Misconceptions in Maths:

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The Problem:

"Misconceptions" is a term used to describe deep, intuitive misunderstandings about mathematics. These form when students get an idea in their heads about how something works that makes intuitive sense to them. Misconceptions heavily influence a student's ability to learn and retain mathematics in class and, in many cases, are the cause of great confusion to the student. Until these misconceptions are diagnosed, confronted and abandoned they cause problems.

When we see students answering questions in really weird ways we tend to respond in one of these two ways:

1. Tell them that they are doing it wrong, and proceed to give them detailed steps to follow to get the answer right. We help them to practice the steps until they can "remember" them. The problem is that students are left trying to memorise steps that are in conflict with their intuitive understanding of maths because they haven't changed their own mind about the misconception first. We can't fix an understanding problem with memorisation.
2. Open up the problem by asking them to "try something else" - randomly guessing until they hit on the correct response and then trying to remember that one.

Misconceptions are a problem with understanding, not with fluency or memory. The kids fundamentally don't get a concept. They need to have this problem fixed before they can successfully learn a new concept. Otherwise we keep chasing our tails as students keep "forgetting" what they have "learned".

We need to create misconception-shattering, mind-altering, light-bulb moments.
And we can't do this by simply telling students that they are wrong
- we need to get them to change their own minds.

The process:

In Back-to-Front Maths, we have developed a unique process of dealing with student misconceptions - instead of telling them that they're wrong, we confront the problem head-on by taking students along a logical-thinking path of answering questions until they decide that their initial idea would never work and abandon it. We call this, taking things to their illogical extreme. Here's how it works:

Confrontational questions:

Firstly, instead of opening up the problem ("why don't you try something else?"), we narrow it. By using a sequence of closed questions we limit the possible responses, and begin to present the student with situations that challenge and confront their thinking. They aim to point out the illogicality of thinking. These questions become more and more narrow, forcing the student to reconsider their initial answer each time until they realise the problem for themselves.

For example,
Students were asked to model 23.7 out of MAB blocks (Grade 5 Journal). James made 23 blocks,

then drew a dot on the page and made another 7 blocks. He did not understand that 7 tenths need to be smaller than ones.

The teacher, following the Back-to-Front Maths questions in the lesson plan did this to confront the misconception:

- Push all of the blocks back together and ask if it is still 23.7. Then separate the 23 blocks from the 7 and ask if it is 23.7 now. Lots of students usually answer *yes*. You will need to repeat the demonstration from the introduction using just 23. If they keep thinking that separating 23 from 7 makes it 23.7 then hold up your hands and ask how many fingers you have. Put one MAB on your head (to be the dot) and hold your hands apart. Ask how many fingers you have now. Watch out because lots of students say 5.5! You can then use this to challenge the idea (*Really? Did I chop one in half?*)
- Make 23 from MAB and move slowly to put just one more MAB with it. Ask if this is ever going to work (they should answer, *no that's 24*). Ask if they think that the "decimals" are going to be as big as this one. They should decide at this point that they will need to be smaller. Then prompt them saying, *"What if you could cut the MAB into as many pieces as you wanted to make the point seven? Could you draw what you would do?"*

These questions forced James to realise that what he had made was actually 30, not 23.7. He realised that the numbers after the decimal point must be SMALLER than ones - he abandoned his misconception and was ready to try something else.

He went back to his initial idea, and decided to use half-sized blocks to model the decimal number (23 normal blocks, and 7 half-blocks). This showed another misconception - the misunderstanding of the "base-ten" nature of place value. The teacher, again following the lesson plan tried these:

- Draw each of the "cutting the MAB" ideas on the board in 3D. Then "glue" them back together by rubbing out the lines. Ask "*is it still 23.7?*" At this point students can exclude any ideas that use a whole block or more than a whole block.

James quickly realised that half-blocks would end up making 3.5, which when put together with the 23 would make 26.5, not something that is between 23 and 24. He then tried cutting the blocks into quarters and sevenths, both of which made 24 or bigger when put back together. He was stuck.

Leading Questions:

Once a student has abandoned their misconceptions, the second step is to lead them to make connections to what they already know. Now was the time to lead him to discover the base-ten nature of decimal numbers, connecting it with his previous knowledge of the base-ten nature of whole numbers. The teacher, again following the Back-to-Front Maths lesson plan did this:

- Focus on the patterns between the MAB using this line of questioning once you get to the point where students are debating between representations such as 3/4, 1/7, 7/8 and 7/10. You will need a 1, 10, 100 and 1000 MAB. It is also helpful to have 10 x 1000 blocks to build a 10 000 stack. Here's how:
    - Hold a one MAB in one of your hands and a ten MAB in the other. Ask: "What is this block called (*one*), and what's this one (*ten*)? How many of these (*ones*) would I use to build this (*ten*)?"
    - Repeat using a ten and a hundred, then a hundred and a thousand.

- Hold a thousand MAB. Ask what the next number is called (*ten thousand*). Ask how many thousands you would need to build a ten thousand. Build it.
- Now work backwards from ten thousand, asking "If I only had this block (*larger one, e.g. ten thousand*), how could I use it to make this one (*small block, e.g. thousand*)?" The kids should tell you "Cut it in ten bits". Repeat all the way back to one MAB.
- Then hold one MAB in one hand, and ask, "So how would I make the next one?" Stop before they tell you and ask if they want to change their minds on how to draw 0.7.

James had a light-bulb moment. It would take 10 tenths to make one! Therefore to make 0.7 he would need 7 tenths! It couldn't possibly be anything else because the size wouldn't work! It fit the pattern, it joined the dots, he had made the connection.

The Outcome:

Light-bulb moments create "forever" learning. They change our fundamental ways of thinking and we are never the same again. Learning does not actually progress "a little bit at a time" in a kind of steady process of building upon previous concepts. Real learning happens in leaps and bounds - all at once we understand something that we previously hadn't. That's why most school learning is ineffective. That's why we have to keep reteaching concepts. That's why students just "don't get maths".

We have to stop evaluating the effectiveness of our teaching by what students can remember immediately after we have taught it, and start thinking in terms of long-term learning. *What will my students still know <u>after</u> the Christmas holidays*? If we can't change that, then we are basically just wasting our time. Someone else is going to have to go back and reteach it all next year anyway.

But we can make misconception-shattering, mind-altering changes to students. We can create life-long understanding! It's just that we can't do it by telling a student that they're wrong or trying to fix it all for them. We have to start realising that the most effective way of teaching is by actually not "teaching", but by helping the student to question what they think. By enabling them to change their own minds, not doing it for them. By using problem-based teaching to create light-bulb moments.