

The Geometer's Sketchpad - [Week10 LT BMA 1303 Linear Algebra.gsp - 5]

File Edit Display Construct Transform Measure Number Graph Window Help

Systems of Linear Equations in n Variables

A linear equation in n variables is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n = b$$

where

$a_1, a_2, a_3, \dots, a_n$ and b are real numbers
and $x_1, x_2, x_3, \dots, x_n$ are variables

MMA | MMA | Lec | system | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | G1 | G2 | G3 | G123 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | break time | matrix tool | < | >

Windows taskbar: 28°C นิมมานเหมินท์ ENG 9:59 AM 9/18/2023

The Geometer's Sketchpad - [Week10 LT BMA 1303 Linear Algebra.gsp - 6]

File Edit Display Construct Transform Measure Number Graph Window Help

Systems of Linear Equation in Two Variables

A systems of linear equations in two variables has the form

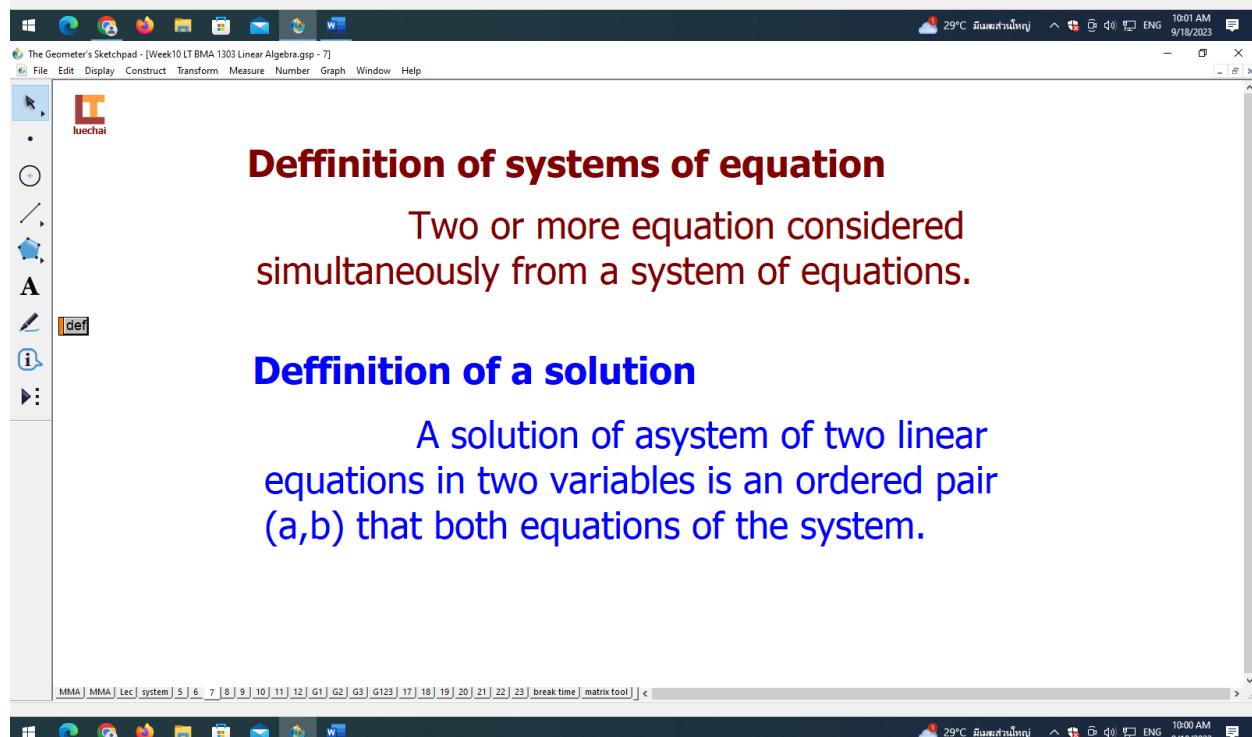
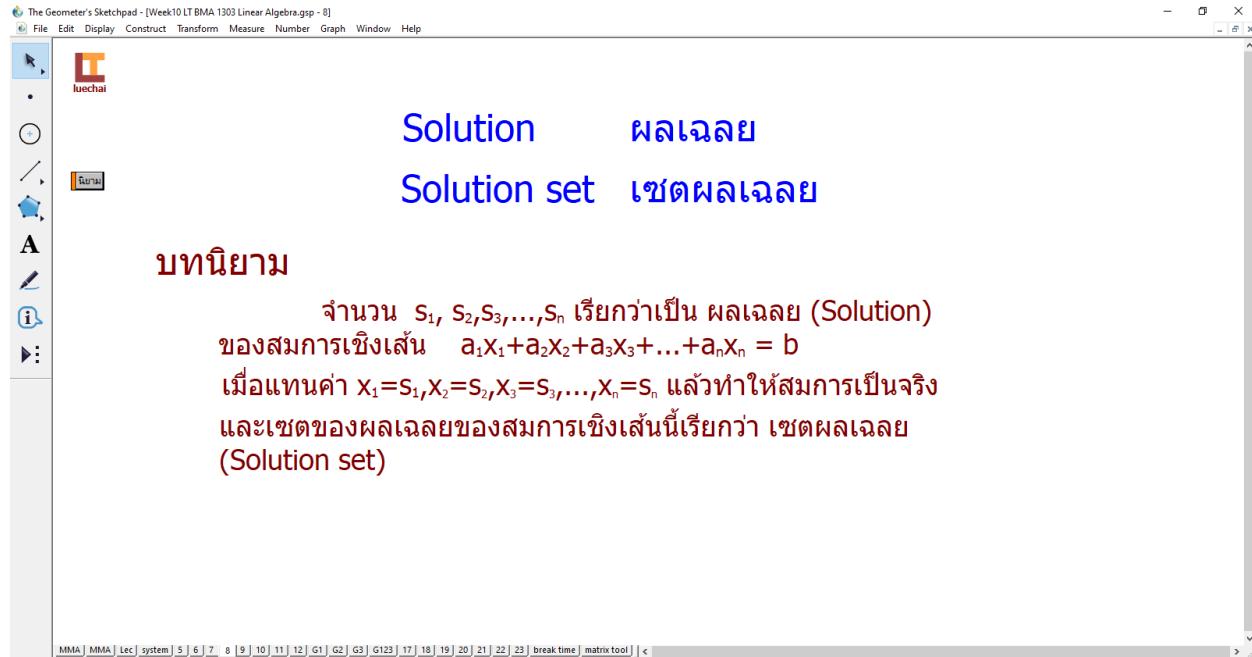
$$a_1x + b_1y = c_1 \quad \text{where } a_1, b_1 \text{ are both non-zero}$$
$$a_2x + b_2y = c_2 \quad \text{and } a_2, b_2 \text{ are both non-zero}$$

The following are some examples.

$$3x - y = 0 \quad 8x - 2y = 5 \quad -2x + 6y = 3$$
$$5x + 2y = 22, \quad -12x + 3y = 7, \quad 4x - 12y = -6$$

MMA | MMA | Lec | system | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | G1 | G2 | G3 | G123 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | break time | matrix tool | < | >

Windows taskbar: 28°C นิมมานเหมินท์ ENG 9:59 AM 9/18/2023



The Geometer's Sketchpad - [Week10 LT BMA 1303 Linear Algebra.gsp - 9]

Example 1 Solve the following system of linear equations using elimination method

$$\begin{aligned} 3x - y &= 0 \\ 5x + 2y &= 22 \end{aligned}$$

Solution 1

We will eliminate y from the equations and solve for x.

$$\begin{aligned} 3x - y &= 0 \quad \dots (1) \\ 5x + 2y &= 22 \quad \dots (2) \\ 2x(1); \quad 6x - 2y &= 0 \quad \dots (3) \\ (2)+(3); \quad 11x &= 22 \\ x &= \frac{22}{11} = 2 \end{aligned}$$

From (1); substitute $x=2$

$$\begin{aligned} 3(2) - y &= 0 \\ y &= 6 \end{aligned}$$

The solution of system is the order pair $(x,y) = (2,6)$
or The solution set is $\{(2, 6)\}$

MMA | MMA | Lec | system | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | G1 | G2 | G3 | G123 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | break time | matrix tool | <

10:02 AM 9/19/2023

The Geometer's Sketchpad - [Week10 LT BMA 1303 Linear Algebra.gsp - 10]

Example 1 Solve the following system of linear equations

$$\begin{aligned} 3x - y &= 0 \\ 5x + 2y &= 22 \end{aligned}$$

Solution2

$$\begin{aligned} 3x - y &= 0 \quad \dots (1) \\ 5x + 2y &= 22 \quad \dots (2) \\ \text{From}(1); \quad y &= 3x \\ \text{substitute } y = 3x \text{ into (2)} \\ 5x + 2(3x) &= 22 \\ 11x &= 22 \\ x &= 2 \end{aligned}$$

substitute $x = 2$ into equation (1)

$$\begin{aligned} 3(2) - y &= 0 \\ y &= 6 \end{aligned}$$

The solution set is $\{(2, 6)\}$

MMA | MMA | Lec | system | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | G1 | G2 | G3 | G123 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | break time | matrix tool | <

10:02 AM 9/18/2023

The Geometer's Sketchpad - [Week10 LT BMA 1303 Linear Algebra.gsp - 11]

File Edit Display Construct Transform Measure Number Graph Window Help

Example 2 Solve the following system of linear equations

$$\begin{aligned} 8x - 2y &= 5 \\ -12x + 3y &= 7 \end{aligned}$$

Solution

$$\begin{aligned} 8x - 2y &= 5 \dots (1) \\ -12x + 3y &= 7 \dots (2) \\ 3x(1); \quad 24x - 6y &= 15 \dots (3) \\ 2x(2); \quad -24x + 6y &= 14 \dots (4) \\ (3)+(4); \quad 0 &= 29 \end{aligned}$$

The resulting false statement indicates that there is no solution of the system of equations.

Therefore, the solution set is the empty set, and the system is inconsistent.

MMA | MMA | Lec | system | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | G1 | G2 | G3 | G123 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | break time | matrix tool | < | >

Windows taskbar: MMA, browser, mail, etc. | ENG 10:03 AM 9/18/2023

The Geometer's Sketchpad - [Week10 LT BMA 1303 Linear Algebra.gsp - 12]

File Edit Display Construct Transform Measure Number Graph Window Help

Example 3 Solve the following system of linear equations

$$\begin{aligned} -2x + 6y &= 3 \\ 4x - 12y &= -6 \end{aligned}$$

Solution

$$\begin{aligned} -2x + 6y &= 3 \dots (1) \\ 4x - 12y &= -6 \dots (2) \\ 2x(1); \quad -4x + 12y &= 6 \dots (3) \\ (2)+(3); \quad 0 &= 0 \end{aligned}$$

Since the two equations are exactly the same. Therefore, the lines coincide; we can conclude that every point on the line is a solution of the system.

Therefore, the system has infinitely many solutions.

MMA | MMA | Lec | system | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | G1 | G2 | G3 | G123 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | break time | matrix tool | < | >

Windows taskbar: MMA, browser, mail, etc. | ENG 10:03 AM 9/18/2023

The Geometer's Sketchpad - [Week10 LT BMA 1303 Linear Algebra.gsp - G1]

File Edit Display Construct Transform Measure Number Graph Window Help

The system of linear equations from example 1

$$3x - y = 0$$

$$5x + 2y = 22$$

We can draw both graphs as follows :

$$y = 3x \quad f(x) = 3 \cdot x$$

$$y = \frac{-5}{2}x + 11 \quad g(x) = \left(\frac{-5}{2}\right)x + 11$$

The two lines intersect at one point. The system has exactly one solution.

MMA MMA Lec system 5 6 7 8 9 10 11 12 G1 G2 G3 G123 17 18 19 20 21 22 23 break time matrix tool < 29°C 29°C 9/18/2023 10:04 AM ENG

The Geometer's Sketchpad - [Week10 LT BMA 1303 Linear Algebra.gsp - G2]

File Edit Display Construct Transform Measure Number Graph Window Help

The system of linear equations from example 2

$$8x - 2y = 5$$

$$-12x + 3y = 7$$

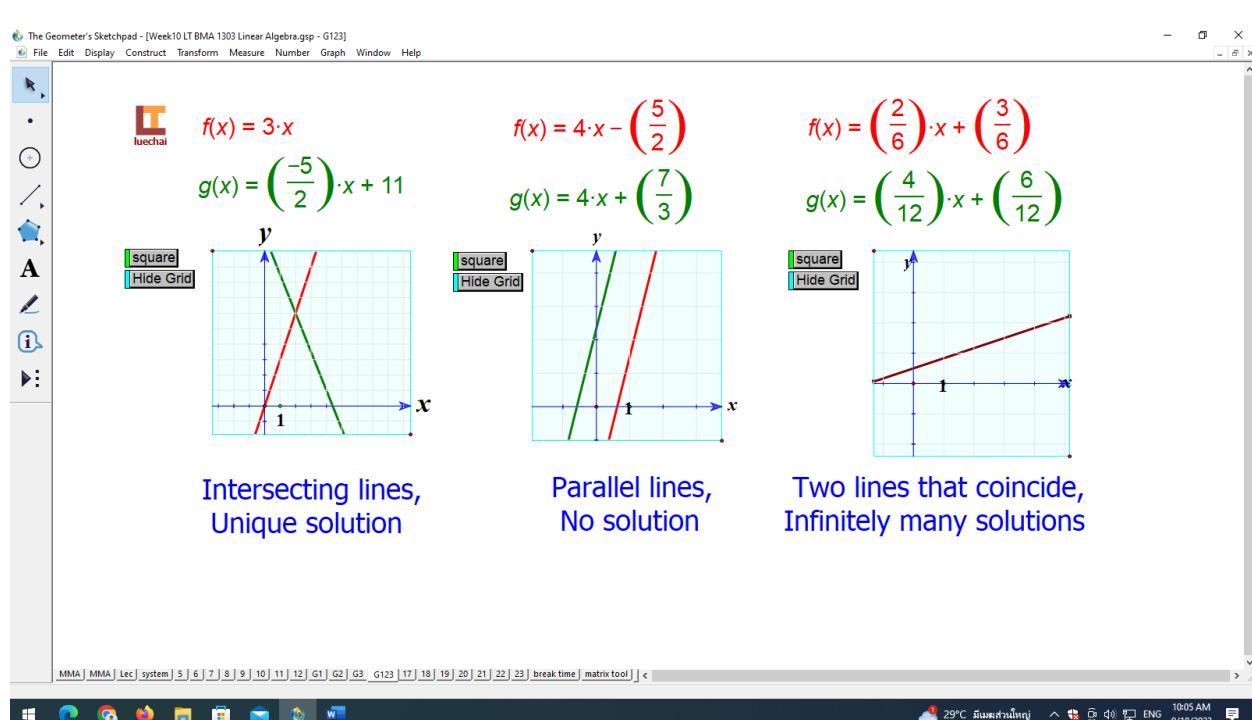
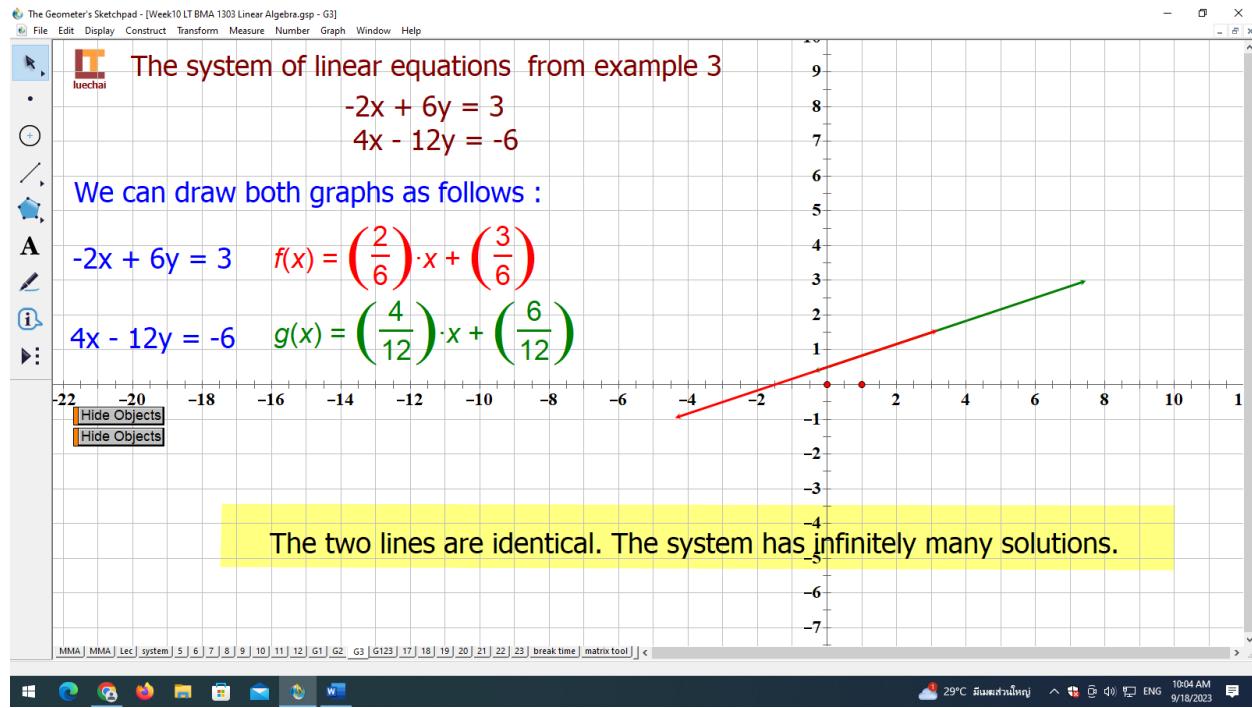
We can draw both graphs as follows :

$$8x - 2y = 5 \quad f(x) = 4 \cdot x - \left(\frac{5}{2}\right)$$

$$-12x + 3y = 7 \quad g(x) = 4 \cdot x + \left(\frac{7}{3}\right)$$

The two lines are parallel. The system has no solution.

MMA MMA Lec system 5 6 7 8 9 10 11 12 G1 G2 G3 G123 17 18 19 20 21 22 23 break time matrix tool < 29°C 29°C 9/18/2023 10:04 AM ENG



The Geometer's Sketchpad - [Week10 LT BMA 1303 Linear Algebra.gsp - 17]

File Edit Display Construct Transform Measure Number Graph Window Help

Exercises

Solve the system of linear equations and graph the lines.

1. $-x + 2y = 2$
 $3x + y = 15$

3. $3x + 2y = 2$
 $6x + 4y = 14$

2. $x - 3y = 5$
 $-2x + 6y = -10$

4. $3x - 2y = 6$
 $-6x + 4y = -12$

MMA MMA Lec system 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | G1 | G2 | G3 | G123 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | break time | matrix tool | <

DII -0.83% ENG 10:05 AM 9/18/2023

The Geometer's Sketchpad - [Week10 LT BMA 1303 Linear Algebra.gsp - 18]

File Edit Display Construct Transform Measure Number Graph Window Help

Systems of Linear Equations in Three or more Variables

Example Solve the following system of linear equations.

Solution

$x - 2y + 3z = 10$
 $y + 3z = 4$
 $z = 3$

sustitute $z=3$ into (2)
 $y + 3(3) = 4 \rightarrow y = -5$

sustitute $y=-5$ and $z=3$ into (1)
 $x - 2(-5) + 3(3) = 10 \rightarrow x = -9$

The solution is
 $x=-9, y=-5$ and $z=3$

which can be written as
the ordered triple $(-9, -5, 3)$

or The solution set is $\{(-9, -5, 3)\}$

MMA MMA Lec system 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | G1 | G2 | G3 | G123 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | break time | matrix tool | <

DII -0.83% ENG 10:06 AM 9/18/2023

The Geometer's Sketchpad - [Week10 LT BMA 1303 Linear Algebra.gsp - 19]
File Edit Display Construct Transform Measure Number Graph Window Help

Example Solve the following system of linear equations.

Solution

$$4x + y - 3z = 11 \quad \dots (1)$$
$$2x - 3y + 2z = 9 \quad \dots (2)$$
$$x + y + z = -3 \quad \dots (3)$$

Using elimination to solve a sistem

The solution set is $\{(2, -3, -2)\}$ **[ans]**

MMA | MMA | Lec | system | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | G1 | G2 | G3 | G123 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | break time | matrix tool | < | >

DII -0.83% ENG 10:06 AM 9/18/2023