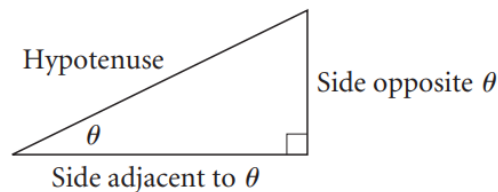


Chapter 1 The Trigonometry of Right Triangle

The trigonometric functions, including *sine*, *cosine*, and *tangent* were first defined as *ratios of sides in a right triangle*. Many applications involve *an angle of elevation* or *an angle of depression*. Suppose that the pilot of the plane is looking forward and then looks up, the pilot's eyes pass through an angle of elevation. If he is looking straight ahead and then he moves his eye down for landing, the pilot's eyes pass through an angle of depression.

The Trigonometric Ratios

Consider a right triangle with one of its acute angles labeled θ . The side opposite the right angle is called the *hypotenuse*. The other sides of the right triangle are referenced by their position relative to the acute angle θ . One side is called *side opposite θ* and one is called *side adjacent to θ* .



Trigonometric ratios are the ratios of the length of sides of a right triangle. In trigonometry, there are six trigonometric ratios, namely, *sine(sin)*, *cosine(cos)*, *tangent(tan)*, *secant(sec)*, *cosecant(csc)*, and *cotangent(cot)*.

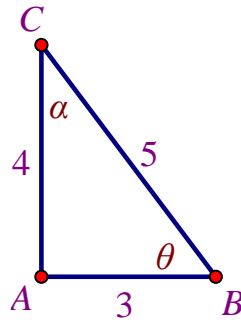
Trigonometric Ratio Values of an Acute Angle θ

Let θ be an acute angle of a right triangle. Then the six trigonometric ratios of θ are as follows:

| | |
|--|--|
| $\sin \theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$ | $\csc \theta = \frac{\text{hypotenuse}}{\text{side opposite } \theta} = \frac{\text{hyp}}{\text{opp}}$ |
| $\cos \theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$ | $\sec \theta = \frac{\text{hypotenuse}}{\text{side adjacent to } \theta} = \frac{\text{hyp}}{\text{adj}}$ |
| $\tan \theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \frac{\text{opp}}{\text{adj}}$ | $\cot \theta = \frac{\text{side adjacent to } \theta}{\text{side opposite } \theta} = \frac{\text{adj}}{\text{opp}}$ |

See the example 1.1 illustrated how to find the value of trigonometric ratios.

Example 1.1 In the right triangle shown below, find the values of six trigonometric ratios values of (a) θ and (b) α .



Solution:

$$\text{a) } \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}, \quad \csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}, \quad \sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}, \quad \cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$$

$$\text{b) } \sin \alpha = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}, \quad \csc \alpha = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$$

$$\cos \alpha = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}, \quad \sec \alpha = \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$$

$$\tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}, \quad \cot \alpha = \frac{\text{adj}}{\text{opp}} = \frac{4}{3} \quad \#$$

Reciprocal Trigonometric Ratios

In Example 1.1(a), there are the reciprocal relationship between the values of $\sin \theta, \frac{4}{5}$ and $\csc \theta, \frac{5}{4}$, the values of $\cos \theta, \frac{3}{5}$ and $\sec \theta, \frac{5}{3}$, and the values of $\tan \theta, \frac{4}{3}$ and $\cot \theta, \frac{3}{4}$. Likewise, for any angle, the cosecant (csc), secant (sec), and cotangent (cot) are the reciprocals of the sine, cosine, and tangent ratio values, respectively. There are the reciprocal identities of the trigonometric ratios as follows:

- The **reciprocal sine** is **cosecant**:

$$\sin \theta = \frac{1}{\csc \theta} \text{ and } \csc \theta = \frac{1}{\sin \theta}$$

- The **reciprocal cosine** is **secant**:

$$\cos \theta = \frac{1}{\sec \theta} \text{ and } \sec \theta = \frac{1}{\cos \theta}$$

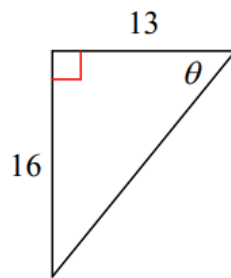
- The *reciprocal tangent* is *cotangent*:

$$\tan \theta = \frac{1}{\cot \theta} \text{ and } \cot \theta = \frac{1}{\tan \theta}$$

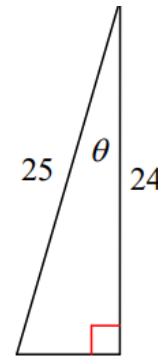
When solving right triangles, the reciprocal identities can be helpful in solving the value of the trigonometric ratios as illustrated in Example 1.2 and Example 1.3.

Example 1.2 Find the values of the trigonometric ratios indicated. (You will need to use Pythagorean theorem to find the missing side length)

a) $\csc \theta$



b) $\sec \theta$



Solution:

a) Using Pythagorean theorem:

$$\text{Hypotenuse} = \sqrt{16^2 + 13^2} = \sqrt{256 + 169} = 5\sqrt{17}$$

$$\text{Since } \csc \theta \text{ is the reciprocal sine, and } \sin \theta = \frac{16}{5\sqrt{17}}$$

$$\text{Therefore, } \csc \theta = \frac{5\sqrt{17}}{16} \quad \#$$

b) Since $\sec \theta$ is the reciprocal cosine, and $\cos \theta = \frac{24}{25}$

$$\text{Therefore, } \sec \theta = \frac{25}{24} \quad \#$$

Example 1.3 Find the values of trigonometric ratios indicated. (You will need to draw a right triangle)

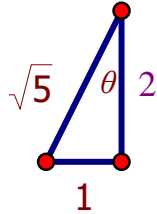
a) Find $\csc \theta$ if $\sec \theta = \frac{\sqrt{5}}{2}$

b) Find $\cot \theta$ if $\sec \theta = \frac{5}{3}$

Solution:

a) Since $\sec \theta$ is the reciprocal cosine, and $\sec \theta = \frac{\sqrt{5}}{2}$

$$\text{Then } \cos \theta = \frac{2}{\sqrt{5}}$$



Using Pythagorean theorem:

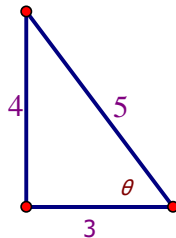
$$\text{Opposite } \theta = \sqrt{(\sqrt{5})^2 - 2^2} = \sqrt{5 - 4} = 1$$

Since $\csc \theta$ is the reciprocal sine, and $\sin \theta = \frac{1}{\sqrt{5}}$

$$\text{Therefore } \csc \theta = \sqrt{5} \quad \#$$

b) Since $\sec \theta$ is the reciprocal cosine, and $\sec \theta = \frac{5}{3}$

$$\text{Then } \cos \theta = \frac{3}{5}$$



Using Pythagorean theorem:

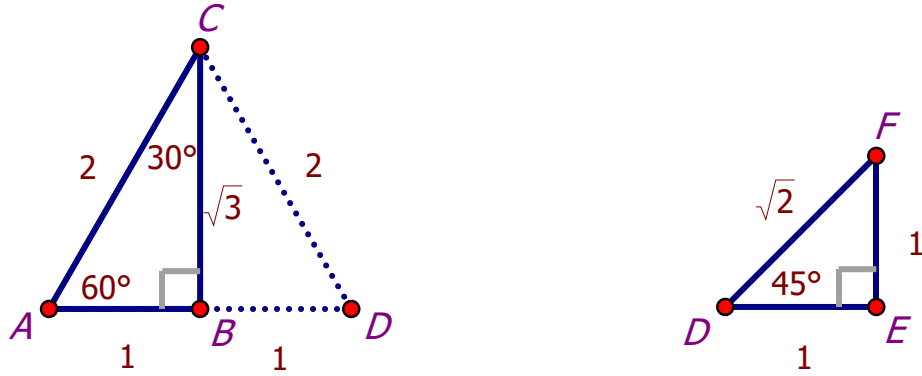
$$\text{Opposite } \theta = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$$

Since $\cot \theta$ is the reciprocal tangent, and $\tan \theta = \frac{4}{3}$

$$\text{Therefore } \cot \theta = \frac{3}{4} \quad \#$$

The Trigonometric Ratio for 30° , 45° , 60°

The trigonometric ratios for 30° , 45° , and 60° are based on some standard triangles as follows:



The left figure shows 30° and 60° based on an equilateral triangle ADC with sides of length 2 and with one of the angles bisected perpendicular to side \overline{AD} . It follows that $\angle ACB = 30^\circ$ and $AB = 1$. By using Pythagorean theorem, $BC = \sqrt{3}$.

The right figure shows 45° angle based on an isosceles with the equal sides ($DE = EF$) having a length of 1. By using Pythagorean theorem, $DF = \sqrt{2}$.

Then, the trigonometric ratios for 30° , 45° , and 60° are listed in the following table:

| θ | 30° | 45° | 60° |
|---------------|----------------------|----------------------|----------------------|
| $\sin \theta$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ |
| $\cos \theta$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ |
| $\tan \theta$ | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ |
| $\csc \theta$ | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ |
| $\sec \theta$ | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 |
| $\cot \theta$ | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ |

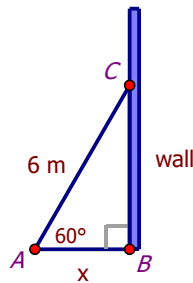
Applications of Right Triangle Trigonometry

Many real situations involve right triangles. In geometry, solving the problems involving distances can be used the Pythagorean theorem. In this topic, solving the problems involving measure of all three angles and the lengths of all three sides of a right triangle can be used the knowledge of trigonometric ratios.



Source: <https://numberdyslexia.com/real-life-applications-of-right-angle-triangle/>

Example 1.4 A 6 meter ladder leans against a brick wall forming angle θ of 60° with the ground. How far is the base of the ladder from the wall?



Solution: Let x be the distance along the ground to the wall, as in the above picture.

Since a 6 meter ladder forms an angle of 60° with the ground against a brick wall.

Then, it forms a right triangle ABC with $\angle ABC$ is a right angle, \overline{AB} is the adjacent side with the length x and \overline{AC} is the hypotenuse with the length 6 meters.

$$\text{Since } \cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\text{Then, } \cos 60^\circ = \frac{AB}{AC} = \frac{x}{6} \quad (\text{equation 1})$$

$$\text{Since } \cos 60^\circ = \frac{1}{2}$$

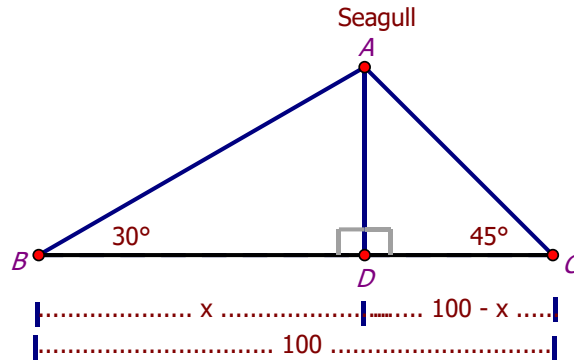
Substitute $\cos 60^\circ$ in equation 1 and solving for x :

$$\frac{1}{2} = \frac{x}{6}$$

$$x = 3$$

Therefore, the base of the ladder 3 meters far from the wall. #

Example 1.5 Two girls have the same height are standing 100 meters apart. They both see a beautiful seagull in the air between them. The angles of elevation from their eyes to the bird are 30° and 45° , respectively. How high up is the seagull?



Solution: In $\triangle ABC$, let \overline{AD} perpendicular to \overline{BC} at point D .

Divide up the 100 meters into x and $100 - x$

$$\text{In } \triangle ABD, \tan 30^\circ = \frac{AD}{x} \quad \dots\dots\dots(\text{eq. 1})$$

$$\text{In } \triangle ACD, \tan 45^\circ = \frac{AD}{100-x} \quad \dots\dots\dots(\text{eq. 2})$$

$$\text{Since, } \tan 30^\circ = \frac{1}{\sqrt{3}} \quad \text{and } \tan 45^\circ = 1$$

Substitute $\tan 30^\circ = \frac{1}{\sqrt{3}}$ in eq. 1 and $\tan 45^\circ = 1$ in eq. 2

$$\text{From eq. 1, } \frac{1}{\sqrt{3}} = \frac{AD}{x} \text{ or } AD = \frac{x}{\sqrt{3}} \quad \dots\dots\dots(\text{eq. 3})$$

$$\text{From eq. 2, } 1 = \frac{AD}{100-x} \text{ or } AD = 100 - x \quad \dots\dots\dots(\text{eq. 4})$$

$$\text{Since eq.3 = eq.4, then } \frac{x}{\sqrt{3}} = 100 - x \quad \dots\dots\dots(\text{eq. 5})$$

Solve eq. 5 for x , $x + \sqrt{3}x = 100\sqrt{3}$

$$(1 + \sqrt{3})x = 100\sqrt{3}$$

$$x = \frac{100\sqrt{3}}{1+\sqrt{3}}$$

$$\text{Thus, } AD = 100 - x = 100 - \frac{100\sqrt{3}}{1+\sqrt{3}}$$

$$AD \approx 36.60$$

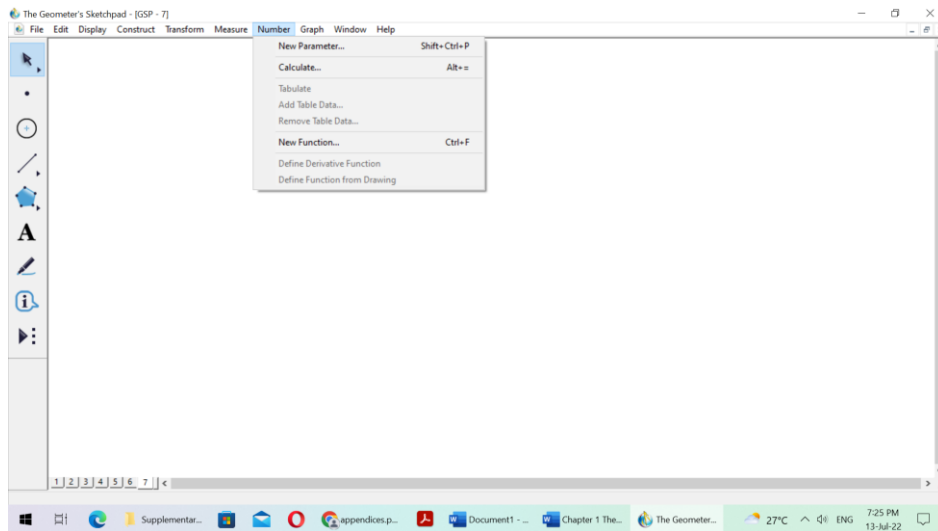
Therefore, the seagull is about 37 meters high.

#

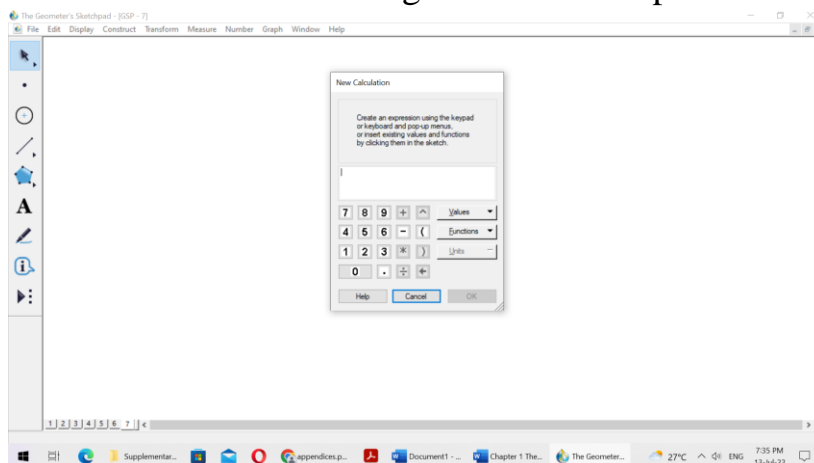
Finding Trigonometric Ratios by Using Calculators

The values of trigonometric ratios of any angles can be found by using a calculator. For this topic, the use of Geometer's Sketchpad (V. 5) to calculate the values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ for any angle θ measured in degree is shown in the following steps.

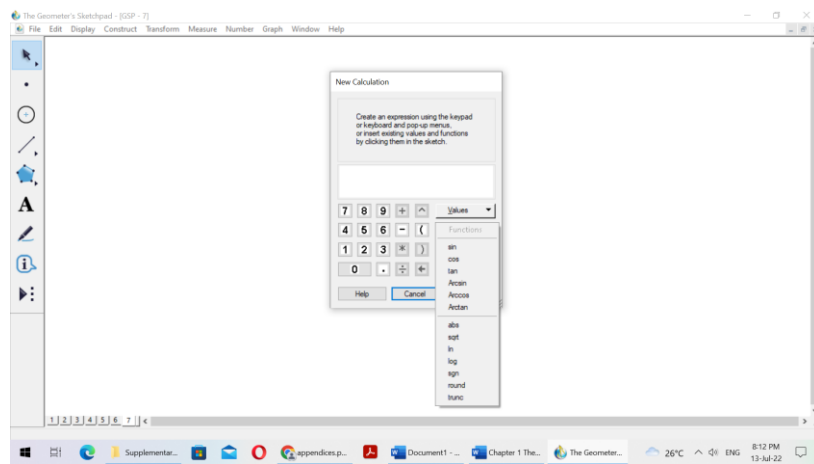
1. To invoke the calculator, choose “Calculate” from under the Number menu.



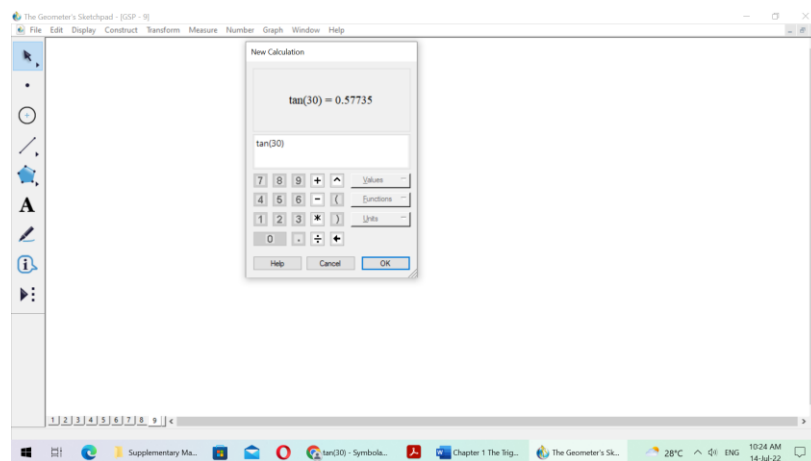
2. Click on “Calculate”, the display area shows the expression being built. Click a value on the keypad or type from keyboard to enter it into the calculator. The keypad has operators for addition (+), subtraction (-), multiplication (*), and division (/). The next column contains parentheses, carat symbol (^) for raising to a power, and arrow to delete or change the value or operator.



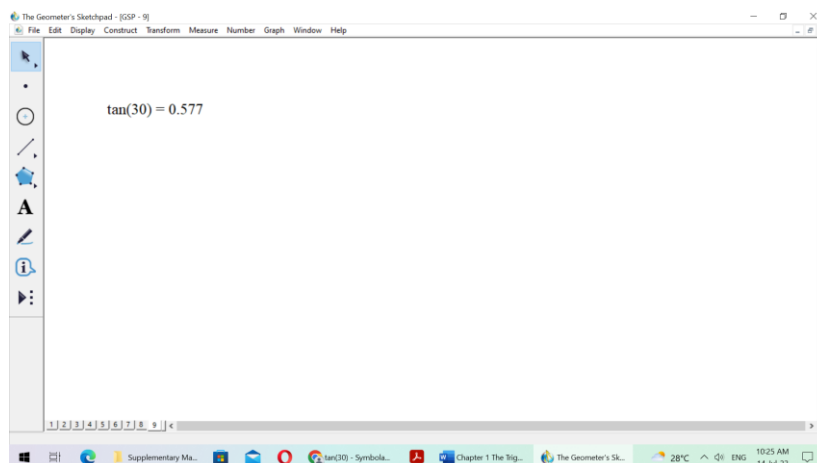
3. Click on “Function” button and enter “sin” or “cos” or “tan” into the calculator.



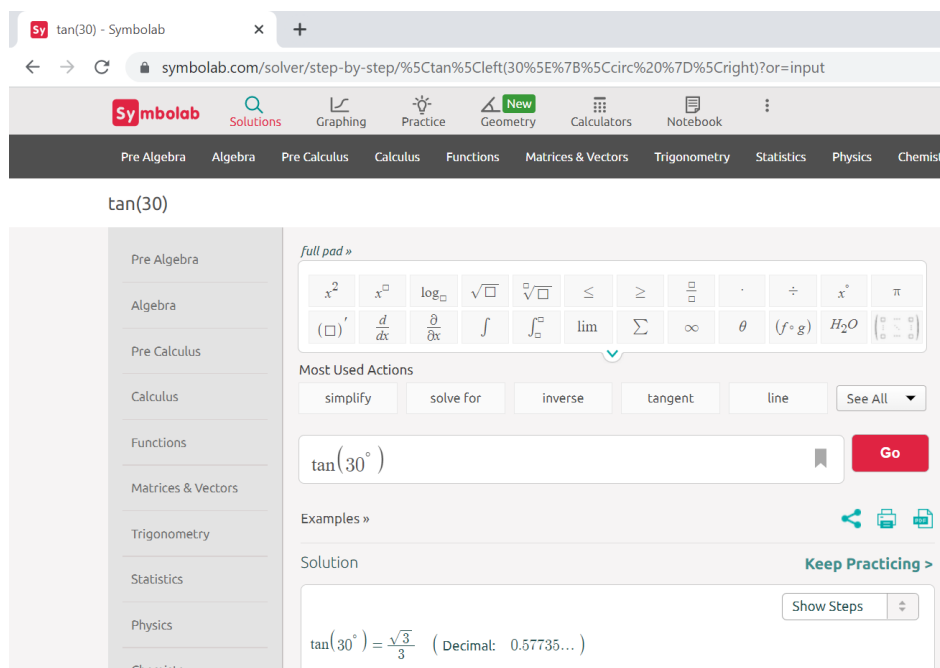
4. Enter the measure of angle to find the value of trigonometric ratios. The angles can be measured in different units, for measuring in “degree” click the value on keypad.



5. Click on “OK”, then the solution will show on GSP screen.



Different calculators may use different keying as shown one online calculator (<https://www.symbolab.com/solver/step-by-step/>).



tan(30)

full pad »

Most Used Actions

simplify solve for inverse tangent line See All

$\tan(30^\circ)$ Go

Examples »

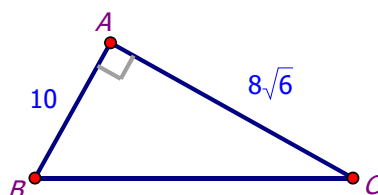
Solution Keep Practicing >

Show Steps

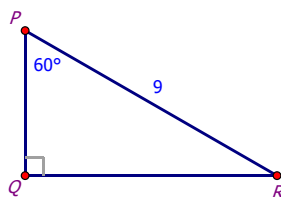
$\tan(30^\circ) = \frac{\sqrt{3}}{3}$ (Decimal: 0.57735...)

Practice 1

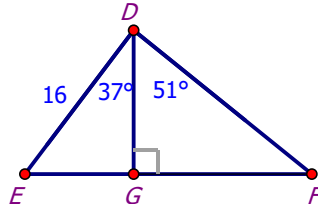
1. In $\triangle ABC$, $AB = 10$ cm, $AC = 8\sqrt{6}$, and $\angle BAC = 90^\circ$.



- (a) Find the length of \overline{BC} .
- (b) Find the values of $\sin B$, $\cos B$, $\sin C$, and $\cos C$, giving the answers as fractions in lowest terms.
- (c) What is the relation between (i) $\sin B$ and $\cos C$, (ii) $\cos B$ and $\sin C$.
2. In the diagram, $PR = 9$ cm, $\angle QPR = 60^\circ$, and $\angle PQR = 90^\circ$. Find the length of \overline{QR} and \overline{PQ} , correct to one decimal place (round to the nearest 10^{th}).



3. In $\triangle ABC$, $\angle BAC = 90^\circ$, $\angle BCA = 60^\circ$ and $AC = 12$ cm. Find the length of \overline{AB} and \overline{BC} , correct to 1 decimal point.
4. In $\triangle DEF$, $\angle EDF = 90^\circ$, $\angle DFE = 28^\circ$, and $DF = 16$ cm. Find the lengths of \overline{DE} and \overline{EF} , correct to 2 decimal places (round to the nearest 100th).
5. In the diagram, $\overline{DG} \perp \overline{EF}$, $\angle EDG = 37^\circ$, $\angle GDF = 51^\circ$ and $DE = 16$ cm. Find the lengths of \overline{DG} and \overline{FG} .



6. The angle of elevation of the top of the building at distance of 50 meters from its foot on a horizontal plane is found to be 60° . Find the height of the building.
7. A ladder placed against a wall such that it reaches the top of the wall of the height 6 meters and the ladder is inclined at an angle of 60° . Find how far the ladder is from the foot of the wall.
8. From the top of the tower 30 meters height, a man is observing the base of the tree at an angle of depression measuring 30° . Find the distance between the tree and the tower.
9. A man wants to determine the height of the light house. He measured the angle at A and found that $\tan A = \frac{3}{4}$. What is the height of the light house if A is 40 meters from the base?
10. A kite is flying at the height of 65 meters attached to a string. If the inclination of the string with the ground is 31° . Find the length of the string.

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