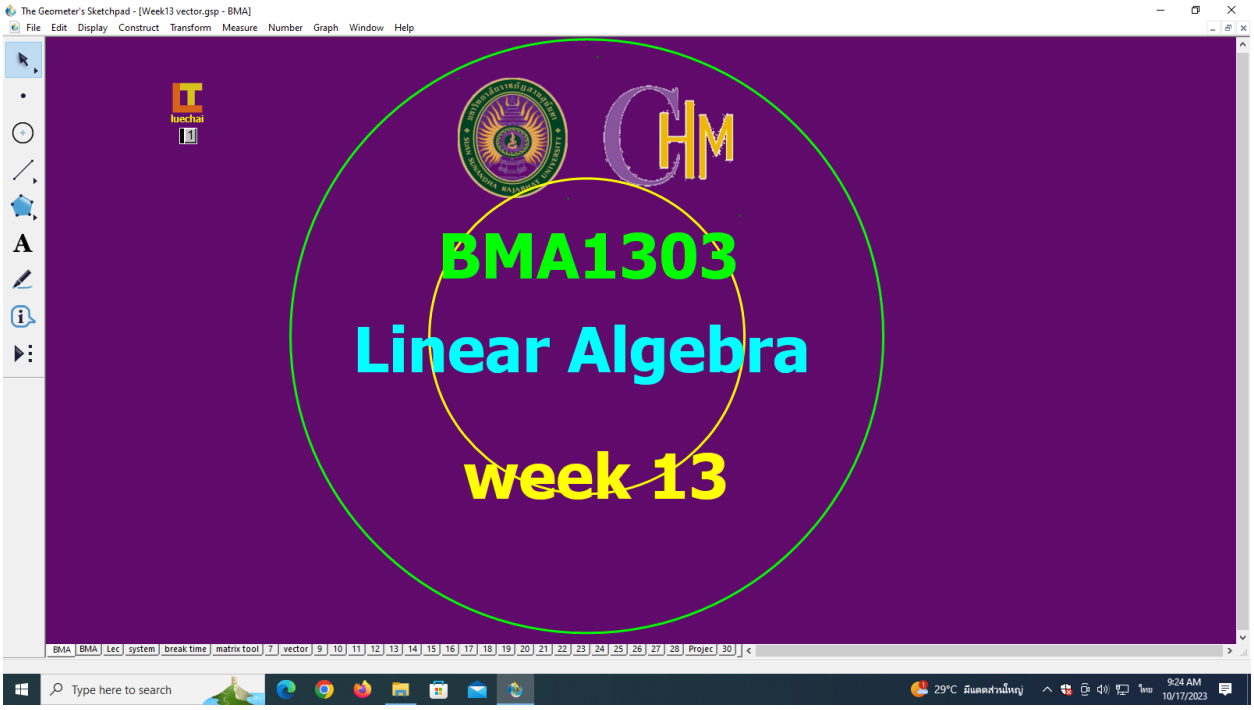
The Geometer's Sketchpad - [Week13 vector.gsp - BMA]
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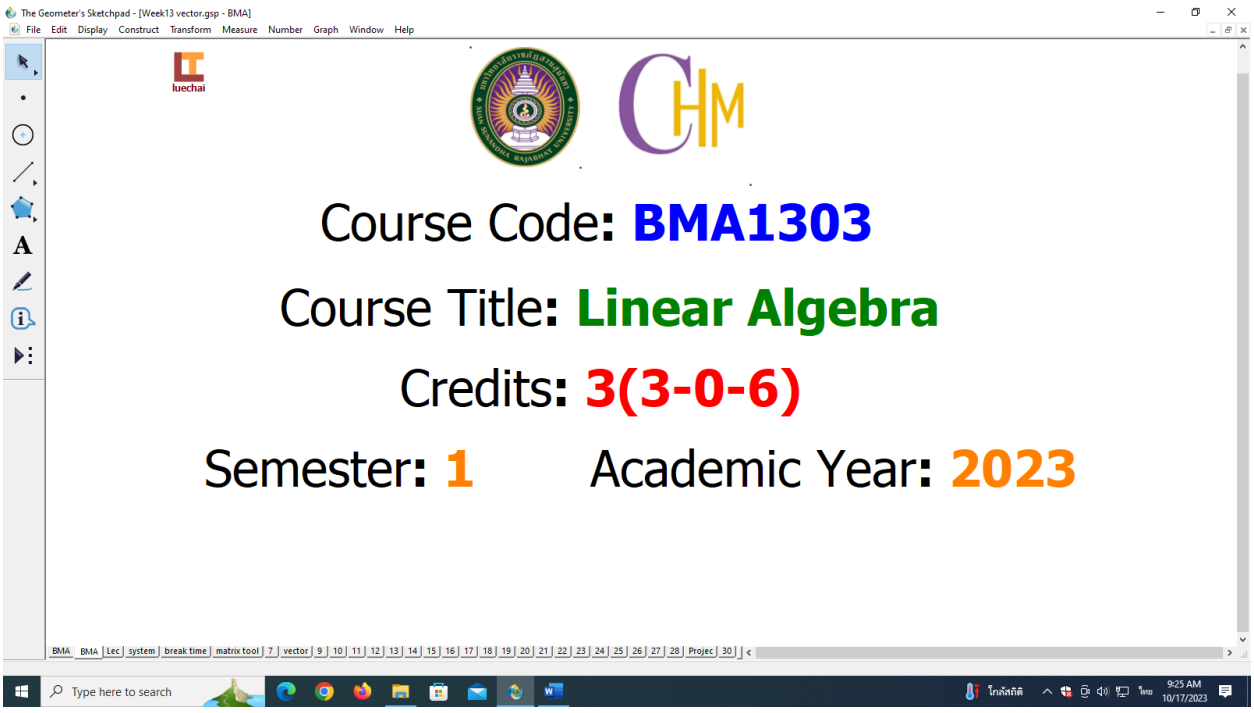
The Geometer's Sketchpad - [Week13 vector.gsp - BMA]
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CHM
Course Code: BMA1303
Course Title: Linear Algebra
Credits: 3(3-0-6)
Semester: 1 Academic Year: 2023
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The Geometer's Sketchpad - [Week13 vector.gsp - Lec]
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luechai
Lecturer responsible for this course:
Mr.Luechai Tiprungsri
Room Number: 305
Tel.: 081-972-5793
Email: luechai.ti@ssru.ac.th
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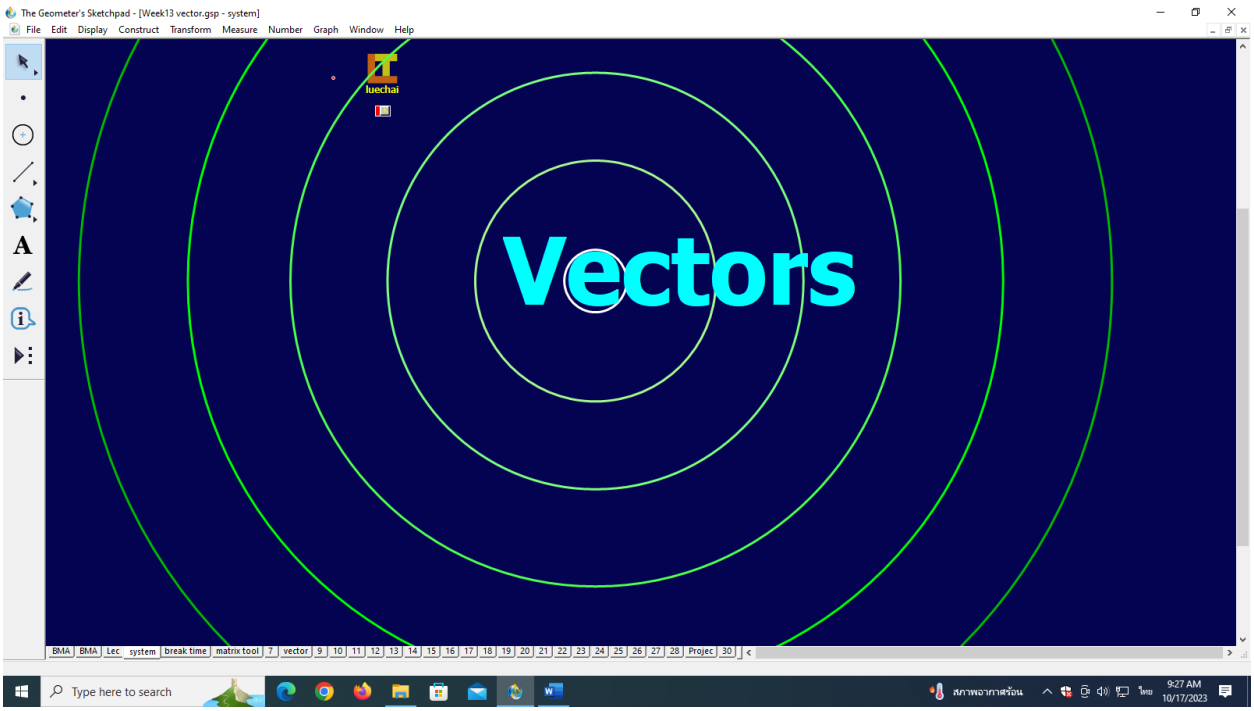
The Geometer's Sketchpad - [Week13 vector.gsp - system]
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Vectors
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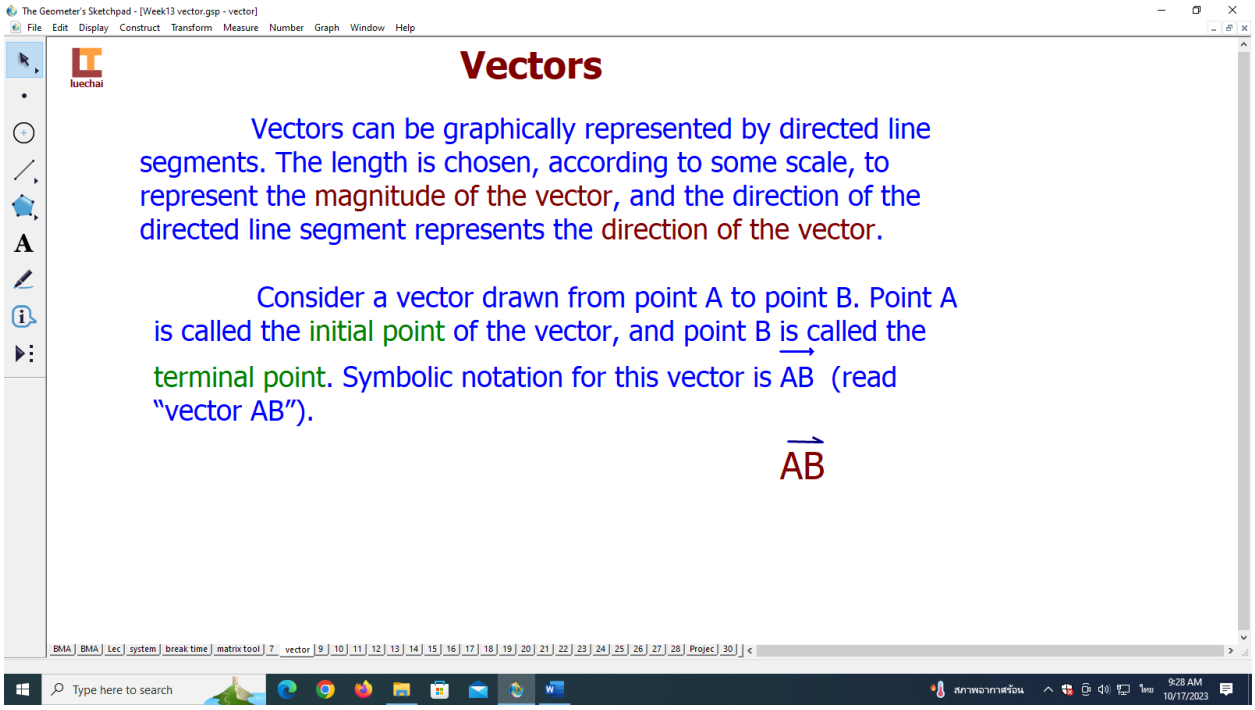
The Geometer's Sketchpad - [Week13 vector.gsp - vector]
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Vectors
Vectors can be graphically represented by directed line segments. The length is chosen, according to some scale, to represent the magnitude of the vector, and the direction of the directed line segment represents the direction of the vector.
Consider a vector drawn from point A to point B. Point A is called the initial point of the vector, and point B is called the terminal point. Symbolic notation for this vector is $\overrightarrow{AB}$ (read "vector AB").
$\overrightarrow{AB}$
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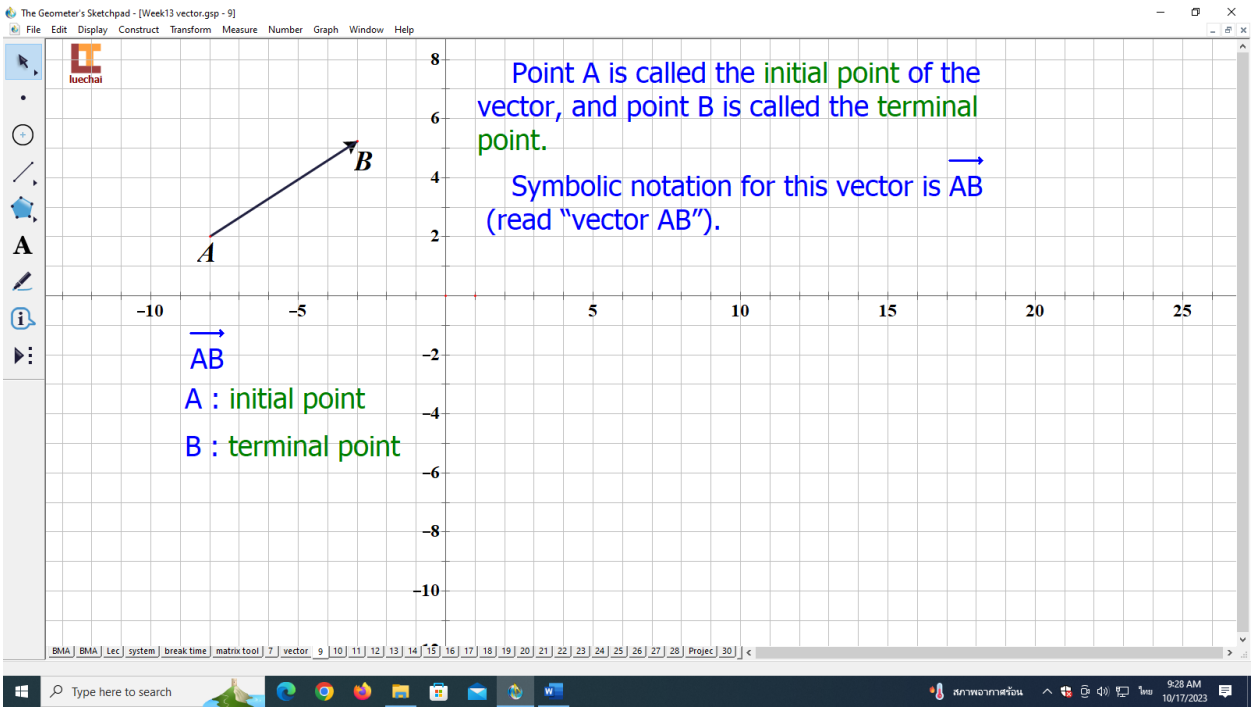
The Geometer's Sketchpad - [Week13 vector.gsp - 9]
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B
A
Point A is called the initial point of the vector, and point B is called the terminal point.
Symbolic notation for this vector is $\overrightarrow{AB}$ (read "vector AB").
$\overrightarrow{AB}$
A : initial point
B : terminal point
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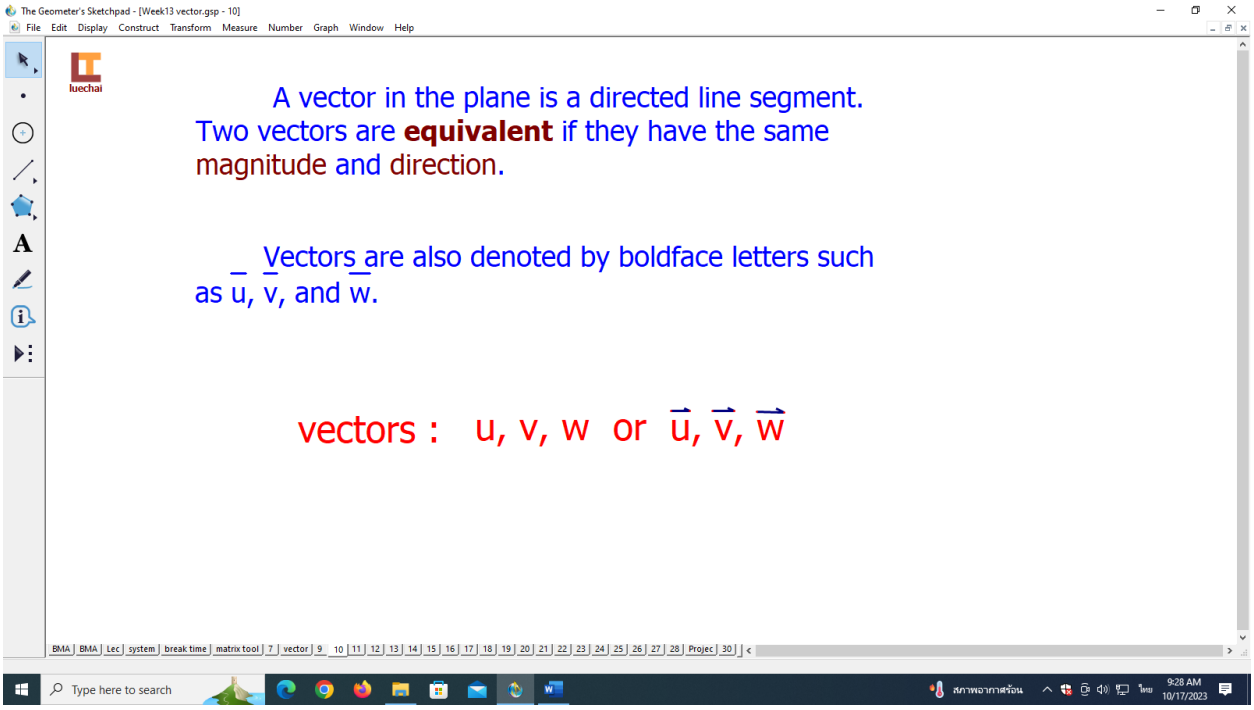
The Geometer's Sketchpad - [Week13 vector.gsp - 10]
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A vector in the plane is a directed line segment. Two vectors are equivalent if they have the same magnitude and direction.
Vectors are also denoted by boldface letters such as u, v, and w.
vectors : u, v, w or $\vec{u}$, $\vec{v}$, $\vec{w}$
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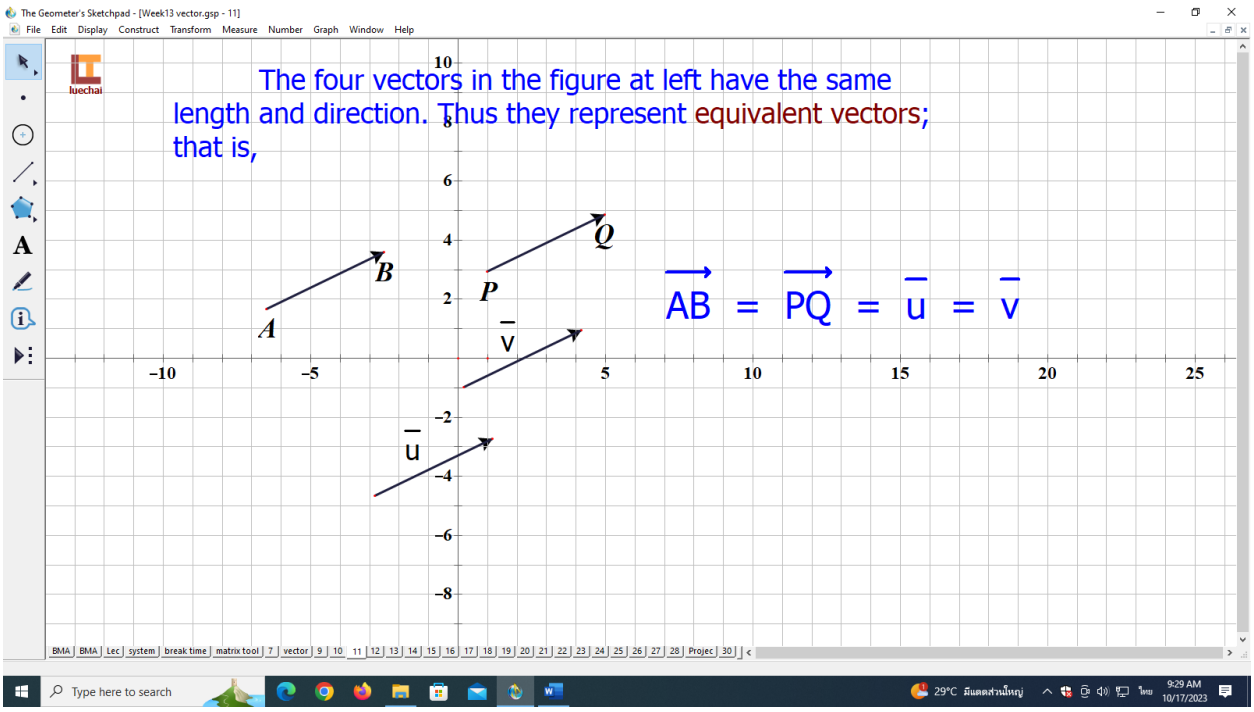
The Geometer's Sketchpad - [Week13 vector.gsp - 11]
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The four vectors in the figure at left have the same length and direction. Thus they represent equivalent vectors; that is,
A
B
P
Q
v
u
$\overrightarrow{AB} = \overrightarrow{PQ} = \bar{u} = \bar{v}$
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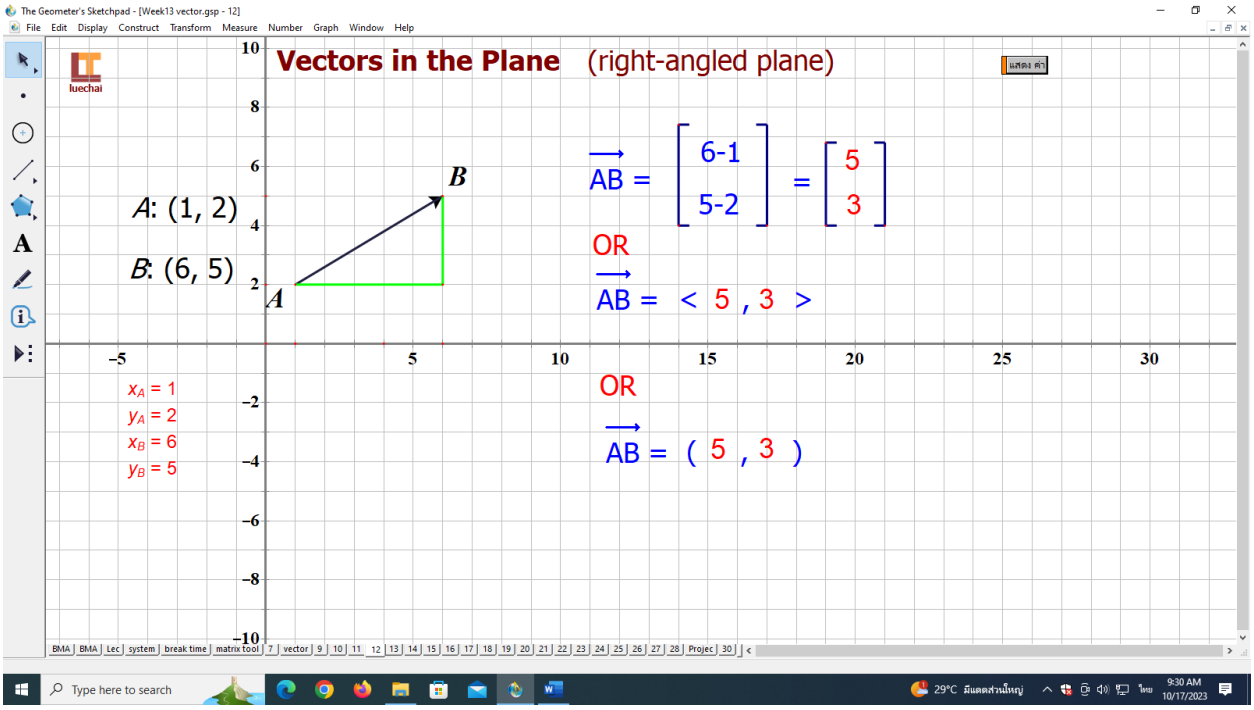
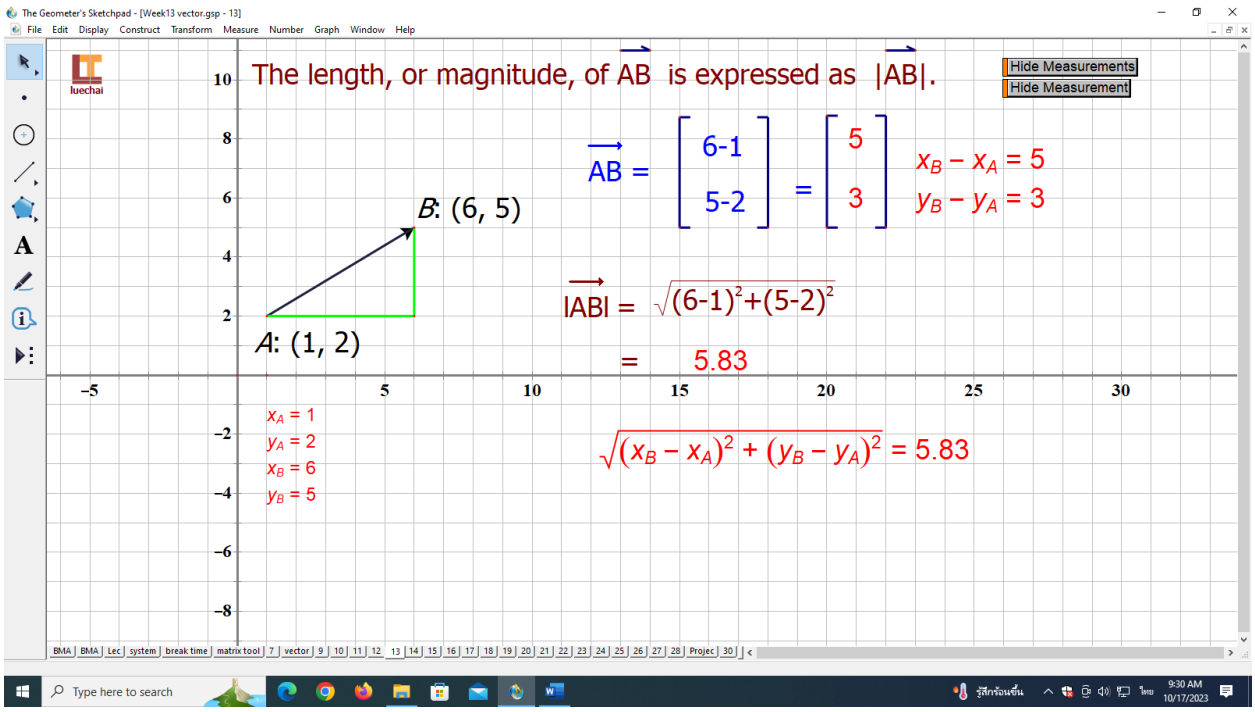
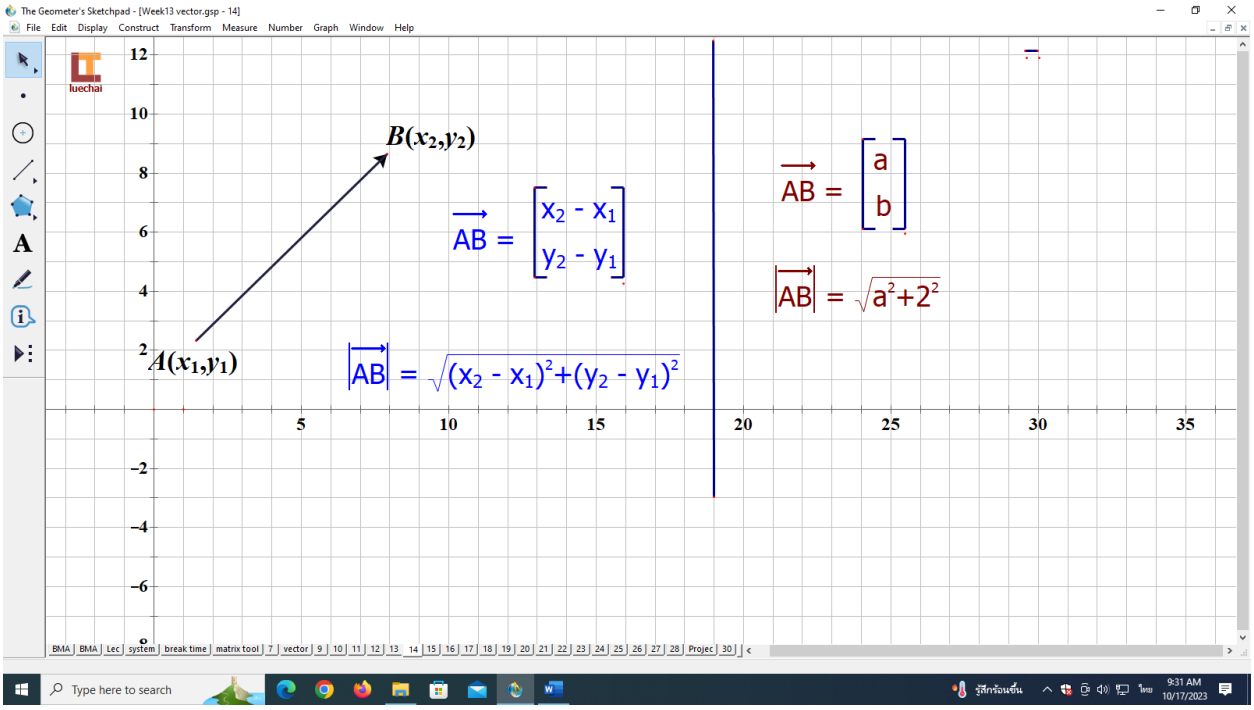

Vectors in the Plane (right-angled plane)
A: (1, 2)
B: (6, 5)
$\overrightarrow{AB} = \begin{bmatrix} 6-1 \\ 5-2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$
OR
$\overrightarrow{AB} = \langle 5, 3 \rangle$
OR
$\overrightarrow{AB} = (5, 3)$
$x_A = 1$
$y_A = 2$
$x_B = 6$
$y_B = 5$


The length, or magnitude, of $\overrightarrow{AB}$ is expressed as $|\overrightarrow{AB}|$.
Hide Measurements
Hide Measurement
B: (6, 5)
A: (1, 2)
$\overrightarrow{AB} = \begin{bmatrix} 6-1 \\ 5-2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$
$x_B - x_A = 5$
$y_B - y_A = 3$
$|\overrightarrow{AB}| = \sqrt{(6-1)^2+(5-2)^2}$
$= 5.83$
$x_A = 1$
$y_A = 2$
$x_B = 6$
$y_B = 5$
$\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = 5.83$


The Geometer's Sketchpad - [Week13 vector.gsp - 14]
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B(x2,y2)
A(x1,y1)
AB = [x2 - x1, y2 - y1]
|AB| = √((x2 - x1)^2+(y2 - y1)^2)
AB = [a, b]
|AB| = √(a^2+2^2)
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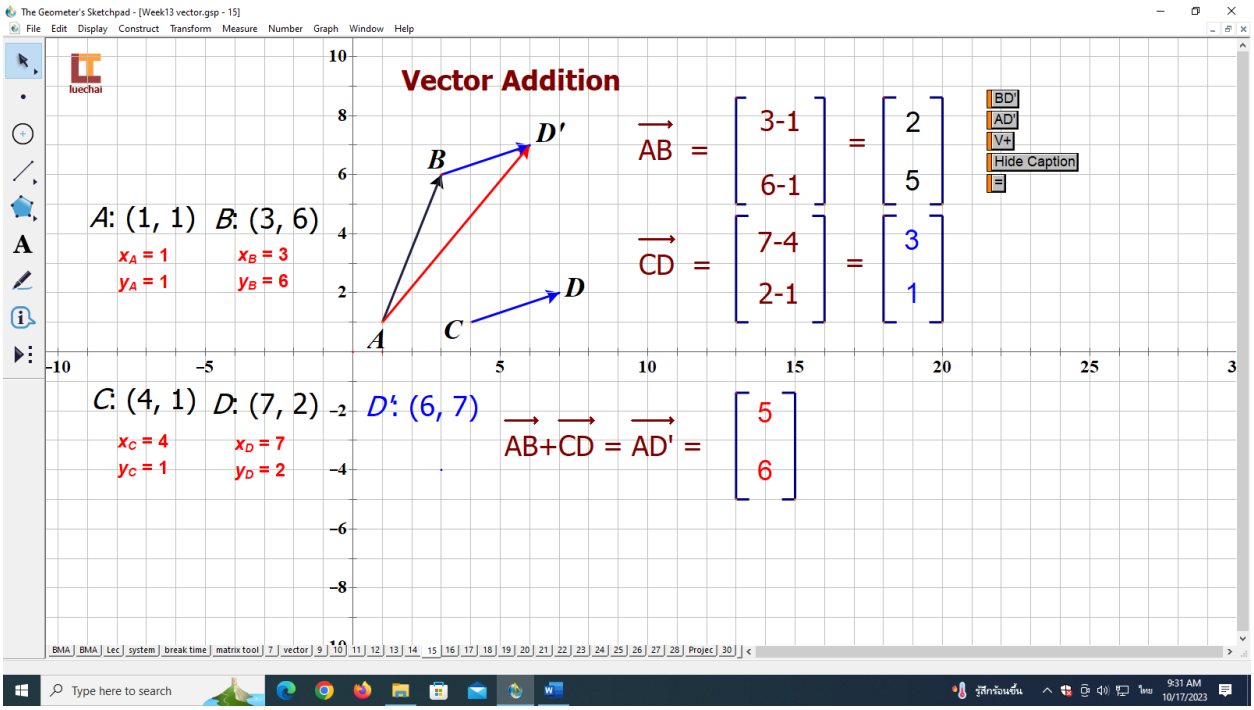
The Geometer's Sketchpad - [Week13 vector.gsp - 15]
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Vector Addition
A: (1, 1) B: (3, 6)
xA = 1 yA = 1
xB = 3 yB = 6
C: (4, 1) D: (7, 2)
xC = 4 yC = 1
xD = 7 yD = 2
D': (6, 7)
AB = [3-1, 6-1] = [2, 5]
CD = [7-4, 2-1] = [3, 1]
AB+CD = AD' = [5, 6]
BD'
AD'
V+
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I=
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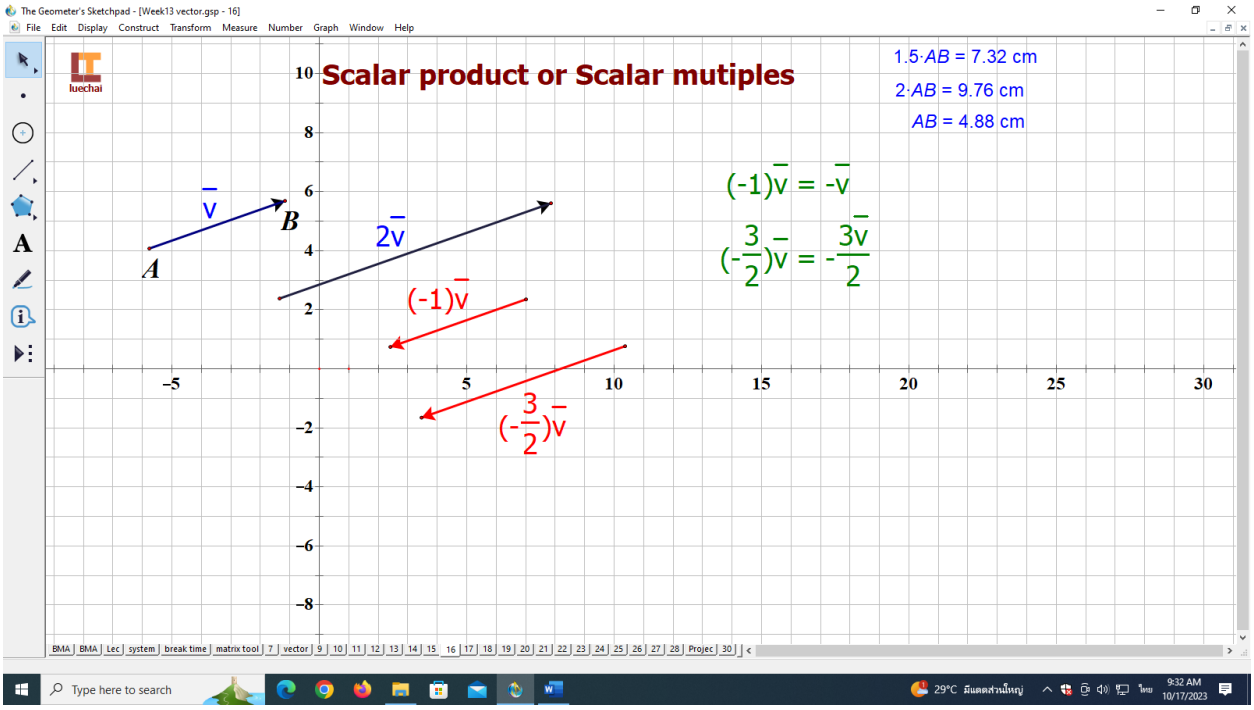
The Geometer's Sketchpad - [Week13 vector.gsp - 16]
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Scalar product or Scalar mutiples
1.5·AB = 7.32 cm
2·AB = 9.76 cm
AB = 4.88 cm
(-1)v = -v
(-3/2)v = -3v/2
v
2v
(-1)v
(-3/2)v
A
B
BMA | BMA | Lec | system | break time | matrix tool | 7 | vector | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | Projec | 30
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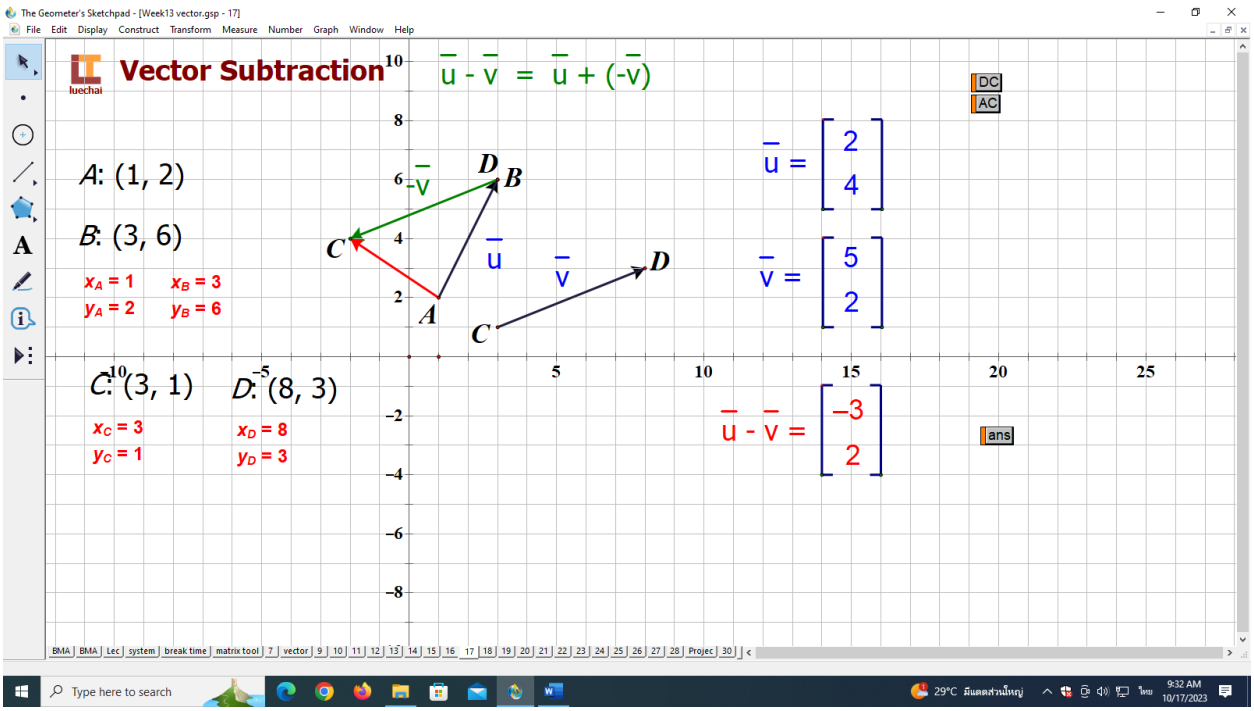
The Geometer's Sketchpad - [Week13 vector.gsp - 17]
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Vector Subtraction
u - v = u + (-v)
DC
AC
A: (1, 2)
B: (3, 6)
xA = 1
yA = 2
xB = 3
yB = 6
C: (3, 1)
D: (8, 3)
xC = 3
yC = 1
xD = 8
yD = 3
u = [2; 4]
v = [5; 2]
u - v = [-3; 2]
ans
-v
u
v
A
B
C
D
BMA | BMA | Lec | system | break time | matrix tool | 7 | vector | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | Projec | 30
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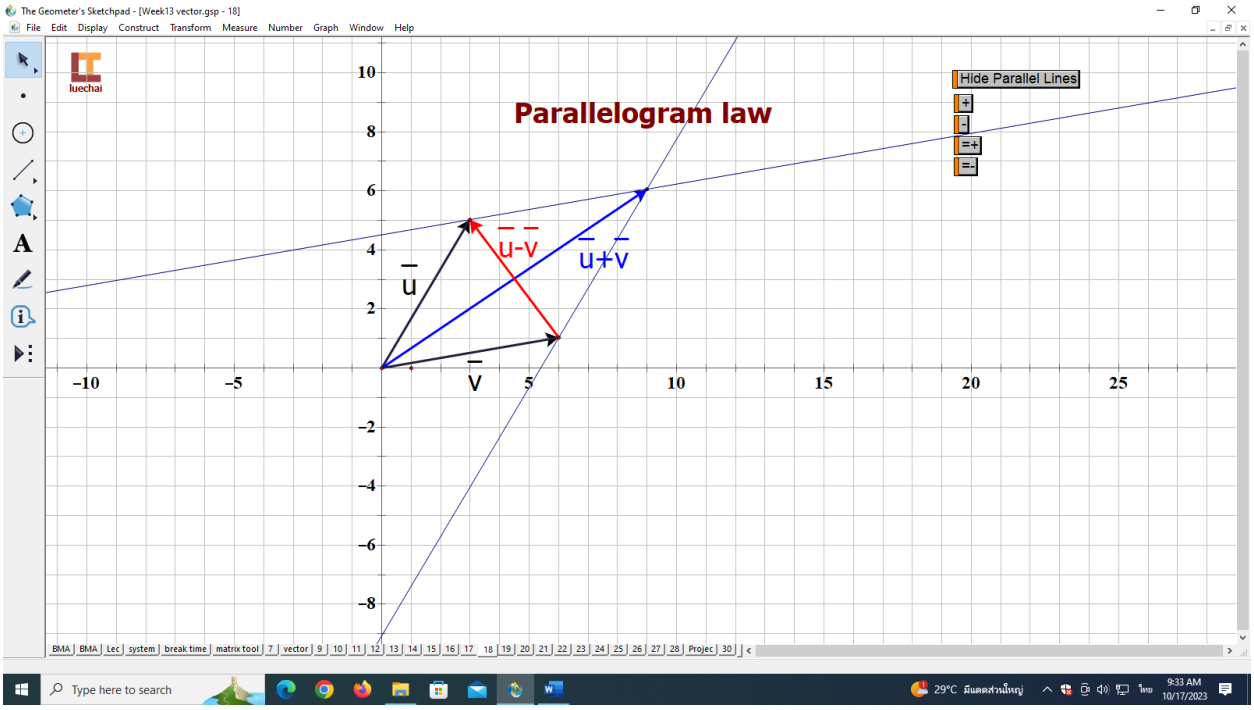
The Geometer's Sketchpad - [Week13 vector.gsp - 18]
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luechai
Parallelogram law
Hide Parallel Lines
u
v
u-v
u+v
10
8
6
4
2
-2
-4
-6
-8
-10
-5
5
10
15
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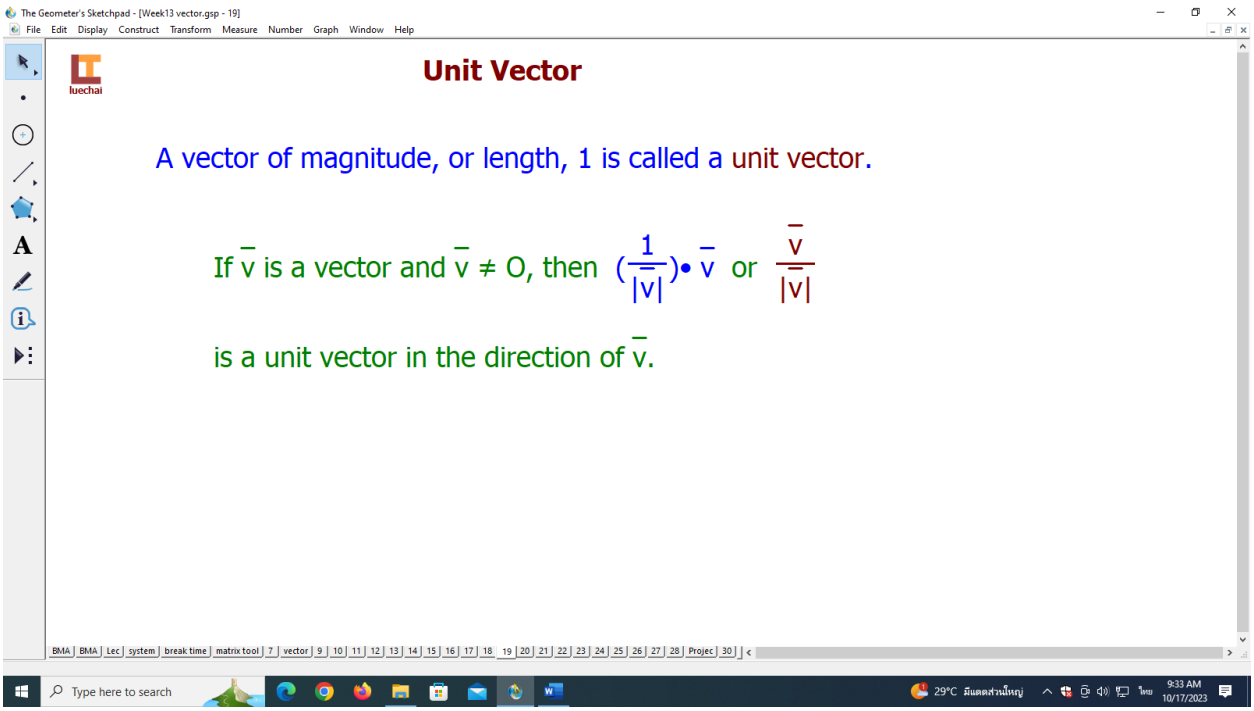
The Geometer's Sketchpad - [Week13 vector.gsp - 19]
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Unit Vector
A vector of magnitude, or length, 1 is called a unit vector.
If v is a vector and v ≠ O, then (1/|v|)• v or v/|v|
is a unit vector in the direction of v.
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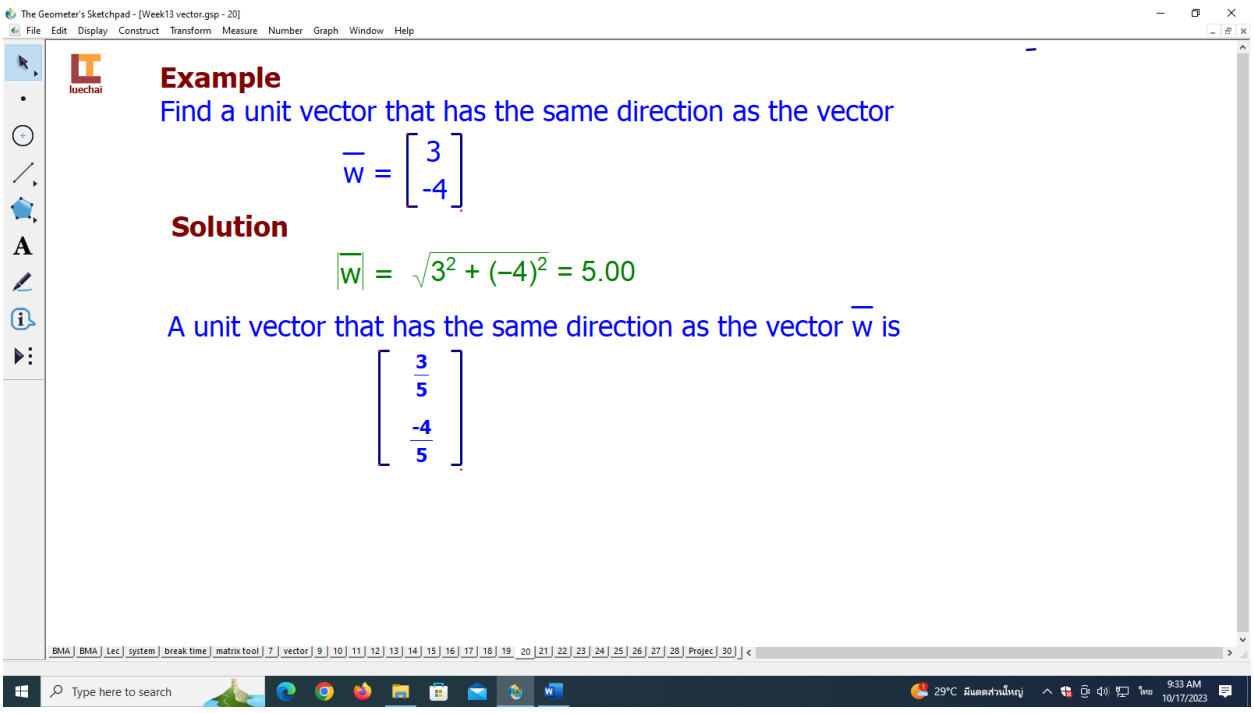
The Geometer's Sketchpad - [Week13 vector.gsp - 20]
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Example
Find a unit vector that has the same direction as the vector
$\overline{w} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$
Solution
$|\overline{w}| = \sqrt{3^2 + (-4)^2} = 5.00$
A unit vector that has the same direction as the vector $\overline{w}$ is
$\begin{bmatrix} \frac{3}{5} \\ \frac{-4}{5} \end{bmatrix}$
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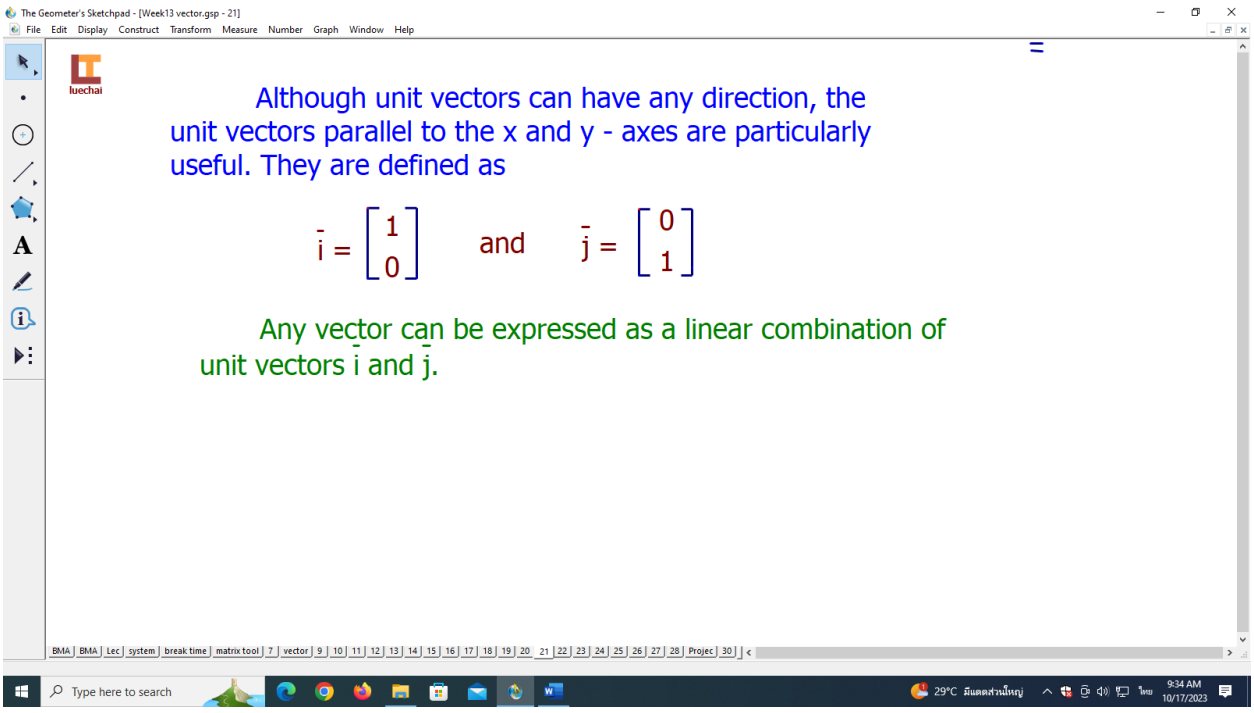
The Geometer's Sketchpad - [Week13 vector.gsp - 21]
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Although unit vectors can have any direction, the unit vectors parallel to the x and y - axes are particularly useful. They are defined as
$\bar{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\bar{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Any vector can be expressed as a linear combination of unit vectors $\bar{i}$ and $\bar{j}$.
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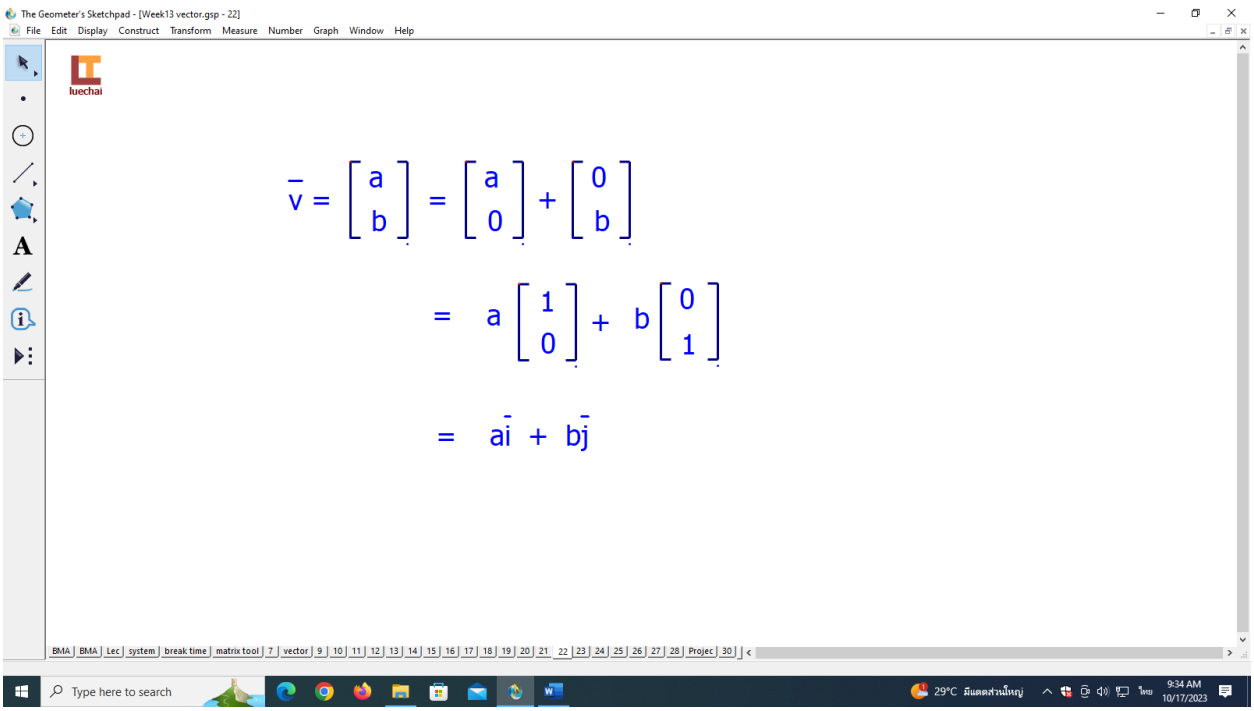
The Geometer's Sketchpad - [Week13 vector.gsp - 22]
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$\bar{v} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix}$
$= a \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
$= a\bar{i} + b\bar{j}$
BMA BMA Lec system break time matrix tool 7 vector 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 Projec 30
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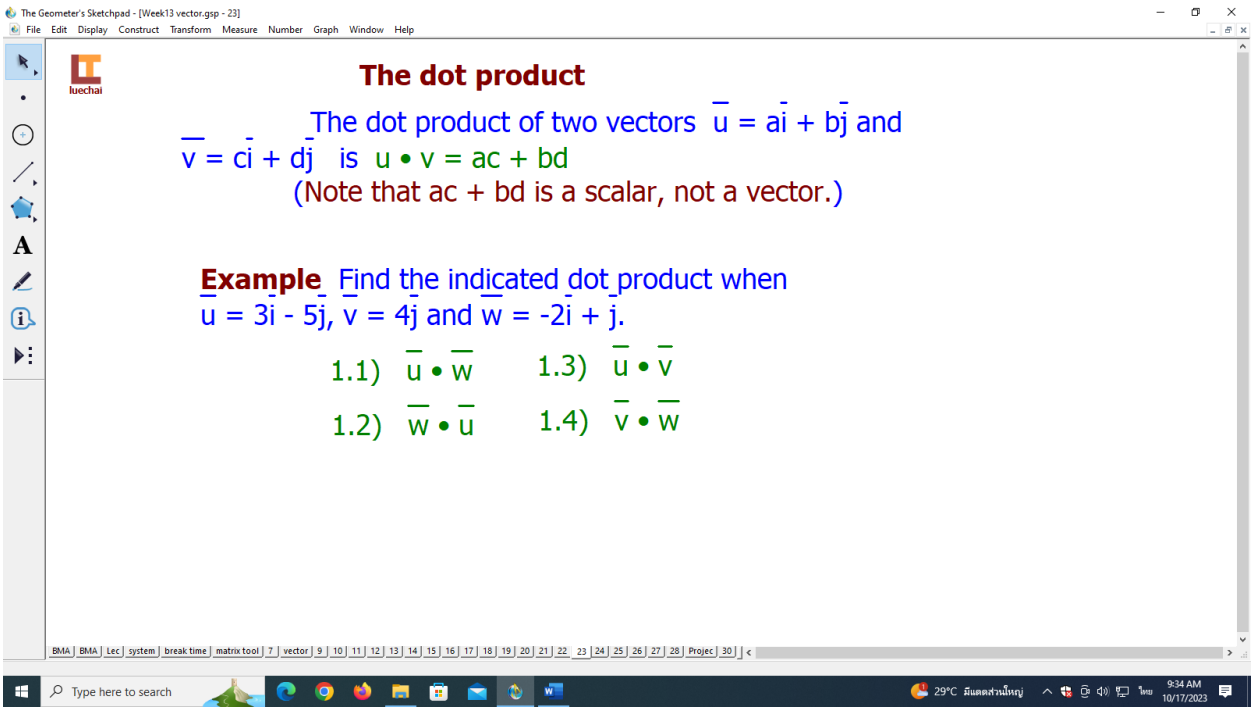
The Geometer's Sketchpad - [Week13 vector.gsp - 23]
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The dot product
The dot product of two vectors $\bar{u} = a\bar{i} + b\bar{j}$ and $\bar{v} = c\bar{i} + d\bar{j}$ is $u \bullet v = ac + bd$
(Note that ac + bd is a scalar, not a vector.)
Example Find the indicated dot product when $\bar{u} = 3\bar{i} - 5\bar{j}$, $\bar{v} = 4\bar{j}$ and $\bar{w} = -2\bar{i} + \bar{j}$.
1.1) $\bar{u} \bullet \bar{w}$
1.2) $\bar{w} \bullet \bar{u}$
1.3) $\bar{u} \bullet \bar{v}$
1.4) $\bar{v} \bullet \bar{w}$
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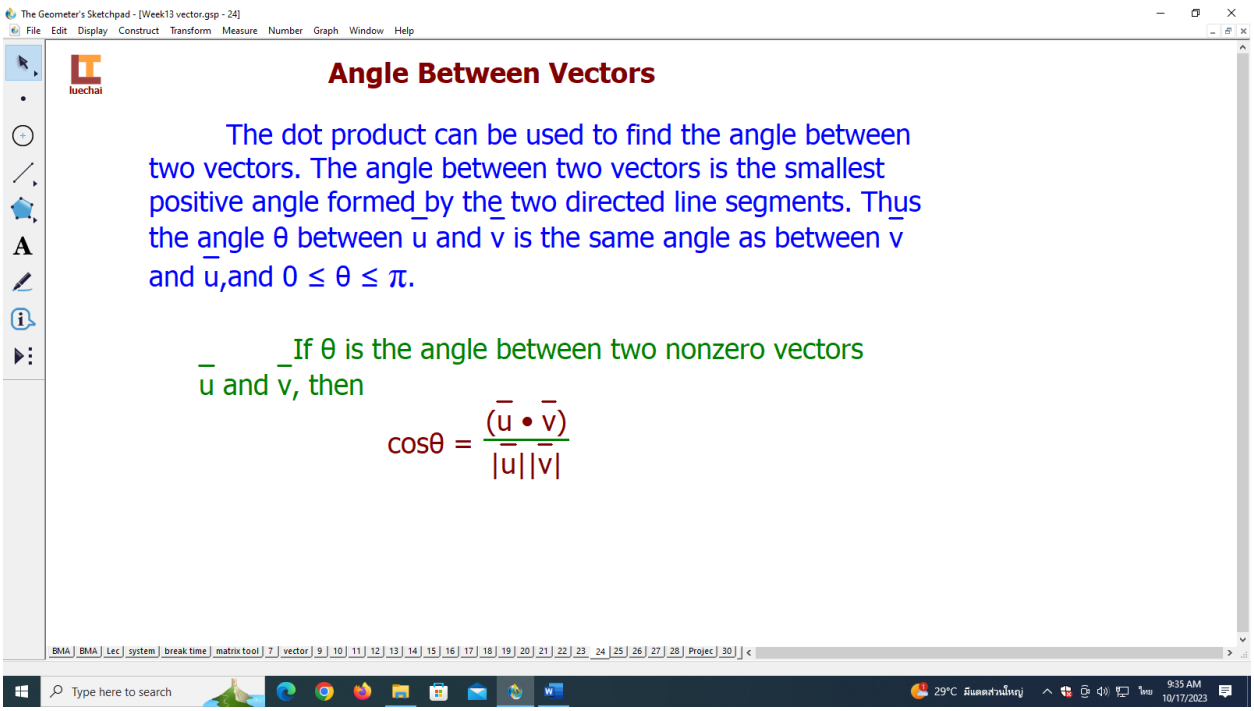
The Geometer's Sketchpad - [Week13 vector.gsp - 24]
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Angle Between Vectors
The dot product can be used to find the angle between two vectors. The angle between two vectors is the smallest positive angle formed by the two directed line segments. Thus the angle θ between u and v is the same angle as between v and u,and 0 ≤ θ ≤ π.
If θ is the angle between two nonzero vectors u and v, then
cosθ = (u • v) / |u||v|
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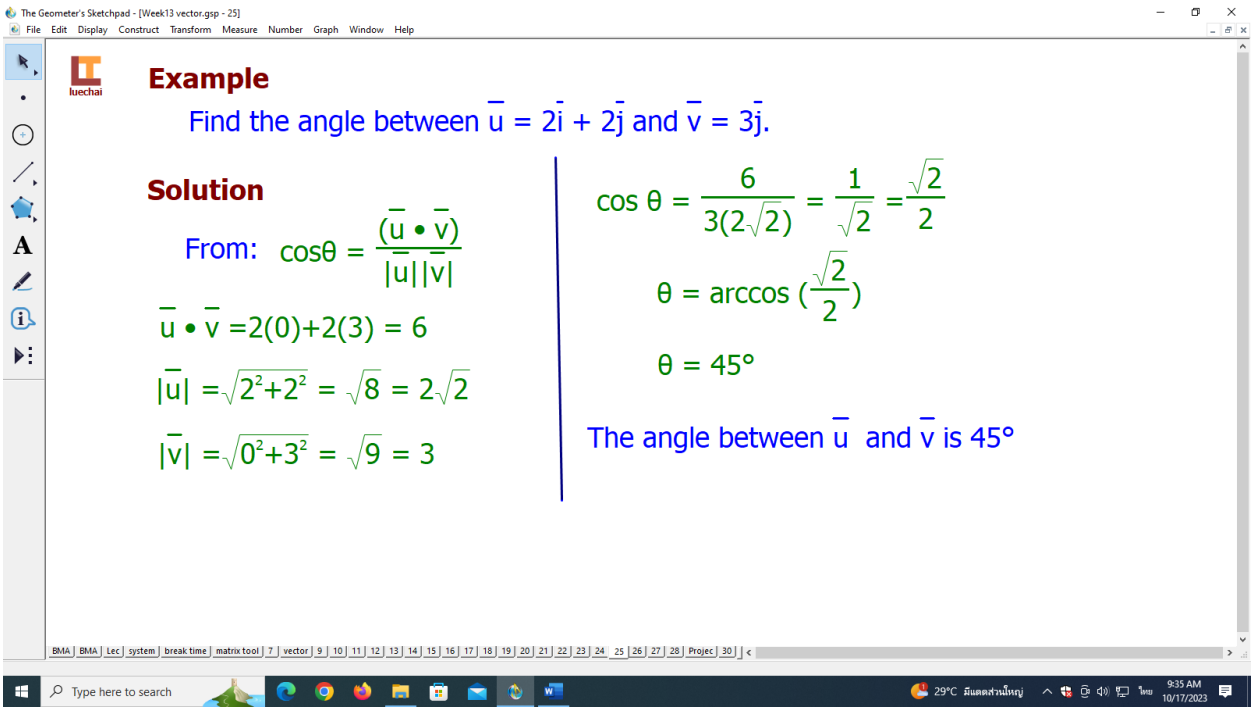
The Geometer's Sketchpad - [Week13 vector.gsp - 25]
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Example
Find the angle between u = 2i + 2j and v = 3j.
Solution
From: cosθ = (u • v) / |u||v|
u • v =2(0)+2(3) = 6
|u| =√(2²+2²) = √8 = 2√2
|v| =√(0²+3²) = √9 = 3
cos θ = 6 / 3(2√2) = 1/√2 = √2/2
θ = arccos (√2/2)
θ = 45°
The angle between u and v is 45°
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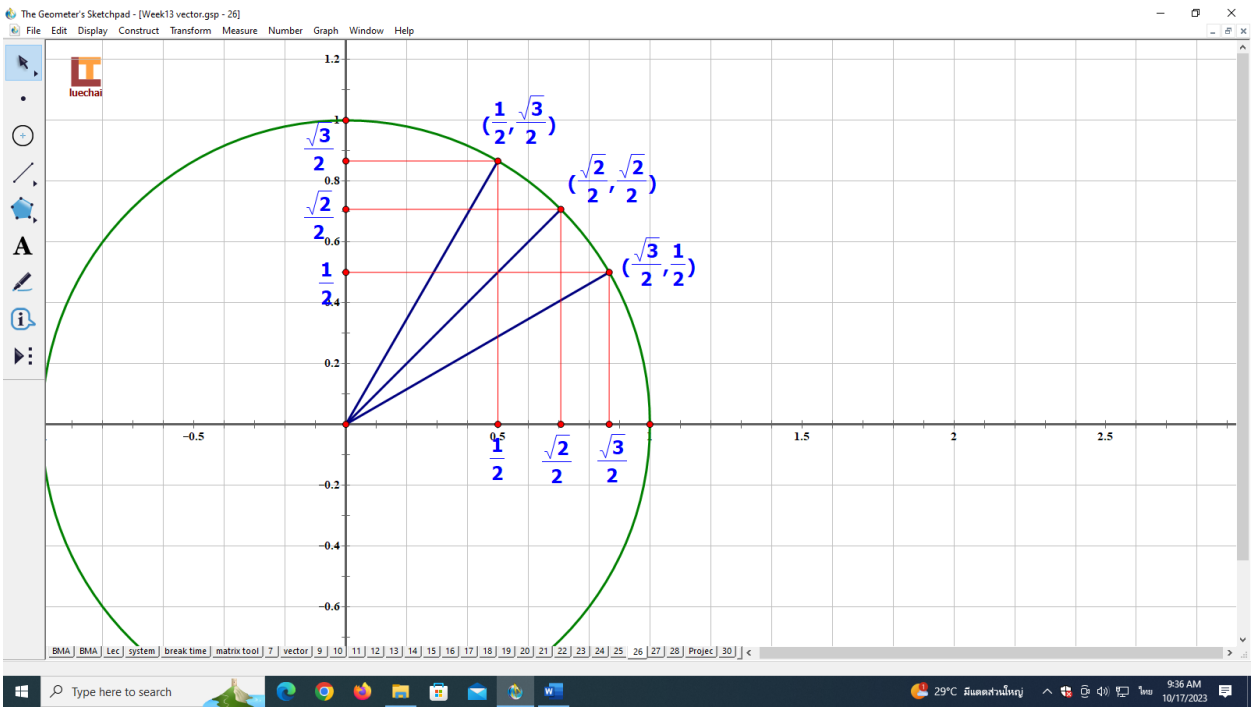
The Geometer's Sketchpad - [Week13 vector.gsp - 26]
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$(\frac{1}{2},\frac{\sqrt{3}}{2})$
$(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$
$(\frac{\sqrt{3}}{2},\frac{1}{2})$
$\frac{\sqrt{3}}{2}$
$\frac{\sqrt{2}}{2}$
$\frac{1}{2}$
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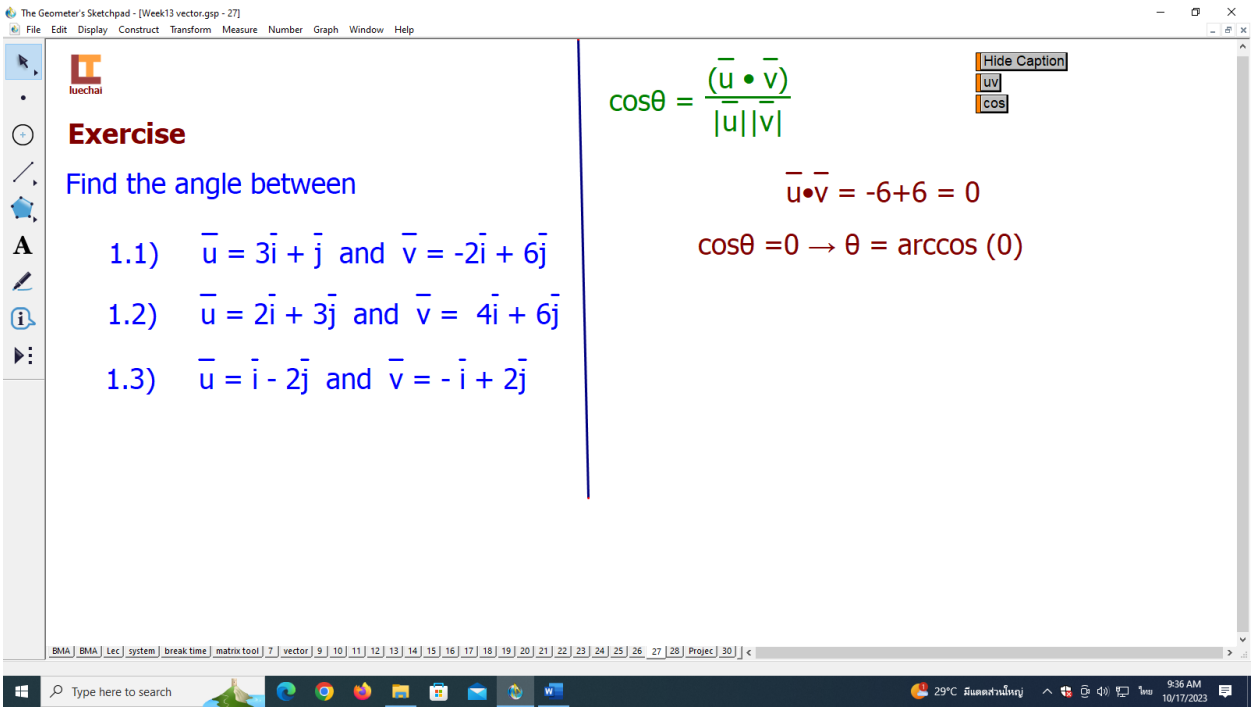
The Geometer's Sketchpad - [Week13 vector.gsp - 27]
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luechai
Exercise
Find the angle between
1.1) $\bar{u} = 3\bar{i} + \bar{j}$ and $\bar{v} = -2\bar{i} + 6\bar{j}$
1.2) $\bar{u} = 2\bar{i} + 3\bar{j}$ and $\bar{v} = 4\bar{i} + 6\bar{j}$
1.3) $\bar{u} = \bar{i} - 2\bar{j}$ and $\bar{v} = -\bar{i} + 2\bar{j}$
$\cos\theta = \frac{(\bar{u} \bullet \bar{v})}{|u||v|}$
Hide Caption
uv
cos
$\bar{u}\bullet\bar{v} = -6+6 = 0$
$\cos\theta = 0 \rightarrow \theta = \arccos(0)$
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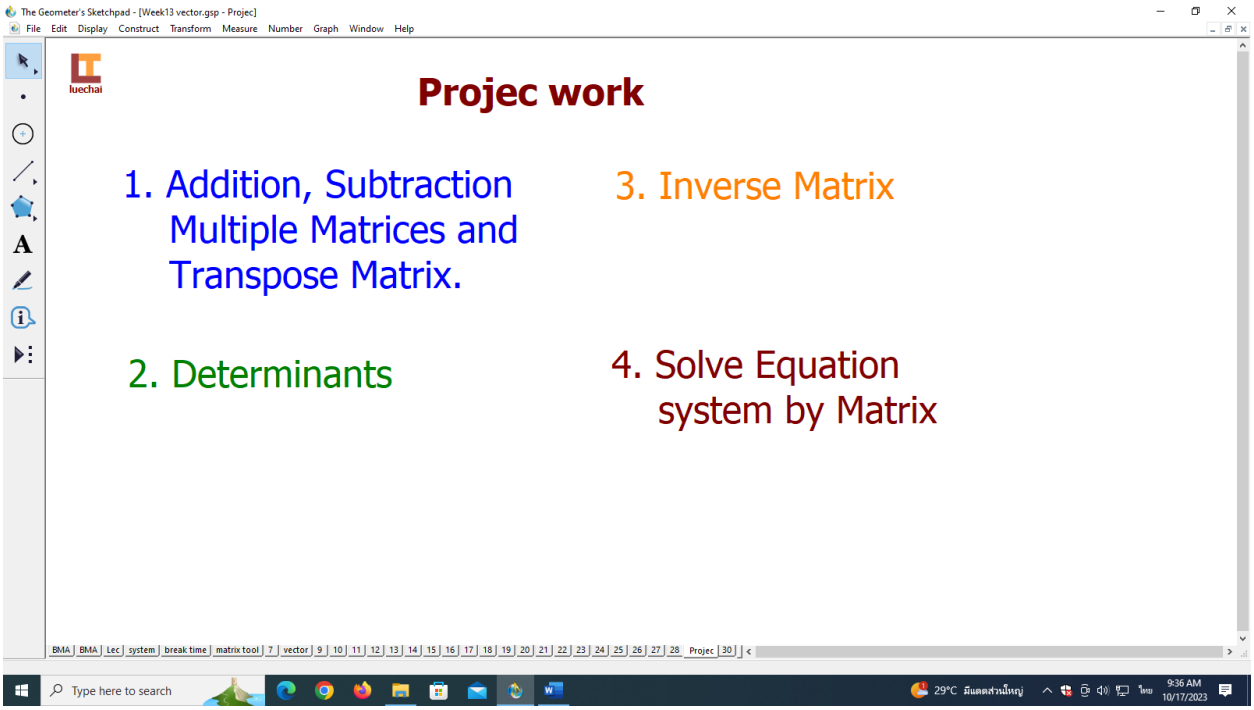
The Geometer's Sketchpad - [Week13 vector.gsp - Projec]
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Projec work
1. Addition, Subtraction Multiple Matrices and Transpose Matrix.
2. Determinants
3. Inverse Matrix
4. Solve Equation system by Matrix
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