right) = f(x) - g(x)$.

EXAMPLE 22.3:

(a) $\int \sqrt[3]{x}\,dx = \int x^{1/3}dx = \frac{x^{4/3}}{4/3} + C = \frac{3}{4}x^{4/3} + C$ by Law (4).

(b) $\int \frac{1}{x^2}dx = \int x^{-2}dx = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$ by Law (4).

(c) $\int 7x^3dx = 7\int x^3dx = 7\left(\frac{x^4}{4}\right) + C = \frac{7}{4}x^4 + C$ by Laws (5), (4).

(d) $\int (x^2+4)\,dx = \int x^2dx + \int 4\,dx = \frac{1}{3}x^3 + 4x + C$ by Laws (6), (4), and (2).

(e) $\int (3x^6 - 4x)\,dx = \int 3x^6dx - \int 4x\,dx = 3\int x^6\,dx - 4\int x\,dx = 3(\frac{1}{7}x^7) - 4(\frac{1}{2}x^2) + C = \frac{3}{7}x^7 - 2x^2 + C.$

EXAMPLE 22.4: Laws (3)–(7) enable us to compute the antiderivative of any polynomial. For instance,

$$\int (6x^8 - \tfrac{2}{3}x^5 + 7x^4 + \sqrt{3})\,dx = 6(\tfrac{1}{9}x^9) - \tfrac{2}{3}(\tfrac{1}{6}x^6) + 7(\tfrac{1}{5}x^5) + \sqrt{3}x + C$$
$$= \tfrac{2}{3}x^9 - \tfrac{1}{9}x^6 + \tfrac{7}{5}x^5 + \sqrt{3}x + C$$

Law (8). (Quick Formula I)

$$\int (g(x))^r g'(x)\,dx = \frac{1}{r+1}(g(x))^{r+1} + C \quad \text{for any rational number } r \neq -1$$

For verification, $D_x\left(\frac{1}{r+1}(g(x))^{r+1}\right) = \frac{1}{r+1}D_x[(g(x))^{r+1}] = \frac{1}{r+1}(r+1)(g(x))^r g'(x) = (g(x))^r g'(x)$ by the power Chain Rule.

EXAMPLE 22.5: $\int (\frac{1}{3}x^3 + 7)^5 x^2\,dx = \frac{1}{6}(\frac{1}{3}x^3 + 7)^6 + C.$

To see this, let $g(x) = (\frac{1}{3}x^3 + 7)$ and $r = 5$ in Quick Formula I.

EXAMPLE 22.6: $\int (x^2+1)^{2/3}x\,dx = \frac{1}{2}\int (x^2+1)^{2/3}2x\,dx = \frac{1}{2}\left(\frac{1}{5/3}\right)(x^2+1)^{5/3} + C = \frac{3}{10}(x^2+1)^{5/3} + C.$

In this case, we had to insert a factor of 2 in the integrand in order to use Quick Formula I.

Law (9). Substitution Method

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

where u is replaced by $g(x)$ after the right-hand side is evaluated. The "substitution" is carried out on the left-hand side by letting $u = g(x)$ and $du = g'(x)\,dx$. (For justification, see Problem 21.)

EXAMPLE 22.7:

(a) Find $\int x\sin(x^2)\,dx$.

Let $u = x^2$. Then $du = 2x\,dx$. So, $x\,dx = \frac{1}{2}du$. By substitution,

$$\int x\sin(x^2)dx = \int \sin u\,(\tfrac{1}{2})du = \tfrac{1}{2}(-\cos u) + C = -\tfrac{1}{2}\cos(x^2) + C$$

(b) Find $\int \sin(x/2)\,dx$.

Let $u = x/2$. Then $du = \frac{1}{2}dx$. So, $dx = 2\,du$. By substitution,

$$\int \sin\left(\frac{x}{2}\right)dx = \int (\sin u)\,2\,du = 2\int \sin u\,du = 2(-\cos u) + C = -2\cos\left(\frac{x}{2}\right) + C$$

Observe that Quick Formula I is just a special case of the Substitution Method, with $u = g(x)$. The advantage of Quick Formula I is that we save the bother of carrying out the substitution.

The known formulas for derivatives of trigonometric and inverse trigonometric functions yield the following formulas for antiderivatives:

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \tan x \sec x\,dx = \sec x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \cot x \csc x\,dx = -\csc x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C \qquad \text{for } a > 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C \qquad \text{for } a > 0$$

$$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C \qquad \text{for } a > 0$$

SOLVED PROBLEMS

In Problems 1–8, evaulate the antiderivative.

1. $\int x^6 dx = \frac{1}{7}x^7 + C$ [Law (4)]

2. $\int \frac{dx}{x^6} = \int x^{-6} dx = \frac{1}{-5}x^{-5} + C = -\frac{1}{5x^5} + C$ [Law (4)]

3. $\int \sqrt[3]{z}\,dz = \int z^{1/3} dz = \frac{1}{4/3}z^{4/3} + C = \frac{3}{4}(\sqrt[3]{z})^4 + C$ [Law (4)]

4. $\int \frac{1}{\sqrt[3]{x^2}}\,dx = \int x^{-2/3} dx = \frac{1}{1/3}x^{1/3} + C = 3\sqrt[3]{x} + C$ [Law (4)]

5. $\int (2x^2 - 5x + 3)\,dx = 2\int x^2 dx - 5\int x\,dx + \int 3\,dx$

$= 2(\frac{1}{3}x^3) - 5(\frac{1}{2}x^2) + 3x + C = \frac{2}{3}x^3 - \frac{5}{2}x^2 + 3x + C$ [Laws (3)–(7)]

6. $\int(1-x)\sqrt{x}\,dx=\int(1-x)x^{1/2}dx=\int(x^{1/2}-x^{3/2})dx$

$$=\int x^{1/2}dx-\int x^{3/2}dx=\frac{1}{3/2}x^{3/2}-\frac{1}{5/2}x^{5/2}+C$$

$$=\tfrac{2}{3}x^{3/2}-\tfrac{2}{5}x^{5/2}+C=2x^{3/2}(\tfrac{1}{3}-\tfrac{1}{5}x)+C \qquad \text{[Laws (4), (7)]}$$

7. $\int(3s+4)^2\,ds=\int(9s^2+24s+16)\,ds$

$$=9(\tfrac{1}{3}s^3)+24(\tfrac{1}{2}s^2)+16s+C=3s^3+12s^2+16s+C \qquad \text{[Laws (3)–(6)]}$$

Note that it would have been easier to use Quick Formula I:

$$\int(3s+4)^2\,ds=\tfrac{1}{3}\int(3s+4)^2 3\,ds=\tfrac{1}{3}(\tfrac{1}{3}(3s+4)^3)+C=(\tfrac{1}{9})(3s+4)^3+C$$

8. $\int\frac{x^3+5x^2-4}{x^2}\,dx=\int(x+5-4x^{-2})\,dx=\tfrac{1}{2}x^2+5x-4\left(\frac{1}{-1}x^{-1}\right)+C$

$$=\tfrac{1}{2}x^2+5x+\frac{4}{x}+C \qquad \text{[Laws (3)–(7)]}$$

Use Quick Formula I in Problems 9–15.

9. $\int(s^3+2)^2(3s^2)\,ds=\tfrac{1}{3}(s^3+2)^3+C$

10. $\int(x^3+2)^{1/2}x^2dx=\tfrac{1}{3}\int(x^3+2)^{1/2}3x^2dx=\tfrac{1}{3}\left(\frac{1}{3/2}(x^3+2)^{3/2}\right)+C=\tfrac{2}{9}(x^3+2)^{3/2}+C$

11. $\int\frac{8x^2}{(x^3+2)^3}\,dx=\tfrac{8}{3}\int(x^3+2)^{-3}3x^2dx=\tfrac{8}{3}\left(\frac{1}{-2}(x^3+2)^{-2}\right)+C=-\frac{4}{3}\frac{1}{(x^3+2)^2}+C$

12. $\int\frac{x^2dx}{\sqrt[4]{x^3+2}}=\tfrac{1}{3}\int(x^3+2)^{-1/4}3x^2dx=\tfrac{1}{3}\left(\frac{1}{3/4}(x^3+2)^{3/4}\right)+C=\tfrac{4}{9}(x^3+2)^{3/4}+C$

13. $\int 3x\sqrt{1-2x^2}\,dx=-\tfrac{3}{4}\int -4x\sqrt{1-2x^2}\,dx$

$$=-\tfrac{3}{4}\int -4x(1-2x^2)^{1/2}dx=-\tfrac{3}{4}\left(\frac{1}{3/2}(1-2x^2)^{3/2}\right)+C$$

$$=-\tfrac{1}{2}(1-2x^2)^{3/2}+C$$

14. $\int\sqrt[3]{1-x^2}\,x\,dx=-\tfrac{1}{2}\int(1-x^2)^{1/3}(-2x)\,dx$

$$=-\tfrac{1}{2}\left(\frac{1}{4/3}(1-x^2)^{4/3}\right)+C=-\tfrac{3}{8}(1-x^2)^{4/3}+C$$

15. $\int\sin^2 x\cos x\,dx=\int(\sin x)^2\cos x\,dx=\tfrac{1}{3}(\sin x)^3+C=\tfrac{1}{3}\sin^3 x+C$

In Problems 16–18, use the Substitution Method.

16. $\int\frac{\cos\sqrt{x}}{\sqrt{x}}\,dx.$

Let $u=\sqrt{x}=x^{1/2}$. Then $du=\tfrac{1}{2}x^{-1/2}dx$. So, $2\,du=\frac{1}{\sqrt{x}}dx$. Thus,

$$\int\frac{\cos\sqrt{x}}{\sqrt{x}}\,dx=2\int\cos u\,du=2\sin u+C=2\sin(\sqrt{x})+C$$

17. $\int x\sec^2(4x^2-5)\,dx$.

Let $u = 4x^2 - 5$. Then $du = 8x\,dx$, $\frac{1}{8}du = x\,dx$. Thus,

$$\int x\sec^2(4x^2-5)\,dx = \tfrac{1}{8}\int \sec^2 u\,du = \tfrac{1}{8}\tan u + C = \tfrac{1}{8}\tan(4x^2-5)+C$$

18. $\int x^2\sqrt{x+1}\,dx$.

Let $u = x + 1$. Then $du = dx$ and $x = u - 1$. Thus,

$$\int x^2\sqrt{x+1}\,dx = \int (u-1)^2\sqrt{u}\,du = \int (u^2-2u+1)u^{1/2}du$$

$$= \int (u^{5/2}-2u^{3/2}+u^{1/2})\,du = \tfrac{2}{7}u^{7/2} - 2(\tfrac{2}{5})u^{5/2} + \tfrac{2}{3}u^{3/2} + C$$

$$= 2u^{3/2}(\tfrac{1}{7}u^2 - \tfrac{2}{5}u + \tfrac{1}{3}) + C$$

$$= 2(x+1)^{3/2}[\tfrac{1}{7}(x+1)^2 - \tfrac{2}{5}(x+1) + \tfrac{1}{3}] + C$$

19. A stone is thrown straight up from the ground with an initial velocity of 64 ft/sec. (a) When does it reach its maximum height? (b) What is its maximum height? (c) When does it hit the ground? (d) What is its velocity when it hits the ground?

In free-fall problems, $v = \int a\,dt$ and $s = \int v\,dt$ because $a = \frac{dv}{dt}$ and $v = \frac{ds}{dt}$. Since $a = -32$ ft/sec^2,

$$v = \int -32\,dt = -32t + C_1$$

Letting $t = 0$, we see that $C_1 = v_0$, the initial velocity at $t = 0$. Thus, $v = -32t + v_0$. Hence,

$$s = \int (-32t + v_0)\,dt = -16t^2 + v_0 t + C_2$$

Letting $t = 0$, we see that $C_2 = s_0$, the initial position at $t = 0$. Hence

$$s = -16t^2 + v_0 t + s_0$$

In this problem, $s_0 = 0$ and $v_0 = 64$. So,

$$v = -32t + 64,\ s = -16t^2 + 64t$$

(a) At the maximum height, $\frac{ds}{dt} = v = 0$. So, $-32t + 64 = 0$ and, therefore, $t = 2$ seconds.
(b) When $t = 2$, $s = -16(2)^2 + 64(2) = 64$ ft, the maximum height.
(c) When the stone hits the ground, $0 = s = -16t^2 + 64t$. Dividing by t, $0 = -16t + 64$ and, therefore, $t = 4$.
(d) When $t = 4$, $v = -32(4) + 64 = -64$ ft/sec.

20. Find an equation of the curve passing through the point (3, 2) and having slope $5x^2 - x + 1$ at every point (x, y).

Since the slope is the derivative, $dy/dx = 5x^2 - x + 1$. Hence,

$$y = \int (5x^2 - x + 1)\,dx = \tfrac{5}{3}x^3 - \tfrac{1}{2}x^2 + x + C$$

Since (3, 2) is on the curve, $2 = \frac{5}{3}(3)^3 - \frac{1}{2}(3)^2 + 3 + C = 45 - \frac{9}{2} + 3 + C$. So, $C = -\frac{83}{2}$. Hence, an equation of the curve is

$$y = \tfrac{5}{3}x^3 - \tfrac{1}{2}x^2 + x - \tfrac{83}{2}$$

21. Justify the Substitution Method: $\int f(g(x))g'(x)dx = \int f(u)\,du$.

Here, $u = g(x)$ and $du/dx = g'(x)$. By the Chain Rule,

$$D_x\left(\int f(u)\,du\right) = D_u\left(\int f(u)\,du\right)\cdot\frac{du}{dx} = f(u)\cdot\frac{du}{dx} = f(g(x))\cdot g'(x)$$

SUPPLEMENTARY PROBLEMS

In Problems 22–44, evaluate the given antiderivative.

22. $\int \frac{(1+x^2)}{\sqrt{x}}\,dx.$ *Ans.* $2x^{1/2}(1+\frac{2}{3}x+\frac{1}{5}x^2)+C$

23. $\int \frac{(x^2+2x)}{(x+1)^2}\,dx$ *Ans.* $\frac{x^2}{x+1}+C$

24. $\int \cos 3x\,dx$ *Ans.* $\frac{1}{3}\sin 3x + C$

25. $\int \frac{\sin y\,dy}{\cos^2 y}$ *Ans.* $\sec y + C$

26. $\int \frac{dx}{1+\cos x}$ (*Hint:* Multiply numerator and denominator by $1 - \cos x$.)

Ans. $-\cot x + \csc x + C$

27. $\int (\tan 2x + \sec 2x)^2\,dx$ *Ans.* $\tan 2x + \sec 2x - x + C$

28. $\int \frac{dx}{\sqrt{4-x^2}}$ *Ans.* $\sin^{-1}\left(\frac{x}{2}\right)+C$

29. $\int \frac{dx}{9+x^2}$ *Ans.* $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

30. $\int \frac{dx}{\sqrt{25-16x^2}}$ (*Hint:* Factor 16 out of the radical.)

Ans. $\frac{1}{4}\sin^{-1}\left(\frac{4x}{5}\right)+C$

31. $\int \frac{dx}{4x^2+9}$ (*Hint:* Either factor 4 out of the denominator or make the substitution $u = 2x$.)

Ans. $\frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)+C$

32. $\int \frac{dx}{x\sqrt{4x^2-9}}$ (*Hint:* Either factor 4 out of the radical or make the substitution $u = 2x$.)

Ans. $\frac{1}{3}\sec^{-1}\left(\frac{2x}{3}\right)+C$

33. $\int \frac{x^2dx}{\sqrt{1-x^6}}$ (*Hint*. Substitute $u = x^3$.) *Ans.* $\frac{1}{3}\sin^{-1}(x^3)+C$

34. $\int \frac{x\,dx}{x^4+3}$ (*Hint:* Substitute $u = x^2$.) *Ans.* $\frac{\sqrt{3}}{6}\tan^{-1}\left(\frac{x^2\sqrt{3}}{3}\right)+C$

35. $\int \frac{dx}{x\sqrt{x^4-1}}$ *Ans.* $\frac{1}{2}\cos^{-1}\left(\frac{1}{x^2}\right)+C$

36. $\int \frac{3x^3 - 4x^2 + 3x}{x^2 + 1} dx$ *Ans.* $\frac{3x^2}{2} - 4x + 4\tan^{-1} x + C$

37. $\int \frac{\sec x \tan x \, dx}{9 + 4\sec^2 x}$ *Ans.* $\frac{1}{6}\tan^{-1}\left(\frac{2\sec x}{3}\right) + C$

38. $\int \frac{(x+3)dx}{\sqrt{1-x^2}}$ *Ans.* $-\sqrt{1-x^2} + 3\sin^{-1} x + C$

39. $\int \frac{dx}{x^2 + 10x + 30}$ *Ans.* $\frac{\sqrt{5}}{5}\tan^{-1}\left(\frac{(x+5)\sqrt{5}}{5}\right) + C$

40. $\int \frac{dx}{\sqrt{20 + 8x - x^2}}$ *Ans.* $\sin^{-1}\left(\frac{x-4}{6}\right) + C$

41. $\int \frac{dx}{2x^2 + 2x + 5}$ *Ans.* $\frac{1}{3}\tan^{-1}\left(\frac{2x+1}{3}\right) + C$

42. $\int \frac{dx}{\sqrt{28 - 12x - x^2}}$ *Ans.* $\sin^{-1}\left(\frac{x+6}{8}\right) + C$

43. $\int \frac{x+3}{\sqrt{5 - 4x - x^2}} dx$ *Ans.* $-\sqrt{5 - 4x - x^2} + \sin^{-1}\left(\frac{x+2}{3}\right) + C$

44. $\int \frac{x+2}{\sqrt{4x - x^2}} dx$ *Ans.* $-\sqrt{4x - x^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C$

In Problems 45–52, use Quick Formula I.

45. $\int (x-2)^{3/2} dx$ *Ans.* $\frac{2}{5}(x-2)^{5/2} + C$

46. $\int \frac{dx}{(x-1)^3}$ *Ans.* $-\frac{1}{2(x-1)^2} + C$

47. $\int \frac{dx}{\sqrt{x+3}}$ *Ans.* $2\sqrt{x+3} + C$

48. $\int \sqrt{3x-1}\, dx$ *Ans.* $\frac{2}{9}(3x-1)^{3/2} + C$

49. $\int \sqrt{2-3x}\, dx$ *Ans.* $-\frac{2}{9}(2-3x)^{3/2} + C$

50. $\int (2x^2+3)^{1/3} x\, dx$ *Ans.* $\frac{3}{16}(2x^2+3)^{4/3} + C$

51. $\int \sqrt{1+y^4}\, y^3\, dy$ *Ans.* $\frac{1}{6}(1+y^4)^{3/2} + C$

52. $\int \frac{x\, dx}{(x^2+4)^3}$ *Ans.* $-\frac{1}{4(x^2+4)^2} + C$

In Problems 53–64, use any method.

53. $\int (x-1)^2 x\,dx$ *Ans.* $\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 + C$

54. $\int (x^2-x)^4(2x-1)\,dx$ *Ans.* $\frac{1}{5}(x^2-x)^5 + C$

55. $\int \frac{(x+1)\,dx}{\sqrt{x^2+2x-4}}$ *Ans.* $\sqrt{x^2+2x-4} + C$

56. $\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}\,dx$ *Ans.* $\frac{2}{3}(1+\sqrt{x})^3 + C$

57. $\int \frac{(x+1)(x-2)}{\sqrt{x}}\,dx$ *Ans.* $\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} - 4x^{1/2} + C = 2x^{1/2}(\frac{1}{5}x^2 - \frac{1}{3}x - 2) + C$

58. $\int \sec 3x \tan 3x\,dx$ *Ans.* $\frac{1}{3}\sec 3x + C$

59. $\int \csc^2(2x)\,dx$ *Ans.* $-\frac{1}{2}\cot 2x + C$

60. $\int x \sec^2(x^2)\,dx$ *Ans.* $\frac{1}{2}\tan(x^2) + C$

61. $\int \tan^2 x\,dx$ *Ans.* $\tan x - x + C$

62. $\int \cos^4 x \sin x\,dx$ *Ans.* $-\frac{1}{5}\cos^5 x + C$

63. $\int \frac{dx}{\sqrt{5-x^2}}$ *Ans.* $\sin^{-1}\left(\frac{x\sqrt{5}}{5}\right) + C$

64. $\int \frac{\sec^2 x\,dx}{1-4\tan^2 x}$ *Ans.* $\frac{1}{2}\sin^{-1}(2\tan x) + C$

65. A stone is thrown straight up from a building ledge that is 120 ft above the ground, with an initial velocity of 96 ft/sec. (a) When will it reach its maximum height? (b) What will its maximum height be? (c) When will it hit the ground? (d) With what speed will it hit the ground?

Ans. (a) $t = 3$ sec; (b) 264 ft; (c) $\frac{6+\sqrt{66}}{2} \sim 7.06$ sec; (d) ~129.98 ft/sec

66. An object moves on the x axis with acceleration $a = 3t - 2$ ft/sec². At time $t = 0$, it is at the origin and moving with a speed of 5 ft/sec in the negative direction. (a) Find a formula for its velocity v. (b) Find a formula for its position x. (c) When and where does it change direction? (d) At what times is it moving toward the right?

Ans. (a) $v = \frac{3}{2}t^2 - 2t - 5$; (b) $x = \frac{1}{2}t^3 - t^2 - 5t$; (c) $\frac{2 \pm \sqrt{34}}{3}$; (d) $t > \frac{2+\sqrt{34}}{3}$ or $t < \frac{2-\sqrt{34}}{3}$

67. A rocket shot straight up from the ground hits the ground 8 seconds later. (a) What was its initial velocity? (b) What was its maximum height?

Ans. (a) 128 ft/sec; (b) 256 ft

68. A driver applies the brakes on a car going at 55 miles per hour on a straight road. The brakes cause a constant deceleration of 11 ft/sec^2. (a) How soon will the car stop? (b) How far does the car move after the brakes were applied?

Ans. (a) 5 sec; (b) 137.5 ft

69. Find the equation of a curve going through the point (3, 7) and having slope $4x^2 - 3$ at (x, y).

Ans. $y = \frac{4}{3}x^3 - 3x - 20$