



# Mathematical Problem Solving

## ■ What is a problem?

A problem is a problem because you don't know straight away how to do it.

- *First*, there may be something about the **wording** that you don't understand.
- *Second*, you may not see how to **get started**. There may be no obvious **strategy** for you to use.
- *Third*, you may not know the **right** piece of **mathematics** to use.
- *Fourth*, you may know the right strategy and the right mathematics but you may not be **using them correctly** or you may not be able to see how to put them together to come up with a solution.

The strange thing about problems is that:  
what is a problem for one person is not necessarily a problem for someone else.

*How to Solve It* (1945) is a small volume by mathematician George Polya describing methods of problem solving.

*How to Solve It* suggests the following steps when solving a [mathematical problem](#):

1. First, you have to *understand the problem*.
2. After understanding, *make a plan*.
3. *Carry out the plan*.
4. *Look back* on your work. How could it be better?

## 1

### First principle: Understand the problem

Pólya taught teachers how to prompt each student with appropriate questions, depending on the situation,

The teacher select the question with the appropriate level of difficulty for each student to ascertain if each student understands at their own level, moving up or down the list to prompt each student, until each one can respond with something constructive; such as:

- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or a diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?
- Do you understand all the words used in stating the problem?
- Do you need to ask a question to get the answer?

## 2

**Second principle: Devise a plan**

Pólya mentions that there are many reasonable ways to solve problems. The skill at choosing an appropriate strategy is best learned by solving many problems. You will find choosing a strategy increasingly easy. A partial list of strategies is included:

- Guess and check
- Make an orderly list
- Eliminate possibilities
- Look for a pattern
- Draw a picture
- Solve a simpler problem
- Use a model
- Work backward
- Use a formula
- Be creative
- Use symmetry
- Consider special cases
- Use direct reasoning
- Solve an equation
- Applying these rules to devise a plan takes your own skill and judgement.

## 3

**Third principle: Carry out the plan**

This step is usually easier than devising the plan. In general, all you need is care and patience, given that you have the necessary skills. Persist with the plan that you have chosen. If it continues not to work, discard it and choose another. Don't be misled; this is how mathematics is done, even by professionals.

## 4

**Fourth principle: Review/extend**


Pólya mentioned that it would be better and more useful. You will gain by:

- taking the time to reflect;
- look back at what you have done;
- what worked and what did not; and
- by thinking about other problems where this could be useful.


Doing this will enable you to predict what strategy to use to solve future problems, if these relate to the original problem.

## Polya's Problem Solving Principles

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From the book  
"How To Solve It"



COMPETITIVE KIDS STEM PROJECTS

○ ○ ○ Understand the problem

- Do you understand all the words used in stating the problem?
- Can you restate the problem in your own words?
- What are you asked to find or show?
- Can you think of a picture or diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?

○ ○ ○ Devise a plan

- Guess and check
- Make an orderly list
- Eliminate possibilities
- Use symmetry
- Consider special cases
- Use direct reasoning
- Solve an equation
- Look for a pattern
- Draw a picture
- Solve a simpler problem
- Use a model
- Work backwards
- Use a formula
- Be ingenious

○ ○ ○ Carry out the plan

- All you need is care and patience, given that you have the necessary skills.
- Persist with the plan that you have chosen. If it continues not to work discard it and choose another.
- Don't be misled, this is how mathematics is done, even by professionals.






○ ○ ○ Look back

- Polya mentions that much can be gained by taking the time to reflect and look back at what you have done, what worked, and what didn't.
- Doing this will enable you to predict what strategy to use to solve future problems.

### What Are Problem Solving Strategies?

Strategies are things that Pólya would have us choose in his second stage of problem solving and use in his third stage (What is Problem Solving?). Polya called them **heuristics**. To Pólya they were things to try that he couldn't guarantee would solve the problem but, of course, he sincerely hoped they would. So they are some sort of general ideas that might work for a number of problems. And then again they might not.

# Problem Solving Strategies

<h3 style="margin: 0;">Make a table</h3> <table border="1" style="font-size: x-small; margin: 5px;"> <tr> <td style="padding: 2px;">4 legs</td> <td style="padding: 2px;">2 legs</td> <td style="padding: 2px;">no legs</td> </tr> <tr> <td style="padding: 2px;">cat</td> <td style="padding: 2px;">boy</td> <td style="padding: 2px;">snake</td> </tr> <tr> <td style="padding: 2px;">dog</td> <td style="padding: 2px;">emu</td> <td></td> </tr> </table>	4 legs	2 legs	no legs	cat	boy	snake	dog	emu		<h3 style="margin: 0;">Make a list</h3> <ul style="list-style-type: none"> <li>4 dogs</li> <li>3 cats</li> <li>7 rabbits</li> </ul>
4 legs	2 legs	no legs								
cat	boy	snake								
dog	emu									
<h3 style="margin: 0;">Draw a picture/diagram</h3> 	<h3 style="margin: 0;">Act it out</h3> 									
<h3 style="margin: 0;">Make a model</h3> 	<h3 style="margin: 0;">Identify a pattern</h3> <p style="margin: 0; font-size: small;">5, 10, 15, 20</p>									
<h3 style="margin: 0;">Write a number sentence</h3> <p style="margin: 0; font-size: small;">3x4=12</p>	<h3 style="margin: 0;">Solve a simpler, related problem</h3> <p style="margin: 0; font-size: x-small;">102, 104 ??? 2, 4, 6, 8, 10...</p>									
<h3 style="margin: 0;">Guess and Check</h3> 	<h3 style="margin: 0;">Work backwards</h3> 									

# The 9 Problem Solving Strategies

## 1 Look for the important words in the question

Write them down.  
Underline them.  
Make sure I know what to do.

## 2 Look for a pattern

Can I see something happening over and over again?  
Will this help me solve the problem?

## 3 Have a go

Try an answer.  
Does the answer make sense?

## 4 Use a table or a chart

Will something like this help?


## 5 Use a drawing

Can I draw something about the problem?  
Will this help me to find the answer?

## 6 Work backwards

Can I start at the end of the question to help work it out?  
Will my answer work?

## 7 Try an easier problem

Can I change the numbers in the question to make it simpler?  
Will this make finding the answer easier?

## 8 Make a model

Can I use paper or blocks to help me find the answer?  
Can I use people to help me find the answer?

## 9 Think logically

Can I tell something about the answer straight away?  
Can I get rid of answers that are not correct?