Chapter- 4 ‖ Special Multiplication Methods

Multiplication in considered as one of the most difficult of the four mathematical operations. Students are scared of multiplication as well as tables. Just by knowing tables up to 5 students can multiply bigger numbers easily by some special multiplication methods of Vedic Mathematics. We should learn and encourage children to look at the special properties of each problem in order to understand it and decide the best way to solve the problem. In this way we also enhance the analytical ability of a child. Various methods of solving the questions /problems keep away the monotonous and charge up student's mind to try new ways and in turn sharpen their brains.

Easy way for multiplication

Sutra:Vertically and Cross wise :

For speed and accuracy tables are considered to be very important. Also students think why to do lengthy calculations manually when we can do them faster by calculators. So friends/ teachers we have to take up this challenge and give our students something which is more interesting and also faster than a calculator. Of course it's us (the teachers/parents) who do understand that more we use our brain, more alert and active we will be for, that is the only exercise we have for our brain.

Example 1: 7 x8

Step 1: Here base is 10,

$7 - 3$ (7 is 3 below 10) also called deficiencies

$\times\ 8 - 2$ (8 is 2 below 10) also called deficiencies

Step 2: Cross subtract to get first figure (or digit) of the answer: $7 - 2 = 5$ or $8 - 3 = 5$, the two difference are always same.

Step 3 : Multiply vertically *i.e.* $-3 \times -2 = 6$ which is second part of the answer.

So, $7 - 3$

$\underline{8 - 2}$ *i.e.* $7 \times 8 = 56$

$5 / 6$

Example 2: To find 6×7

Step 1 : Here base is 10,

$6 - 4$ (6 is 4 less than 10) *i.e.* deficiencies

$7 - 3$ (7 is 3 less than 10) *i.e.* deficiencies

Step 2: Cross subtraction : $6 - 3 = 3$ or $7 - 4 = 3$ (both same)

Step 3: $- 3 \times - 4 = + 12$, but 12 is 2 digit number so we carry this 1 over to 3 (obtained in 2 step)

$6 - 4$

$\underline{7 - 3}$

$3 / (1)\ 2$ *i.e.* $6 \times 7 = 42$

Try these : (1) 9×7 (ii) 8×9 (iii) 6×9 (iv) 8×6 (v) 7×7

Second Method:

Same Base Method :

When both the numbers are more than the same base. This method is extension of the above method i.e. we are going to use same sutra here and applying it to larger numbers.

Example 1: 12 × 14

Step 1: Here base is 10

12 + 2 [12 is 2 more than 10 also called surplus]

14 + 4 [14 is 4 more than 10also called surplus]

Step 2: Cross add: 12 + 4 =16 or 14 + 2 = 16,(both same) which gives first part of answer = 16

Step 3: Vertical multiplication: 2 × 4 = 8

So, 12 + 2

<u>14 +4</u>

16 / 8So, 12 × 14 = 168

(14 + 2 = 12 + 4)

Example 2:105x 107

Step1: Here base is 100

105 + 05 [105 is 5 more than 100 or 5 is surplus]

107 + 07 [107 is 7 more than 100 or 7 is surplus]

Base here is 100 so we will write 05 in place of 5and 07 in place of 7

Step 2: Cross add: 105 + 7 = 112 or 107 + 5 = 112 which gives first part of the answer = 112

Step 3: Vertical multiplication: 05 × 07 = 35 (two digits are allowed)

As the base in this problem is 100 so two digits are allowed in the second part.

So, 105 × 107 = 11235

Example 3: 112 x 115

Step 1: Here base is 100

112 + 12 [2 more than 100 i.e. 12 is surplus]

115 + 15 [15 more than 100 i.e. 15 is surplus]

Step 2: Cross add: 112 + 15 = 127 = 115 + 12 to get first part of answer

*i.e.*127

Step 3: Vertical multiplication 12 × 15 = ? Oh, my god!It's such a big number. How to get product of this? Again use the same method to get the product.

12 + 2

<u>15 + 5</u>

12 + 5 = 15 + 2 = 17/ (1) 0, 17 + 1 / 0 = 180 *i.e.* 12 × 15 = 180

But only two digits are allowed here, so 1 is added to 127 and we get (127 + 1) = 128

So, 112 × 115 = 128, 80

Try these: (i)12 × 14 (ii) 14 × 17 (iii) 17 × 19 (iv) 19 × 11 (v) 11 × 16 (vi) 112 × 113 (vii) 113 × 117 (viii) 117 × 111 (ix) 105 × 109 (x) 109 × 102 (xi) 105 × 108 (xii) 108 × 102 (xiii) 102 × 112 (xiv) 112 × 119 (xv) 102 × 115

Both numbers less than the same base:

Same sutra applied to bigger numbers which are less than the same base.

Example1: 99 × 98

Step 1: Check the base: Here base is 100 so we are allowed to have two digits on the right hand side.

∴ 99 – 01 (1 less than 100) i.e. 01 deficiency

98 – 02 (2 less than 100) i.e. 0 2 deficiency

Step 2: Cross – subtract: 99 – 02 = 97 = 98 – 01 both same so first part of answer is 97

Step3: Multiply vertically – 01 × – 02 = 02 (As base is 100 so two digits are allowed in second part

So, 99 × 98 = 9702

Example 2 : 89 × 88

Step1: Here base is 100

So, 89 – 11 (i.e. deficiency = 11)

88 – 12 (i.e. deficiency = 12)

Step2: Cross subtract: 89 – 12 = 77 = 88 – 11(**both same)**

So, first part of answer can be 77

Step 3: Multiply vertically – 11 × – 12

Again to multiply 11 × 12 apply same rule

11 + 1 (10 + 1)

12 + 2 (10 + 2)

11 + 2 = 13 = 12 + 1 / 1 × 2 = 12 so, 11 × 12 = (1) 32 as only two digits are allowed on right hand side so add 1to L.H.S.

So, L.H.S. = 77 + 1 = 78

Hence 89 × 88 = 7832

Example 3: 988 × 999

Step 1: As the numbers are near 1000 so the base here is 1000 and hence three digits allowed on the right hand side

988 – 012 (012 less than 1000) i.e. deficiency = 0 12

999 – 001 (001 less than 1000) i.e. deficiency = 00 1

Step 2: Cross – subtraction: 988 – 001 = 987 = 999 – 012 = 987

So first part of answer can be 987

Step 3: Multiply vertically: –012 xs – 001 = 012 (three digits allowed)

∴ 988 × 999 = 987012

How to check whether the solution is correct or not by 9 – check method.

Example 1: 99 × 98 = 9702 Using 9 – check method.

As, $\cancel{9}\cancel{9} = 0$ Product (L.H.S.) = 0 × 8 = 0 [taking 9 = 0]

$\cancel{9}8 = 8$

R.H.S. = $\cancel{9}702 = 7 + 2 = \cancel{9} = 0$ $\cancel{9}702 = 9$ both are same

As both the sides are equal answer may be correct.

Example 2: 89 × 88 = 7832

$8\cancel{9} = 8$

88 = 8 + 8 = 16 = 1 + 6 = 7 (add the digits)

L.H.S. = 8 × 7 = 56 = 5 + 6 = 11 = 2 (1 + 1)

R.H.S. = $\cancel{7}83\cancel{2}$ = 8 + 3 = 11 = 1 + 1 = 2

As both the sides are equal, so answer is correct

Example 3: 988 × 999 = 987012

$\cancel{9}88$ = 8 + 8 = 16 = 1 + 6 =7

$\cancel{9}\cancel{9}\cancel{9}$ = 0

As 0 × 7 =0 = LHS

$\cancel{9}\cancel{8}\cancel{7}0\cancel{1}\cancel{2}$ = 0 (As 7 + 2 = 9 = 0 , 8 + 1 = 9 = 0 also 9 = 0)

∴ RHS = 0

As LHS = RHS So, answer is correct.

Try These:

(i) 97 × 99 (ii) 89 × 89 (iii) 94 × 97 (iv) 89 × 92 (v) 93 × 95 (vi) 987 × 998 (vii) 997 × 988 (viii) 988 × 996 (ix) 983 × 998 (x) 877 × 996 (xi) 993 × 994 (xii) 789 × 993 (xiii) 9999 × 998 (xiv) 7897 × 9997 (xv) 8987 × 9996.

Multiplying bigger numbers close to a base: (number less than base)

Example 1: 87798 x 99995

Step1: Base here is 100000 so five digits are allowed in R.H.S.

87798 – 12202 (12202 less than 100000) deficiency is 12202

<u>99995 – 00005</u> (00005 less than100000) deficiency is 5

Step 2: Cross – subtraction: 87798 -00005 =87793

Also 99995 – 12202 = 87793 (both same)

So first part of answer can be 87793

Step 2 : Multiply vertically: –12202 × – 00005 = + 61010

∴ 87798 × 99995 = 8779361010

Checking:

$877\cancel{9}8$ total 8 + 7 + 7 + 8 = 30 = 3 (single digit)

$\cancel{9}\cancel{9}\cancel{9}\cancel{9}5$ total = 5

LHS = 3 x 5 =15 total = 1 + 5 = 6

RHS = product = $\cancel{8}\cancel{7}\cancel{7}\cancel{9}\cancel{3}\cancel{6}\cancel{1}0\cancel{1}0$ total = 15 = 1 + 5 = 6

L.H.S = R.H.S. So, correct answer

Example 2 : 88777 × 99997

Step 1: Base have is 100000 so five digits are allowed in R.H.S.

88777 – 11223 i.e. deficiency is 11223

<u>99997 – 00003</u> i.e. deficiency is 3

Step 2: Cross subtraction: 88777 – 00003 = 88774 = 99997 – 11223

So first part of answer is 88774

Step 3: Multiply vertically: – 11223 × – 00003 = + 33669

∴ 88777 × 99997 = 8877433669

Checking:

88777 total 8 + 8 + 7 + 7 + 7 = 37 = + 10 = 1

$\cancel{9}\cancel{9}\cancel{9}\cancel{9}7$ total = 7

∴ LHS = 1 × 7 = 7

RHS = $88774\cancel{3}\cancel{3}\cancel{6}6\cancel{9}$ =8 + 8 + 7 + 7 + 4 = 34 = 3 + 4 = 7

i.e. LHS = RHS So, correct answer

Try These:

(i) 999995 × 739984 (ii) 99837 × 99995 (iii) 99998 × 77338 (iv) 98456 × 99993 (v) 99994 × 84321

Multiply bigger number close to base (numbers more than base)

Example 1: 10021 × 10003

Step 1: Here base is 10000 so four digits are allowed

10021 + 0021 (Surplus)

<u>10003 + 0003</u> (Surplus)

Step 2: Cross – addition 10021 + 0003 = 10024 = 10003 + 0021 (both same)

∴ First part of the answer may be 10024

Step 3: Multiply vertically: 10021 × 0003 = 0063 which form second part of the answer

∴ 10021 × 10002 = 100240063

Checking:

10021 = 1+ 2 + 1 + 1 = 4

10003 = 1 + 3 = 4

∴ LHS = 4 × 4 = 16 = 1 + 6 = 7

RHS = $1002400\cancel{63}$ = 1 + 2 + 4 = 7

As LHS = RHS So, answer is correct

Example 2: 11123 × 10003

Step 1: Here base is 10000 so four digits are allowed in RHS

11123 + 1123 (surplus)

<u>10003 + 0003</u> (surplus)

Step 2: Cross – addition: 11123 + 0003 = 11126 = 10003 + 1123 (both equal)

∴ First part of answer is 11126

Step 3: Multiply vertically: 1123 × 0003 = 3369 which form second part of answer

∴ 11123 × 10003 = 111263369

Checking:

11123 = 1 + 1 + 1 + 2 + 3 = 8

10003 = 1 + 3 = 4 and 4 × 8 = 32 = 3 + 2 = 5

∴ LHS = 5

R.H.S = $1112\cancel{63369}$ = 1 + 1 + 1 + 2 = 5

As L.H.S = R.H.S So, answer is correct

Try These:

(i) 10004 × 11113 (ii) 12345 × 111523 (iii) 11237 × 10002 (iv) 100002 × 111523 (v) 10233 × 10005

Numbers near different base: **(Both numbers below base)**

Example 1: 98 × 9

Step 1: 98 Here base is 100 deficiency = 02

9 Base is 10 deficiency = 1

∴ 98 – 02 Numbers of digits permitted on R.H.S is 1 (digits in lower base)

Step 2: Cross subtraction: 98

<u>-1</u>

88

It is important to line the numbers as shown because 1 is not subtracted from 8 as usual but from 9 so as to get 88 as first part of answer.

Step 3: Vertical multiplication: (-02) x (-1) = 2 (one digits allowed)

∴ Second part = 2

∴ 98 × 9 = 882

Checking:

(Through 9 – check method)

$\cancel{9}8 = 8$, $\cancel{9} = 0$, LHS = 98 × 9 = 8 × 0 = 0

RHS = 882 = 8 + 8 + 2 = 18 = 1 + 8 = $\cancel{9}$ = 0

As LHS = RHS So, correct answer

Example 2: 993 × 97

Step 1: 993 base is 1000 and deficiency is 007

97 base is 100 and deficiency is 03

∴ 993 – 007 (digits in lower base = 2 So, 2 digits are permitted on

× 97 – 03 RHS or second part of answer)

Step 2: Cross subtraction:

993

<u>– 03</u>

963

Again line the number as shown because 03 is subtracted from 99 and not from 93 so as to get 963 which from first part of the answer.

Step 3: Vertical multiplication: (–007) – (–03) = 21 only two digits are allowed in the second part of answer So, second part = 21

∴ 993 × 97 = 96321

Checking: (through 9 – check method)

$\cancel{9}\cancel{9}3 = 3$ $\cancel{9}7 = 7$

∴ L.H.S. = 3 × 7 = 21 = 2 + 1 = 3

R.H.S. = $\cancel{9}\cancel{6}\cancel{3}21$ = 2 + 1 = 3

As LHS =RHS so, answer is correct

Example 3 : 9996 base is 10000 and deficiency is 0004

988 base is 1000 and deficiency is 012

∴ 9996 – 0004 (digits in the lower base are 3 so,3digits

<u>× 988 – 012</u> permitted on RHS or second part of answer)

Step 2 : Cross – subtraction:

9996

<u>– 012</u>

9876

Well, again take care to line the numbers while subtraction so as to get 9876 as the first part of the answer.

Step3 : Vertical multiplication: (–0004) × (–012) = 048

(Remember, three digits are permitted in the second part i.e. second part of answer = 048

∴ $9996 \times 988 = 9876048$

Checking:(9 – check method)

$\cancel{9}\cancel{9}\cancel{9}6 = 6$, $\cancel{9}88 = 8 + 8 + = 16 = 1 + 6 = 7$

∴ LHS = $6 \times 7 = 42 = 4 + 2 = 6$

RHS = $\cancel{9}87\cancel{6}0\cancel{4}5 = 8 + 7 = 15 = 1 + 5 = 6$

As, LHS =RHS so, answer is correct

When both the numbers are above base

Example 1: 105×12

Step 1: 105 base is 100 and surplus is 5

12 base is 10 and surplus is 2

∴ 105 + 05 (digits in the lower base is 1 so, 1 digit is permitted in the second part of answer)

12 + 2

Step 2: Cross – addition:

105

<u>+ 2</u>

125 (again take care to line the numbers properly so as to get 125)

∴ First part of answer may be <u>125</u>

Step 3: Vertical multiplication : $05 \times 2 = (1)0$ but only 1 digit is permitted in the second part so 1 is shifted to first part and added to 125 so as to get 126

∴ $105 \times 12 = 1260$

Checking:

105 = 1 + 5 = 6 , 12 = 1 + 2 = 3

∴ LHS = $6 \times 3 = 18 = 1 + 8 = 9 = 0$

∴ RHS = 1260 = 1 + 2 + 6 = 9=0

Example 2: 1122×104

Step1: 1122 – base is 1000 and surplus is 122

104 – base is 100 and surplus is 4

∴ 1122 + 122

<u>104 + 04</u> (digits in lower base are 2 so, 2-digits are permitted in the second part of answer)

Step 2: Cross – addition

1122

<u>+ 04</u> (again take care to line the nos. properly so as to get 1162)

1162

∴ First part of answer may be 1162

Step 3: Vertical multiplication: 122 × 04 = 4, 88

But only 2 – digits are permitted in the second part, so, 4 is shifted to first part and added to 1162 to get 1166 (1162 + 4 = 1166)

∴ 1122 × 104 = 116688

Can be visualised as: 1122 + 122

<u>104 + 04</u>

1162 / ← (4) 88 = 116688

+ 4 /

Checking:

1122 = 1 + 1 + 2 + 2 + = 6, 104 = 1 + 4 =5

∴ LHS = 6 × 5 = 30 = 3

RHS = $\cancel{1}\cancel{1}66\cancel{8}\cancel{8}$ = 6 + 6 = 12 = 1 + 2 = 3

As LHS = RHS So, answer is correct

Example 3: 10007 × 1003

Now doing the question directly

10007 + 0007 base = 10000

<u>× 1003 + 003</u> base = 1000

10037 / 021 (three digits per method in this part)

∴ 10007 × 10003 = 10037021

Checking : 10007 = 1 + 7 = 8 , 1003 = 1 + 3 = 4

∴ LHS = 8 × 4 = 32 = 3 + 2 = 5

RHS = $1003\cancel{7}$ $0\cancel{2}1$ = 1 + 3 + 1 = 5

As LHS = RHS so, answer is correct

Try These:

(i) 1015 × 103 (ii) 99888 × 91 (iii) 100034 × 102 (iv) 993 × 97 (v) 9988 × 98 (vi) 9995 × 96 (vii) 1005 × 103 (viii) 10025 × 1004 (ix) 102 × 10013 (x) 99994 × 95

VINCULUM: "Vinculum" is the minus sign put on top of a number e.g. $\overline{5}, 4\overline{1}, 6\overline{3}$ etc. which means (–5), (40 – 1) , (60 – 3) respectively

Advantages of using vinculum:

(1) It gives us flexibility, we use the vinculum when it suits us .

(2) Large numbers like 6, 7, 8, 9 can be avoided.

(3) Figures tend to cancel each other or can be made to cancel.

(4) 0 and 1 occur twice as frequently as they otherwise would.

Converting from positive to negative form or from normal to vinculum form:

Sutras: All from 9 the last from 10 and one more than the previous one

$9 = 1\overline{1}$ (i.e. $10 - 1$), $8 = 1\overline{2}$, $7 = 1\overline{3}$, $6 = 1\overline{4}$, $19 = 2\overline{1}$, $29 = 3\overline{1}$

$28 = 3\overline{2}$, $36 = 4\overline{4}$ $(40 - 4)$, $38 = 4\overline{2}$

Steps to convert from positive to vinculum form:

(1) Find out the digits that are to be converted i.e. 5 and above.

(2) Apply "all from 9 and last from 10" on those digits.

(3) To end the conversions "add one to the previous digit".

(4) Repeat this as many times in the same number as necessary.

Numbers with several conversions:

$159 = 2\overline{41}$ (i.e. $200 - 41$)

$168 = 2\overline{32}$ (i.e. $200 - 32$)

$237 = 2\overline{43}$ (i.e. $240 - 7$)

$1286 = 13\overline{14}$ (i.e. $1300 - 14$)

$2387129 = 24\overline{13}13\overline{1}$ (here, only the large digits are be changed)

From vinculum back to normal form:

Sutras: "All from 9 and last from ten" and "one less than then one before".

$1\overline{1} = 09$ $(10 - 1)$, $1\overline{3} = 07$ $(10 - 3)$, $2\overline{4} = 16$ $(20 - 4)$, $2\overline{41} = 200 - 41 = 159$, $16\overline{2} = 160 - 2 = 158$

$2\overline{22} = 200 - 22 = 178$ $13\overline{14} = 1300 - 14 = 1286$, $24\overline{13}131 = 2387129$ can be done in part as

$131 = 130 - 1 = 129$ and $24\overline{13} = 2400 - 13 = 2387$

$\therefore \quad 24\overline{13}131 = 2387129.$

Steps to convert from vinculum to positive form:

(1) Find out the digits that are to be converted i.e. digits with a bar on top.

(2) Apply "all from 9 and the last from 10" on those digits

(3) To end the conversion apply "one less than the previous digit"

(4) Repeat this as many times in the same number as necessary

Try These: Convert the following to their vinculum form:

(i) 91 (ii) 4427 (iii) 183 (iv) 19326 (v) 2745 (vi) 7648 (vii) 81513 (viii) 763468 (ix) 73655167 (x) 83252327

Try These: From vinculum back to normal form.

(i) $\overline{14}$ (i) $\overline{21}$ (iii) $\overline{23}$ (iv) $2\overline{31}$ (v) $17\overline{2}$ (vi) $14\overline{13}$ (vii) $23\overline{12}13\overline{2}$ (viii) $24\overline{12}\overline{31}$

(ix) $6\overline{322}33\overline{1}$ (x) $14\overline{14}23\overline{23}$

When one number is above and the other below the base

Example1: 102 × 97

Step 1: Here, base is 100

102 + 02 (02 above base i.e. 2 surplus)

97 – 03 (03 below base i.e. 3 deficiency)

Step 2: Divide the answer in two parts as 102 / + 02

97 / – 03

Step 3: Right hand side of the answer is (+ 02) × (– 03) = – 06 = $\overline{06}$

Step 4: Left hand side of the answer is 102 – 3 = 99 = 97 + 02 (same both ways)

∴ 102 × 97 = 99$\overline{06}$ = 9894 (i.e. 9900 – 6 = 9894)

Checking: 102 = 1 + 2 = 3, 9 7 = 7

∴ L.H.S. = 3 × 7 = 21 = 1 + 2 = 3

∴ R.H.S = 9894 = 8 + 4 = 12 = 1 + 2 = 3

As L.H.S. = R.H.S. So, answer is correct

Example 2 : 1002 × 997

1002 / + 002 (006 = 1000 – 6 = 994 and 1 carried from 999 to 999 reduces to 998)

997 / – 003

999 / $\overline{006}$

∴ 1002 × 997 = 998 994

When base is not same:

Example1: 988 × 12

988 / – 012 base is 1000 deficiency 12

12 / + 2 base is 10 surplus is 2, 1 digit allowed in R.H.S.

1188 – 2 / $\overline{024}$

= 1186 / = (2)$\overline{4}$

∴ 988 × 12 = 1186 $\overline{4}$ = 11856 (because $\overline{4}$ = 10 – 4 = 6)

Checking: $\cancel{9}$88 = 8 + 8 = 16 = 1 + 6 = 7, 12 = 1 + 2 = 3

∴ LHS = 7 × 3 = 21 = 2 + 1 = 3

R.H.S = 11856 = 1 + 5 + 6 = 12 = 1 + 2 = 3

As LHS = RHS So, answer is correct

Example 2: 1012 × 98

1012	1012	+ 012		(base is 1000, 12 surplus (+ve sign)
– 02	98	– 02		(base is 100, 2 deficiency (–ve sign)
992	992	$\overline{24}$		[As 012 × (– 02) = – 24] 2 digits allowed in RHS of

Answer

∴ 1012 × 98 = 99224 = 99176 [As 992200 – 24 = 99176]

Checking:1012 = 1 + 1 + 2 = 4, 98 = 8

LHS = 4 × 8 = 32 = 3 + 2 = 5

RHS = 99176 = 1 + 7 + 6 = 14 = 1 + 4 = 5

As RHS = LHS so, answer is correct

Try These:

(i) 1015 × 89 (ii) 103 × 97 (iii) 1005 × 96 (iv) 1234 × 92 (v) 1223 × 92 (vi) 1051 × 9 (vii) 9899 × 87 (viii) 9998 × 103 (ix) 998 × 96 (x) 1005 × 107

Sub – base method:

Till now we have all the numbers which are either less than or more than base numbers. (i.e.10, 100, 1000, 10000 etc. , now we will consider the numbers which are nearer to the multiple of 10, 100, 10000 etc. i.e. 50, 600, 7000 etc. these are called sub-base.

Example: 213 × 202

Step1: Here the sub base is 200 obtained by multiplying base 100 by 2

Step 2: R. H. S. and L.H.S. of answer is obtained using base- method.

213	+ 13
202	+ 02

215 13 × 02 = 26

Step 3: Multiply L.H.S. of answer by 2 to get 215 × 2 = 430

∴ 213 × 202 = 43026

∴

Example 2: 497 × 493

Step1: The Sub-base here is 500 obtained by multiplying base 100 by 5.

Step2: The right hand and left hand sides of the answer are obtained by using base method.

Step3: Multiplying the left hand side of the answer by 5.

	497	–03
	493	–07
Same	497–07 = 490	21
	493 – 03 = 490	
	490 × 5	
	= 2450	

∴ 497 × 493 = 245021

Example 3: 206 × 197

Sub-base here is 200 so, multiply L.H.S. by 2

$$
\begin{array}{r|l}
206 & +\,06 \\
197 & -\,03 \\
\hline
206 - 3 = 203 & -18 \\
197 + 06 = 203 \times 2 & = 18 \\
= 406 &
\end{array}
$$

∴ 206 × 197 = $406\overline{18}$ = 40582

Example 4: 212 × 188

Sub – base here is 200

$$
\begin{array}{r|l}
212 & +\,12 \\
188 & -12 \\
\hline
200 - 12 = 200 & (1)44 \\
188 + 12 = 200 & \\
\times\, 2 & \\
\hline
400 - 1 = 399 &
\end{array}
$$

∴ 212 × 188 = 399 $\overline{44}$ = 39856

Checking:(11 – check method)

+ – +

2 1 2 = 2 + 2 – 1 = 3

+ – +

1 8 8 = 1 – 8 + 8 = 1

L.H.S. = 3 × 1 = 3

+ – + – +

R.H.S. = 3 9 8 5 6 = 3

As L.H.S = R.H.S. So, answer is correct.

Try these

(1) 42 × 43	(2) 61 × 63	(3) 8004 × 8012	(4) 397 × 398	(5) 583 × 593
(6) 7005 × 6998	(7) 499 × 502	(8) 3012 × 3001	(9) 3122 × 2997	(10) 2999 × 2998

Doubling and Making halves

Sometimes while doing calculations we observe that we can calculate easily by multiplying the number by 2 than the larger number (which is again a multiple of 2). This procedure in called **doubling:**

$35 \times 4 = 35 \times 2 + 2 \times 35 = 70 + 70 = 140$

$26 \times 8 = 26 \times 2 + 26 \times 2 + 26 \times 2 + 26 \times 2 = 52 + 52 + 52 + 52$

$= 52 \times 2 + 52 \times 2 = 104 \times 2 = 208$

$53 \times 4 = 53 \times 2 + 53 \times 2 = 106 \times 2 = 212$

Sometimes situation is reverse and we observe that it is easier to find half of the number than calculating 5 times or multiples of 5. This process is called

Making halves:

4. (1) $87 \times 5 = 87 \times 5 \times 2/2 = 870/2 = 435$

(2) $27 \times 50 = 27 \times 50 \times 2/2 = 2700/2 = 1350$

(3) $82 \times 25 = 82 \times 25 \times 4/4 = 8200/4 = 2050$

Try These:

(1) 18×4

(2) 14×18

(3) 16×7

(4) 16×12

(5) 52×8

(6) 68×5

(7) 36×5

(8) 46×50

(9) 85×25

(10) 223×50

(11) 1235×20

(12) 256×125

(13) 85×4

(14) 102×8

(15) 521×25

Multiplication of Complimentary numbers :

Sutra: **By one more than the previous one.**

This special type of multiplication is for multiplying numbers whose first digits(figure) are same and whose last digits(figures)add up to 10,100 etc.

Example 1: 45×45

Step I: $5 \times 5 = 25$ which form R.H.S. part of answer

Step II: $4 \times$ (next consecutive number) i.e. $4 \times 5 = 20$, which form L.H.S. part of answer

$\therefore$ $45 \times 45 = 2025$

Example 2: $95 \times 95 = 9 \times 10 = 90/25 \longrightarrow (5^2)$

i.e. $95 \times 95 = 9025$

Example 3: $42 \times 48 = 4 \times 5 = 20/16 \longrightarrow (8 \times 2)$

$\therefore$ $42 \times 48 = 2016$

Example 4: $304 \times 306 = 30 \times 31 = 930/24 \longrightarrow (4 \times 6)$

$\therefore$ $304 \times 306 = 93024$

Try These:

(1) 63×67

(2) 52×58

(3) 237×233

(4) 65×65

(5) 124×126

(6) 51×59

(7) 762×768

(8) 633×637

(9) 334×336

(10) 95×95

Multiplication by numbers consisting of all 9's :

Sutras: 'By one less than the previous one' and 'All from 9 and the last from 10'

When number of 9's in the multiplier is same as the number of digits in the multiplicand.

Example 1 : 765×999

Step I : The number being multiplied by 9's is first reduced by 1

i.e. $765 - 1 = 764$ This is first part of the answer

Step II : "All from 9 and the last from 10" is applied to 765 to get 235, which is the second part of the answer.

$\therefore$ $765 \times 999 = 764235$

When 9's in the multiplier are more than multiplicand

Example II : 1863×99999

Step I : Here 1863 has 4 digits and 99999 have 5-digits, we suppose 1863 to be as 01863. Reduce this by one to get 1862 which form the first part of answer.

Step II: Apply 'All from 9 and last from 10' to 01863 gives 98137which form the last part of answer

∴ 1863 x 99999 = 186298137

When 9's in the multiplier are less than multiplicand

Example 3 : 537 x 99

Step I: Mark off two figures on the right of 537 as 5/37, one more than the L.H.S. of it i.e. (5+1) is to be subtracted from the whole number, 537 – 6 = 531this forms first part of the answer

Step II: Now applying "all from 9 last from 10" to R.H.S. part of 5/37 to get 63 (100 – 37 = 63)

∴ 537 x 99 = 53163

Try these

(1) 254 × 999 (2) 7654 × 9999 (3) 879 × 99 (4) 898 × 9999

(5) 423 × 9999 (6) 876 × 99 (7) 1768 × 999 (8) 4263 × 9999

(9) 30421 × 999 (10) 123 × 99999

Multiplication by 11

Example 1: 23 × 11

Step 1 : Write the digit on L.H.S. of the number first. Here the number is 23 so, 2 is written first.

Step 2 : Add the two digits of the given number and write it in between. Here 2 + 3 = 5

Step 3 : Now write the second digit on extreme right. Here the digit is 3. So, 23 × 11 = 253

OR

23 × 11 = 2 / 2+3 / 3 = 253

(Here base is 10 so only 2 digits can be added at a time)

Example 2: 243 × 11

Step 1: Mark the first, second and last digit of given number

First digit = 2, second digit = 4, last digit = 3

Now first and last digits of the number 243 form the first and last digits of the answer.

Step 2: For second digit (from left) add first two digits of the number i.e. 2 + 4 = 6

Step 3: For third digit add second and last digits of the number i.e. 3 + 4 = 7

So, 243 × 11 = 2673

OR

243 × 11 = 2 / 2 + 4 / 4 + 3 / 3 = 2673

Similarly we can multiply any bigger number by 11 easily.

Example 3: 42431 × 11

42431 × 11 = 4 / 4 + 2 / 2 + 4 / 4 + 3 / 3 + 1 / 1 = 466741

If we have to multiply the given number by 111

Example 1: 189 × 111

Step 1: Mark the first, second and last digit of given number

First digit = 1, second digit = 8, last digit = 9

Now first and last digits of the number 189 may form the first and last digits of the answer

Step 2: For second digit (from left) add first two digits of the number i.e. 1 + 8 = 9

Step 3: For third digit add first, second and last digits of the number to get 1 + 8 + 9 = 18 (multiplying by 111, so three digits are added at a time)

Step 4: For fourth digit from left add second and last digit to get, 8 + 9 = 17

As we cannot have two digits at one place so 1 is shifted and added to the next digit so as to get 189 × 111 = 20979

OR

1	1 + 8 = 9	1 + 8 + 9	8 + 9	9
	9 + 1 =	= 18	= 1 7	
1 + 1 = 2	= 1 0	= 18 + 1		
		= 1 9		

∴ 189 × 111 = 20979

Example 2 : 2891 × 111

2	2 + 8	2 + 8 + 9	8 + 9 + 1	9 + 1
10 +	2 = 1	9 + 1 18	+ 1 = 1	0 1
	= 1 2	= 2 0	= 1 9	

2891 × 111 = 320901

Try These:

(1) 107 × 11 (2) 15 × 11 (3) 16 × 111 (4) 112 × 111

(5) 72 × 11 (6) 69 × 111 (7) 12345 × 11 (8) 2345 × 111

(9) 272 × 11 (10) 6231 × 111.

Note: This method can be extended to number of any size and to multiplying by 1111, 11111 etc. This multiplication is useful in percentage also. If we want to increase a member by 10% we multiply it by 1.1

General Method of Multiplication.

<u>Sutra</u>: Vertically and cross-wise.

Till now we have learned various methods of multiplication but these are all special cases, wherenumbers should satisfy certain conditions like near base, or sub base, complimentary to each other etc. Now we are going to learn about a general method of multiplication, by which we can multiply any two numbers in a line. Vertically and cross-wise sutra can be used for multiplying any number.

For different figure numbers the sutra works as follows:

Two digit – multiplication

Example: Multiply 21 and 23

Step1: Vertical (one at a time)

2 [1]
2 [3]

$1 \times 3 = 3$

| 3

Step2: Cross –wise (two at a time)

2 1
2 3

$(2 \times 3 + 2 \times 1) = 8$

/ 8 / 3

Step3: Vertical (one at a time)

[2] 1
[2] 3

$2 \times 2 = 4$

4 / 8 / 3

$\therefore$ $21 \times 23 = 483$

Multiplication with carry:

Example: Multiply 42 and 26

Step1: Vertical

42
<u>26</u>

$2 \times 6 \;\; = 12$

/ / 12

Step2: Cross-wise

4 2
2 6

$4 \times 6 + 2 \times 2$

$24 + 4 = 28$

/ 2_8 / $_1 2$

Step3: Vertical

42
<u>26</u>

$4 \times 2 = 8$

8 / 8 / 2

+ 2 1

= 10 = / 2 / 9

$\therefore$ $42 \times 26 = 1092$

Three digit multiplication:

Example: 212×112

Step1: Vertical (one at a time)

212 ↓
<u>112</u>

$2 \times 2 = 4$

/4

Step2: Cross-wise (two at a time)

2 1 2
<u>1 1 2</u>

$2 \times 1 + 2 \times 1 = 2 + 2 = 4$

/4/4

Step3: Vertical and cross-wise (three at a time)

2 1 2
1 1 2

$2 \times 2 + 2 \times 1 + 1 \times 1 = 4 + 2 + 1 = 7$

/7/4/4

Step4: cross wise (Two at a time)

2 1 2
<u>1 1 2</u>

$2 \times 1 + 1 \times 1 = 2 + 1 = 3$

/3/7/4/4

Step 5: vertical (one at a time)

↓ 2 1 2
<u>1 1 2</u>

$2 \times 1 = 2$

2/3/7/4/4

∴ $212 \times 112 = 23744$

Three digits Multiplication with carry:

Example: 816×223

8 1 6
2 2 3

8×2	$8 \times 2 + 2 \times 1 = 16 + 2 = 18$	$8 \times 3 + 6 \times 2 + 1 \times 2 = 24 + 12 + 2 = 38$	$3 \times 1 + 2 \times 6$ $3 + 12 = 15$	$6 \times 3 = 18$
16				
16 + 2	18 + 3 = 21	38 + 1 = 39	15 + 1 = 16	18

∴ $816 \times 223 = 181968$

Checking by 11 – check method

+ – + – +

8 1 6 = 14 – 1 = 1 3 = 3 – 1 = 2

+ - +

2 2 3 = 3

∴ L.H.S. = $3 \times 2 = 6$

$$\begin{array}{cccccc} - & + & - & + & - & + \\ 1 & 8 & 1 & 9 & 6 & 8 \end{array} \quad = \quad \begin{array}{cc} - & + \\ 1 & 7 \end{array} = 7 - 1 = 6$$

As L.H.S. = R.H.S.

$\therefore$ Answer is correct

Try These:

(1) 342×514 (2) 1412×4235 (3) 321×53 (4) 2121×2112 (5) 302×415

(6) 1312×3112 (7) 5123×5012 (8) 20354×131 (9) 7232×125 (10) 3434×4321

Number Split Method

As you have earlier used this method for addition and subtraction, the same may be done for multiplication also.

For example :

$$\begin{array}{r} 263 \\ \times\ 2 \\ \hline \\ \hline \end{array} \longrightarrow \begin{array}{r|l} 26 & 3 \\ \times\ 2 & \times 2 \\ \hline 52 & 6 \end{array}$$

Note : The split allows us to add 36 + 24 and 42 + 39 both of which can be done mentally

Multiplication of algebraic expressions:

Sutra: Vertically and cross-wise

Example1 : $(x + 3)\ (x + 4)$

$$\begin{array}{c} x + 3 \\ x + 4 \\ \hline x^2 + 7x + 12 \end{array}$$

$$\begin{array}{c|c|c} x \times x & 4x + 3x & 4 \times 3 \\ x^2 & = 7x & = 12 \\ \hline x^2 & 7 \times 12 & \end{array}$$

Example2: $(2x + 5)\ (3x + 2)$

$$\begin{array}{ccc} 2x & + & 5 \\ 3x & + & 2 \\ \hline 6x^2 & +19x\ + & 10 \end{array}$$

$$\begin{array}{c|c|c} (2x) \times (3x) & 4x + 15x & 10 \\ \hline =6x^2 & = 19x & \end{array}$$

Example3: $(x^2 + 2x + 5)\ (x^2 - 3x + 1)$

$$\begin{array}{cccc} x^2 & +\ 2x & -\ 5 & \\ x^2 & -\ 3x & +\ 1 & \\ \hline x^4 & -\ x^3 & -\ 13x & +\ 5 \end{array}$$

x^4	$-3x^3 + 2x^3$	$x^2 + 5x^2 - 6x^2$	$2x - 15x$	5×1
	$= -x^3$	$= 0$	$= -13x$	$= 5$
x^4	$-x^3$	0	$-13x$	5

Try These:

(1) $(2x - 1)\ (3x + 2)$

(2) $(2x + 1)\ (x^2 + 3x - 5)$

(3) $(5x + 5)\ (7x - 6)$

(4) $(x + 5)\ (x^2 - 2x + 3)$

(5) $(x - 4)\ (x^2 + 2x + 3)$

(6) $(x^2 + 4x - 5)\ (x + 5)$

(7) $(x^3 - 5)\ (x^2 + 3)$

(8) $(x^2 - 2x + 8)\ (x^4 - 2)$

(9) $(x^2 - 7x + 4)\ (x^3 - 1)$

(10) $(x^3 - 5x^2 + 2)\ (x^2 + 1)$