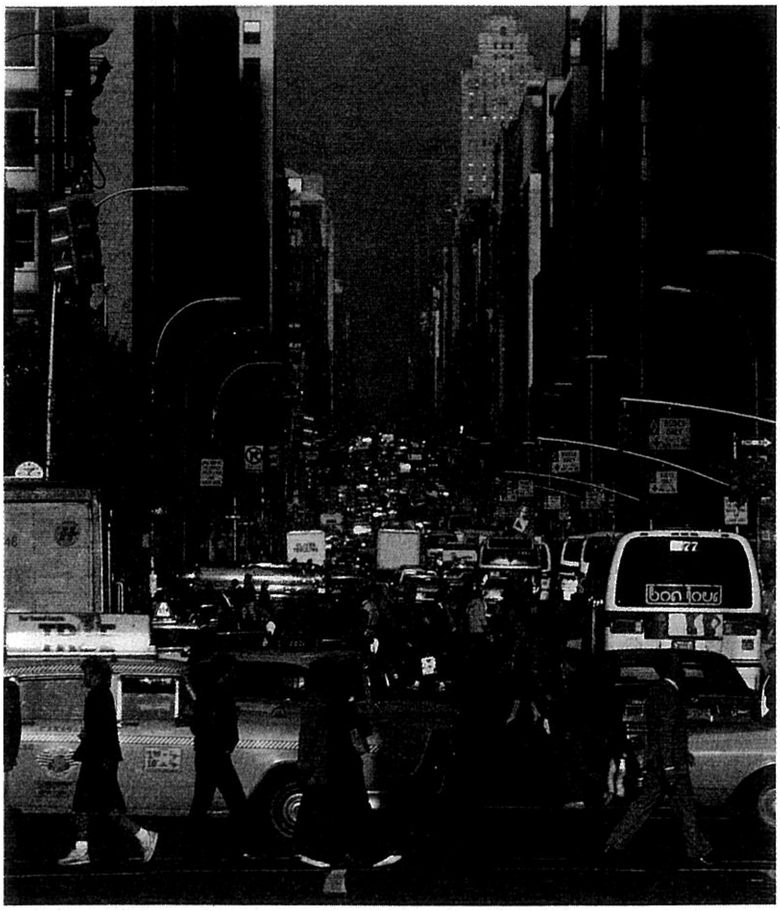
1

Basic Ideas of Geometry

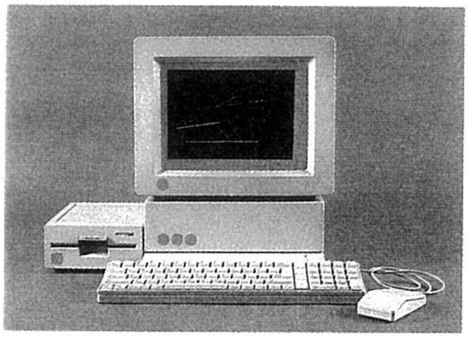


What concepts of geometry can you find in this street scene?

OBJECTIVE: *Name, describe, and draw models for points, lines, and planes and use these terms to define some basic relationships.*

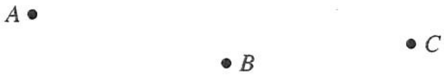
1-1 Points, Lines, Planes, and Space

As you learn about geometry, you will see that it is useful in many occupations and can help you develop your ability to discover, organize, and reason carefully. To begin your study of geometry, you need to understand the meaning of three basic terms—point, line, and plane.

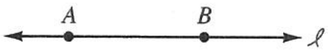


Some computers use a string of very small dots, or "bits," to show lines. This suggests the idea that a line is a set of points.

Think of a **point** as a location, such as a dot on a map or on a computer. A point has no size but can be represented by a dot labeled with a capital letter. Point A, point B, point C are shown below.



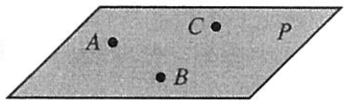
A **line,** like all geometric figures, is an infinite set of points. This idea is understood by thinking of straightness as suggested by the string of very small dots on the computer screen. A line has no thickness. A line is named by a lower-case letter or by any two points on the line. The double arrow indicates that the line continues without end in both directions.



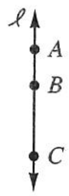
line ℓ, $\overleftrightarrow{AB}$, or $\overleftrightarrow{BA}$

A **plane** is also an infinite set of points. This idea is understood by thinking of flatness as suggested by a table top extending in all directions without bound. A plane has no thickness and is named by a capital letter or three points.

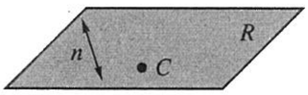
Point, *line*, and *plane* are called undefined terms in geometry. Some relationships between points, lines, and planes are also described using terms that are not defined, such as those in italics below.



plane P or plane ABC



Point A is *on* line ℓ.
A, B, and C, are *in* line ℓ.
ℓ *contains* A, B, and C.
ℓ *passes through* A, B, and C.
B is *between* A and C.



Point C and line n are *in* plane R.
Plane R *contains* C and n.

It is important to understand that for a point to be *between* two other points as described above, all three points must be on the same line.

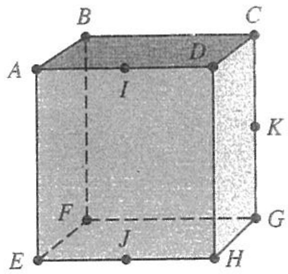
Definitions use undefined terms, previously defined terms, and ordinary words to give clear meaning to terms used in geometry. For example, the undefined term "point" is used to define space: **space** is the set of all points.

The following definitions describe other relationships with points, lines, and planes.

Collinear points are points all in one line. Points that are not in one line are called **noncollinear points.**

Coplanar points are points all in one plane. Points that are not in the same plane are called **noncoplanar points.** Similarly, **coplanar lines** are lines all in one plane. These ideas are illustrated in this figure.

The **intersection** of two figures is the set of points both figures have in common.



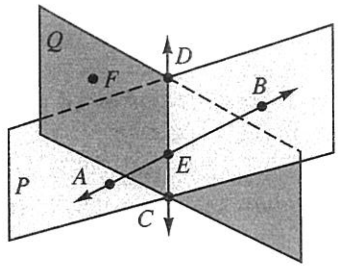
A, I, and D are collinear.
A, B, and C are noncollinear.
E, F, G, and H are coplanar.
A, B, D, and E are noncoplanar.
$\overleftrightarrow{AB}$ and $\overleftrightarrow{AD}$ are coplanar.

Example

Use symbols to describe the intersection of planes P and Q.

Solution

The intersection of planes P and Q is $\overleftrightarrow{CD}$.



Try This

Use symbols to describe the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

A **postulate** is a statement that is accepted as true without proof. The following postulates give some basic relationships among points, lines, planes, and space. Note that "two points (lines, planes)" means "two *different* points (lines, planes)."

POSTULATE 1

A line, a plane, and space each contain an infinite number of points. Some points in a plane are noncollinear. Some points in space are noncoplanar.

POSTULATE 2

For any two points, there is exactly one line containing them.

POSTULATE 3

For any three noncollinear points, there is exactly one plane containing them.

POSTULATE 4

If two points are in a plane, then the line containing them is in the plane.

POSTULATE 5

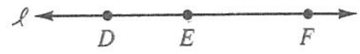
If two planes intersect, then they intersect in exactly one line.

The following is a **convincing argument** that two lines cannot intersect in more than one point.

Two lines, ℓ and m, cannot intersect in two or more points. If they could, you would have two points that have more than one line containing them. This contradicts Postulate 2.

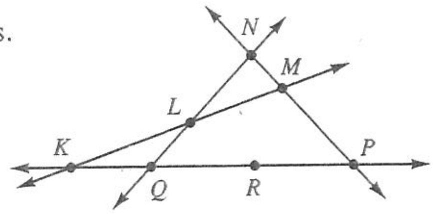
Class Exercises

Short Answer

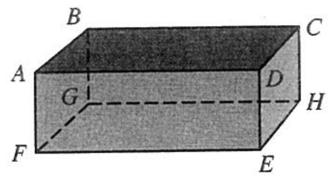


1. Name real-world objects that suggest points, lines, and planes.
2. Give five different names for this line.
3. Two lines that have a point in common are called __?__ lines.
4. Three points that are all on a line are __?__ points.
5. Four points that are not in the same plane are __?__ points.

Sample Exercises

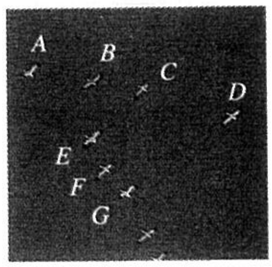


6. Name four sets of three collinear points.
7. Name two sets of three noncollinear points.
8. Name three pairs of intersecting lines. Give the points of intersection.
9. Name two points between K and P. Why is point L not between points K and P?



10. Name three sets of four points that are coplanar.
11. Name two coplanar lines.
12. Use three points in each plane to name a pair of intersecting planes. Name the line of intersection.
13. Draw and label a diagram showing four coplanar points, G, H, I, and J.
14. State the postulate that asserts that this statement is true. You can name as many points as you wish on a line.

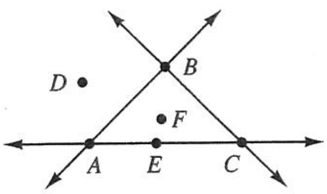
Discussion



15. Think of this formation of geese flying south in the winter as a set of points. Use the words collinear, noncollinear, coplanar, and noncoplanar to describe the situation.
16. Use the postulates to give a convincing argument that the existence of a line and a point not on the line is enough to establish the existence of an infinite number of lines.

Exercises

A



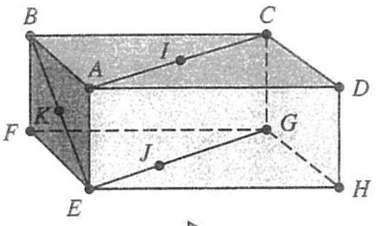
Write each of the following using symbols.

1. the line containing points A and B
2. a point not on $\overleftrightarrow{AC}$
3. a pair of lines that intersect at point A
4. a point on $\overleftrightarrow{AC}$ but not on $\overleftrightarrow{BC}$
5. a point between A and C

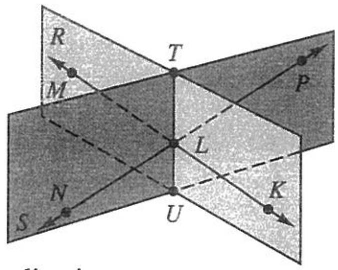
Draw and label each figure.

6. two points, J and K
7. three noncollinear points: R, S, and T
8. $\overleftrightarrow{GH}$
9. line ℓ containing points M, N, and Q
10. plane R containing points A and B
11. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$, interesting at point Z
12. three collinear points: C, D, and E
13. point L between points S and T

Determine whether each statement is true or false.



14. Points E, J, and G are collinear.
15. Points A, F, and H are coplanar.
16. Plane ABC intersects plane AIJ in $\overleftrightarrow{AC}$.
17. $\overleftrightarrow{IJ}$ intersects $\overleftrightarrow{EG}$ at point I.



Complete each statement.

18. The intersection of $\overleftrightarrow{MK}$ and $\overleftrightarrow{NP}$ is ___.
19. The intersection of plane R and plane S is ___.
20. Line ___ intersects plane S in only one point.

State the postulate that asserts that each statement is true.

21. Point A and B are on $\overleftrightarrow{AB}$ and are not both on any other line.
22. When planes P and Q have line ℓ in common, they have no other line in common.
23. When points A and B are in plane R, $\overleftrightarrow{AB}$ is also in plane R.
24. When three noncollinear points A, B, and C are in plane P, all three are in no other plane.
25. A carpenter often needs to make a straight line. To do this, he stretches a taut chalkline from point A to point B and snaps the string to form a straight line. Which postulate does this suggest?

B

Determine whether each statement is true or false. If false, explain why.

26. Three collinear points are always coplanar.
27. Three coplanar points are always collinear.
28. Any three points are coplanar.
29. When A is between C and D, the three points are collinear.
30. It is possible to have four coplanar points, no three of which are collinear.
31. Space may contain more, but not less, than four noncoplanar points.

Given the first statement, state the postulate that allows you to conclude that the second statement is true.

32. Two planes, M and N, both contain line ℓ. Therefore it is impossible for planes M and N both to contain another line, m.
33. Two lines, ℓ and m, both contain point A. Therefore it is impossible for lines ℓ and m both to contain another point, B.
34. Three noncollinear points A, B, and C are contained in plane M. Therefore points A, B, and C cannot all be contained in another plane, N.
35. A and B are points in plane M. Therefore, every point of $\overleftrightarrow{AB}$ is in plane M.
36. A photographer uses a tripod to hold a camera. Explain why a tripod has three legs instead of four. Which postulate applies to this situation?

C

37. Make drawings, when possible, to show that four coplanar lines can be drawn to intersect in only

    a. 1 point **b.** 2 points **c.** 3 points **d.** 4 points **e.** 5 points **f.** 6 points