Probability Tree Diagram

A tree diagram is a way of representing a sequence of events. Tree diagram records *all possible outcomes in a clear and uncomplicated manner*. Tree diagrams give us a visual and simple way to solve multiple events and complex probability problem.

Probability Tree diagrams

Probability Tree diagrams are useful for calculating combined probabilities. Probability Tree diagrams consist of 3 main items:

- Branches,
- Probability: the probability of each branch;
- Outcomes: the outcome is written on the ends of the branch.

How do we calculate the overall probabilities?

- We **multiple** each probability along the branches of the tree.
- We **add** probabilities down **columns.**

Note:

First we show the two possible events:

wake up late and *wake up on time*,

- post the probability,
- draw the next branch,
- multiply the probabilities of the first branch that produces the desired result together,
- multiply along the branches and add the columns,
- Make sure all probabilities add to **1.**

Example 1

If the probability that it rains on any day is 0.2, draw a tree diagram and find the probability of:

a) it rains on two consecutive days

b) it rains on only one of two consecutive days.

Solution

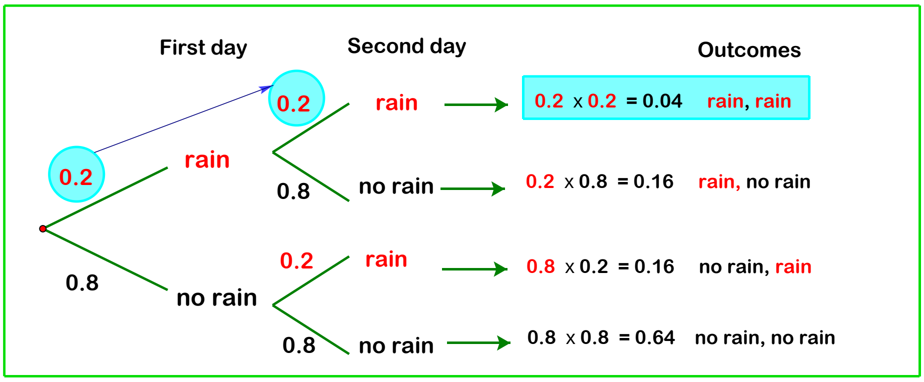
The tree diagram shows all the possible outcomes.

The probability of each event can be placed on the appropriate branch of the tree.

The probability of rain on any day = 0.2

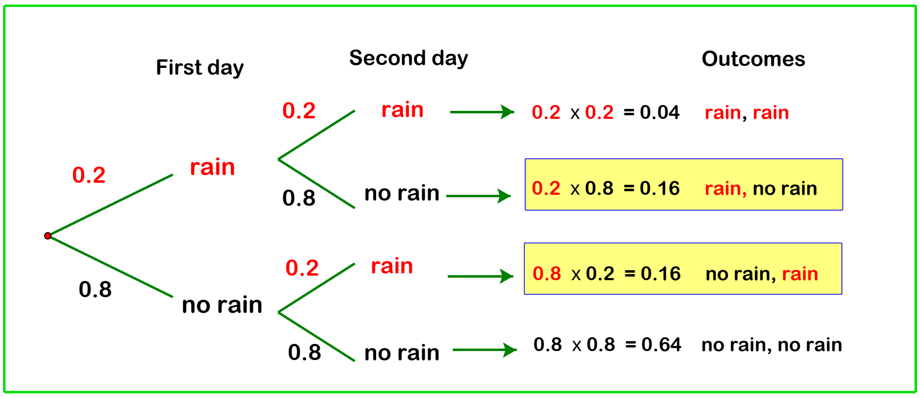
The probability of no rain on any day = 1 – 0.2 = 0.8

a) Tree diagrams: Graph out all possible outcomes.



The probability of rain on two consecutive days = 0.2 × 0.2 = 0.04 ▣

b) it rains on only one of two consecutive days.



The probability of rain only one of two consecutive days = 0.16 + 0.16

= 0.32 ▣

Example 2

Bunpot wakes up late on the average 3 days in every 5. If Bunpot wakes up late, the probability he's late for school is $\frac{9}{10}$. If Bunpot does not wakes up late, the probability he's late for school is $\frac{3}{10}$. What is the probability that Bunpot get to school on time?

Let event A = Bunpot wakes up late on the average 3 days in every 5.

Solution

P (wake up late) or P(A) $= \frac{3}{5}$

$\therefore$ P(wake up on time) or P(A′) $= 1 - \frac{3}{5} = \frac{2}{5}$

(i) If Bunpot wakes up late, the probability he's late for school is $\frac{9}{10}$.

Let event B = Bunpot's late for school

P(School late) or P(B) $= \frac{9}{10}$

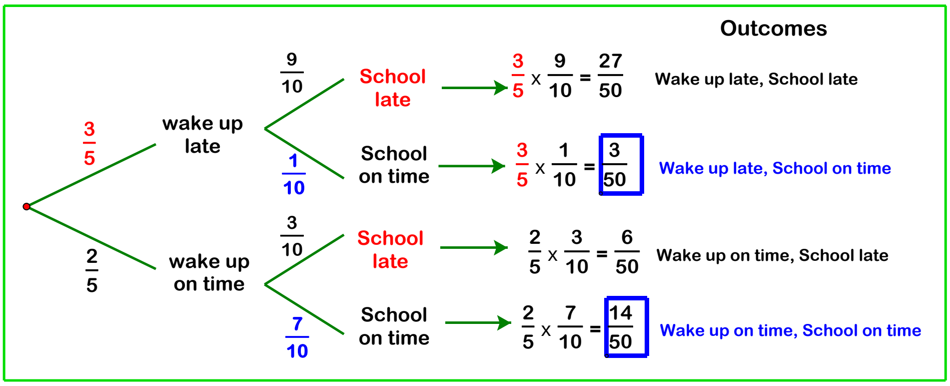
$\therefore$ P(School on time) or P(B′) $= 1 - \frac{9}{10} = \frac{1}{10}$

(ii) If Bunpot does not wakes up late, the probability he's late for school is $\frac{3}{10}$

P(School late) or P(B) $= \frac{3}{10}$

$\therefore$ P(School on time) or P(B′) $= 1 - \frac{3}{10} = \frac{7}{10}$

Draw the probability tree diagram as following:



The probability that Bunpot get to school on time = $\frac{3}{50} + \frac{14}{50} = \frac{17}{50}$ ▣

Example 3

A ball is draw from a bag containing 2 white balls, 3 red balls and 5 pink balls. Find the probability of getting a pink or a white ball.

Solution

A bag a bag containing 2 white balls, 3 red balls and 5 pink balls.

The total balls in a bag = 2 + 3 +5 = 10 balls

$\therefore$ The total possible outcomes = 10

Let ***A*** = Event of getting 5 pink balls

n(A) = 5

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{10}$

Let *B* be the event of getting 2 white balls

n(B) = 2

$P(B) = \frac{2}{10}$

Because we cannot draw one ball and getting a pink ball and a white ball at the same time.

The probability of getting a pink **or** a white ball = P(A) + P(B)

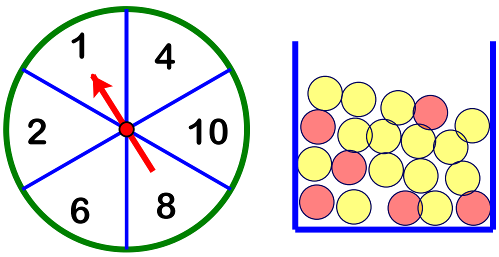
$= \frac{5}{10} + \frac{2}{10}$

$= \frac{7}{10}$ ▣

Example 4

From PISA 2006: Question M471: Spring Fair (http://www.oecd.org/pisa) Spring fair

A game in a booth at spring fair involves using a spinner first. Then, if the spinner stops on an even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bag are represented in the diagram below.



Prizes are given when a blue marble is picked. Sue plays the game once. How likely is it that Sue will win a prize?

a) Impossible
b) Not very likely
c) About 50% likely
d) Very likely
e) Certain.

Solution

Sample space = All possible outcomes of an experiment or *sample space*

S = 1, 2, 4, 6, 8, 10

n(S) = 6

A = An favourable outcomes which the spinner stops on an even numbers.

A = 2, 4, 6, 8, 10

n(A) = 5

P(A) = $\frac{n(A)}{n(S)}$ = $\frac{5}{6}$

Prizes are given when a blue marble is picked.

Sample space = Total number of marbles

n(S) = 20

B = An favourable outcomes which blue marble is picked.

B = $\{B_1, B_2, B_3, B_4, B_5, B_6\}$

n(B) = 6

P(B) = $\frac{n(B)}{n(S)}$

= $\frac{6}{20}$

Prizes are given when a blue marble is picked. Sue plays the game once.

Probability of spinner stops on an even numbers and

picking a blue marble = P (A and B)

P (A and B) = P(A) × P(B)

= $\frac{5}{6} \times \frac{6}{20}$

= $\frac{1}{4}$

= 0.25

∴ Probability of spinner stops on an even numbers and

picking a blue marble = 0.25.

Therefore, it is **not very likely** for Sue to win a prize.

The answer is item b) Not very likely. ▣

Conditional Probability

Two or more Events

We can calculate the probability of two or more events by multiplying the individual probabilities.

So for Independent Events:

$$P(\text{ A and B}) \quad = \quad P(A) \times P(B)$$

Independent Events

When the probability of two events occur together or in sequence is the product of the probability of each of the individual events, then the individual events are said to be independent events.

Dependent Events

Dependent Events are effected by previous events.

Two or more Events

We can calculate the probability of two or more events by **multiplying** the individual probabilities.

So for Independent Events:

$$\mathbf{P(A \text{ and } B)} \quad = \quad \mathbf{P(}A \cap B\mathbf{)} \quad = \quad \mathbf{P(A) \times P(B)}$$

Note

Independent or Dependent Events

- **Independent events**: each event is **not affected** by any other events.
- **Dependent events** or Conditional: each event **can be affected** by previous events.

Replacement:

- **With Replacement**: the events are independent.
- **Without Replacement**: the events are dependent.

Example 5 There are 3 blue and 2 red balls in a bag. What is the probability of drawing a blue ball on the first and second draw?

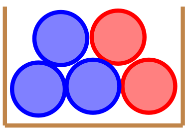
a) with replacement

b) without replacement

Solution

a) With Replacement:

First draw: There are 3 blue (B) and 2 red (R) balls in the bag.

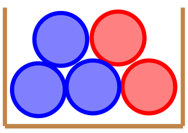


Event A = drawing a *blue* ball on the **first draw**

= $\{B_1, B_2, B_3\}$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{5}$

Second draw: There are 3 blue (B) and 2 red (R) balls in a bag.



Event B = drawing a *blue* ball on the **second draw**

= $\{B_1, B_2, B_3\}$

$P(B) = \frac{n(B)}{n(S)} = \frac{3}{5}$

The probability of drawing a blue ball on the **first and second** draw:

$P(A \text{ and } B) = P(A) \times P(B)$

$P(A \text{ and } B) = \frac{3}{5} \times \frac{3}{5}$

$P(A \text{ and } B) = \frac{9}{25}$

$= 0.36$

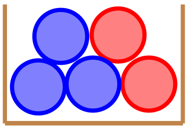
The probability of drawing a blue ball on the first and second draw = 0.36

b) **Without Replacement:**

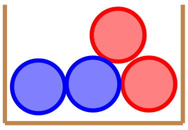
First draw: There are 3 blue (B) and 2 red (R) balls in the bag.

Event A = drawing a *blue* ball on the first draw

= $\{B_1, B_2, B_3\}$



P(A) $= \frac{n(A)}{n(S)} = \frac{3}{5}$



Second draw: There are 2 blue (B) and 2 red (R) balls in a bag.

Event B = drawing a *blue* ball on the second draw

$= \{B_1, B_2\}$

P(B) $= \frac{n(B)}{n(S)} = \frac{2}{4}$

The probability of drawing a blue ball on the **first and second** draw:

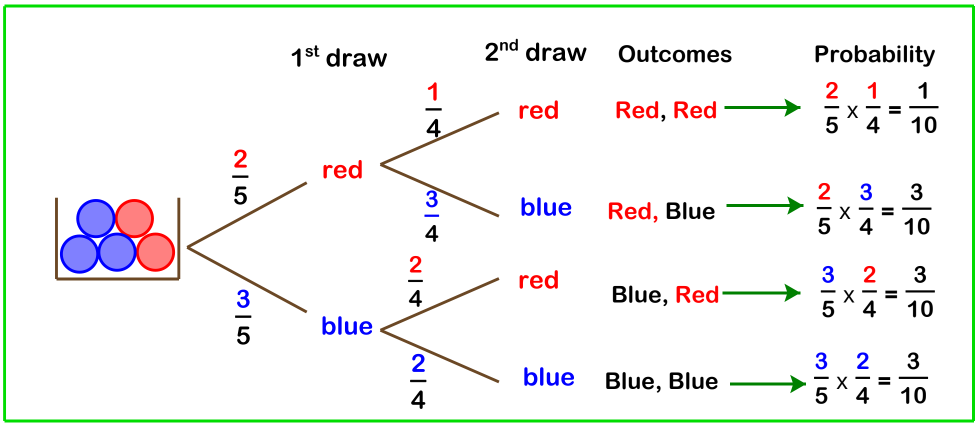
$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(A \text{ and } B) = \frac{3}{5} \times \frac{2}{4}$$

$$P(A \text{ and } B) = \frac{6}{20}$$

$$= 0.3$$

$\therefore$ Probability of drawing a blue ball on the first and second draw = 0.3 ▣



The probability of drawing two red balls from the bag *without* replacements

$$= \frac{2}{5} \times \frac{1}{4} = \frac{1}{10} = 0.1$$ ▣

Example 6

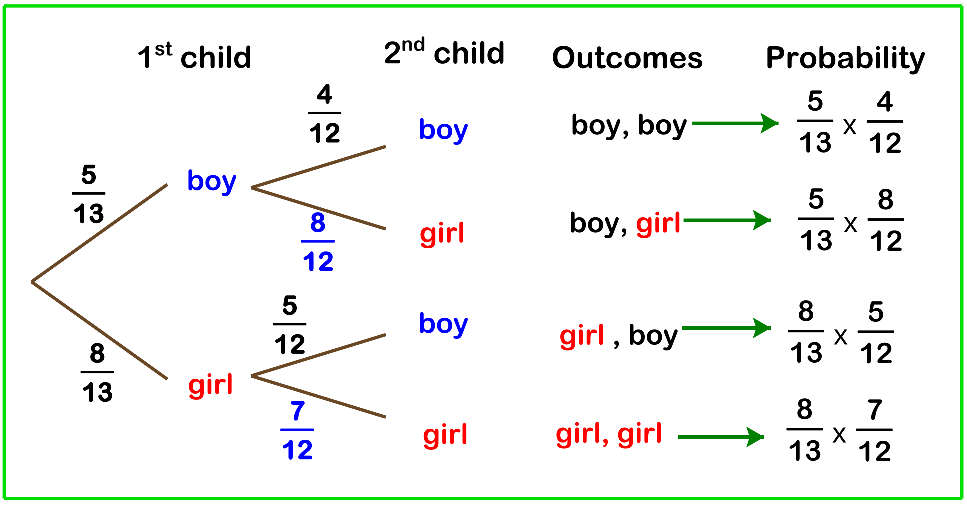
At a tennis club, there are 13 children, 5 of whom are boys and 8 are girls. Two children are selected at random. Find the probability that both are the same gender.

Solution

At a tennis club, there are 13 children, 5 of whom are boys and 8 are girls.

Two children are selected at random. This is the case that we select one after the other without replacing the first one.

We can draw probability tree diagrams as follows.



For both child to be the same gender, we can have both boys or both girls.

For the tree diagram,

P(both child are same gender) = P (both are boys **or** both are girls)

= P (both are boys) + P (both are girls)

$= \left(\frac{5}{13} \times \frac{4}{12}\right) + \left(\frac{8}{13} \times \frac{7}{12}\right)$

$= \frac{5}{39} + \frac{14}{39}$

P(both child are same gender) $= \frac{19}{39}$

Exercises

1. There are two boxes each containing five tickets numbered from 1 to 5. Two tickets are drawn at random, one from each box.

a) Draw a draw probability tree diagrams

b) Calculate the probability that the sum of the numbers on the two tickets will be 6 or more,

c) Calculate the probability that the product of these numbers will be 7 or more.

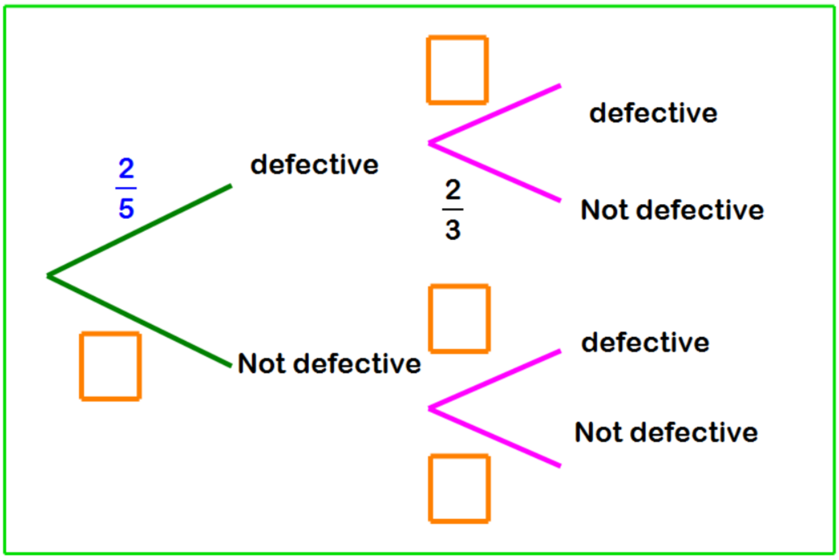
2. The numbers 1 to 13 are written on individual cards and put in a box. A number is selected at random. Find the probability of selecting

a) a prime number or an even number greater than 5,

b) a number greater than 10 or less than 5.

3. A box contains 10 light bulbs of which 4 are defective. Two of the bulbs are chosen at random and tested.

a) Copy and complete the tree diagram given below.



b) Find the probability that

i) both bulbs are defective,

ii) neither bulb is defective,

iii) one is defective and the other is not defective.

4. An integer between one and two thousand is randomly chosen.

a) What is the probability that is a perfect square?

b) If it is found to be a perfect square, what is the probability that it is also a perfect cube?

5. In an inter-school mathematics quiz, the probability of school ***A*** winning the competition is $\frac{1}{3}$, the probability of school ***B*** wining is $\frac{1}{6}$ and the probability of school ***C*** wining is $\frac{1}{10}$. Find the probability that

a) ***B*** or ***C*** wins the competition.

b) ***A***, ***B***, or ***C*** wins the competition,

c) None of these schools wins the competition.

6. At a tennis club, there are 13 children, 5 of whom are boys and 8 are girls. Two children are selected at random. Find the probability that both are the same gender.

7. In one class, $\frac{1}{3}$ of the students read newspaper and $\frac{2}{3}$ watch the news on television. None of these students read newspaper and also watch the news on television. What is the probability that a student chosen at random reads the newspaper or watches the news on television?