Chapter 5 Squaring and square Roots

Square of numbers ending in 5 :

Sutra: 'By one more than previous one"

Example: 75×75 or 75^2

As explained earlier in the chapter of multiplication we simply multiply 7 by the next number i.e. 8 to get 56 which forms first part of answer and the last part is simply $25 = (5)^2$. So, $75 \times 75 = 5625$

This method is applicable to numbers of any size.

Example: 605^2

$60 \times 61 = 3660$ and $5^2 = 25$

$\therefore\ 605^2 = 366025$

Square of numbers with decimals ending in 5

Example : $(7.5)^2$

$7 \times 8 = 56$, $(0.5^2) = 0.25$

$(7.5)^2 = 56.25$ (Similar to above example but with decimal)

Squaring numbers above 50:

Example: 52^2

Step1: First part is calculated as $5^2 + 2 = 25 + 2 = 27$

Step2: Last part is calculated as $(2)^2 = 04$ (two digits)

$\therefore\ 52^2 = 2704$

Squaring numbers below 50

Example : 48^2

Step1: First part of answer calculated as: $5^2 - 2 = 25 - 2 = 23$

Step2: second part is calculated as : $2^2 = 04$

$\therefore\quad 48^2 = 2304$

Squaring numbers near base :

Example : 1004^2

Step1: For first part add 1004and 04 to get 1008

Step2: For second part$4^2 = 16 = 016$ (as,base is 1000 a three digit no.)

$\therefore\quad (1004)^2 = 1008016$

Squaring numbers near sub - base:

Example $(302)^2$

Step1: For first part = 3 (302 + 02) = 3 × 304 = 912 [Here sub – base is 300 so multiply by 3]
Step2: For second part = 2^2 = 04

$\therefore$ $(302)^2 = 91204$

General method of squaring:

The Duplex

Sutra: "Single digit square, pair multiply and double" we will use the term duplex,` D' as follows:

For <u>1 figure</u>(or digit) Duplex is its squaree.g. $D(4) = 4^2 = 16$

For<u>2 digits</u>Duplex is twice of the product e.g. D(34) = 2 (3 x 4) = 24

For <u>3 digit number</u>: e.g. $(341)^2$

$D(3) = 3^2 = 9$

$D(34) = 2(3 \times 4) = 24$

$D(341) = 2(3 \times 1) + 4^2 = 6 + 16 = 22$

9 / 4 /2 /8 /1
/ 2/ 2/ /
=116281

$D(41) = 2(4 \times 1) = 8$

$D(1) = 1^2 = 1$

$\therefore$ $(341)^2 = 116281$

Algebraic Squaring :

Above method is applicable for squaring algebraic expressions:

Example: $(x + 5)^2$

$D(x) = x^2$

$D(x + 5) = 2(x \times 5) = 10x$

$D(5) = 5^2 = 25$

$\therefore$ $(x + 5)^2 = x^2 + 10x + 25$

Example: $(x - 3y)^2$

$D(x)= x^2$

$D(x - 3y) = 2(x) \times - 3y) = - 6xy$

$D(-3y) = (-3y)^2 = 9y^2$

$\therefore$ $(x - 3y)^2 = x^2 - 6xy + 9y^2$

Try these:

(I) 85^2	(II) $(8_2^1)^2$	(III) $(10.5)^2$	(IV) 8050^2
(V) 58^2	(VI) 52^2	(VII) 42^2	(VIII) 46^2
(IX) 98^2	(X) 106^2	(XI) 118^2	(XII) $(x + 2)^2$
(XIII) $(y - 3)^2$	(XIV) $(2x - 3)^2$	(XV) $(3y - 5)^2$	

SQUARE ROOTS:

General method:

As $1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 1[6]$ $5^2 = 2[5]$ $6^2 = 3[6]$

$7^2 = 4[9]$ $8^2 = 6[4]$ $9^2 = 8[1]$ i.e. square numbers only have digits 1,4,5,6,9,0 at the units place (or at the end)

Also in 16, digit sum = 1 + 6 = 7, 25 = 2 + 5 =7 , 36 = 3 + 6 = 9 , 49 = 4 + 9 = 13

13 = 1 + 3 =4, 64 = 6 + 4 = 10 = 1 + 0 = 1 , 81 = 8 + 1 = 9 i.e. square number only have digit sums of 1, 4, 7 and 9.

This means that square numbers cannot have certain digit sums and they cannot end with certain figures (or digits) using above information which of the following are not square numbers:

(1) 4539 (2) 6889 (3) 104976 (4) 27478 (5) 12345

Note: If a number has a valid digit sum and a valid last figure that does not mean that it is a square number. If 75379 is not a perfect square in spite of the fact that its digit sum is 4 and last figure is 9.

Square Root of Perfect Squares:

Example1: √5184

Step 1: Pair the numbers from right to left 5184 two pairs

Therefore answer is 2 digit numbers

$7^2 = 49$ and $8^2 = 64$

49 is less than 51

Therefore first digit of square root is 7.

Look at last digit which is 4

As $2^2 = 4$ and $8^2 = 64$ both end with 4

Therefore the answer could be 72 or 78

As we know $75^2 = 5625$ greater than 5184

Therefore √5184 is below 75

Therefore √5184 =72

Example 2: √9216

Step 1: Pair the numbers from right to left 9216two pairs

Therefore answer is 2 digit numbers

$9^2 = 81$ and $10^2 = 100$

81 is less than 92

Therefore first digit of square root is 9.

Look at last digit which is 6

As $4^2 = 16$ and $6^2 = 36$ both end with 6

Therefore the answer could be 94 or 96

As we know $95^2 = 9025$ less than 9216

Therefore $\sqrt{9216}$ is above 95

Therefore $\sqrt{9216} = 96$

General method

Example 1 : $\sqrt{2809}$

Step1: Form the pairs from right to left which decide the number of digits in the square root. Here 2 pairs therefore 2 - digits in thesquare root

Step 2: Now $\sqrt{28}$, nearest squares is = 25

So first digit is 5 (from left)

Step3: As 28 – 25 = 3 is reminder which forms 30 with the next digit 0.

Step 4: Multiply 2 with 5 to get 10 which is divisor 10 $\sqrt{\underline{2809}}$

30

Now $3 \times 10 = 30$ $\frac{30}{10} = \frac{Q}{3} \frac{R}{0}$

Step 5: As $3^2 = 9$ and 9 – 9 (last digit of the number) = 0

$\therefore$ 2809 is a perfect square and $\sqrt{2809} = 53$

Example 2:3249

Step1: Form the pairs form right to left which decided the number of digits in the square root. Here 2 pairs therefore 2 digits in the square root.

Step2: Now $32 > 25 = 5^2$ so the first digit in 5 (from left)

Step 3: 32 – 25 = 7 is remainder which form 74 with the next digit 4

$\underline{5\ \ 7}$

Step 4: Multiply 2 with 5 to get 10 which is divisor $10\sqrt{\underline{3249}}$

Now $\frac{74}{107}$ = Q R 7 4

4

Step5: $7^2 = 49$ and 49 – 49 =0 (remainder is 4 which together with9 form49)

$\therefore$ 3249 is a perfect square and $\sqrt{3249} = 57$

Example 3: $\sqrt{\underline{54\ 75\ 6}}$

Step1: Form the pairs from right to left therefore the square root of 54756 has 3-digits.

Step2: $5 > 4 = 2^2$ i.e. nearest square is $2^2 = 4$

So first digit is 2 (from left)

Step3: As 5 – 4 = 1 is remainder which form 14 with the next digit 4.

Step4: Multiply 2 with 2 to get 4, which is divisor

2

4 $\underline{5_1 4\ _2 75\ 6}$ Now $\underline{14}$ = Q R

4 3 2

Step 5: Start with remainder and next digit, we get 27.

Find $27 - 3^2 = 27 - 9 = 18$ [square of quotient]

234

Step 6: $\underline{18}$ = Q R 4 $\underline{5_1 4_2\ 7_2 5_1\ 6}$

4 4 2

Now $25 - (3 \times 4 \times 2) = 25 - 24 = 1$

$\underline{1}$ = Q R

4 0 1

$16 - 4^2 = 16 - 16 = 0$

$\therefore$ 54756 is a perfect square and so $\sqrt{5\ 4\ 7\ 5\ 6} = 234$

Try These:

1. 2116
2. 784
3. 6724
4. 4489
5. 9604
6. 3249
7. 34856
8. 1444
9. 103041
10. 97344