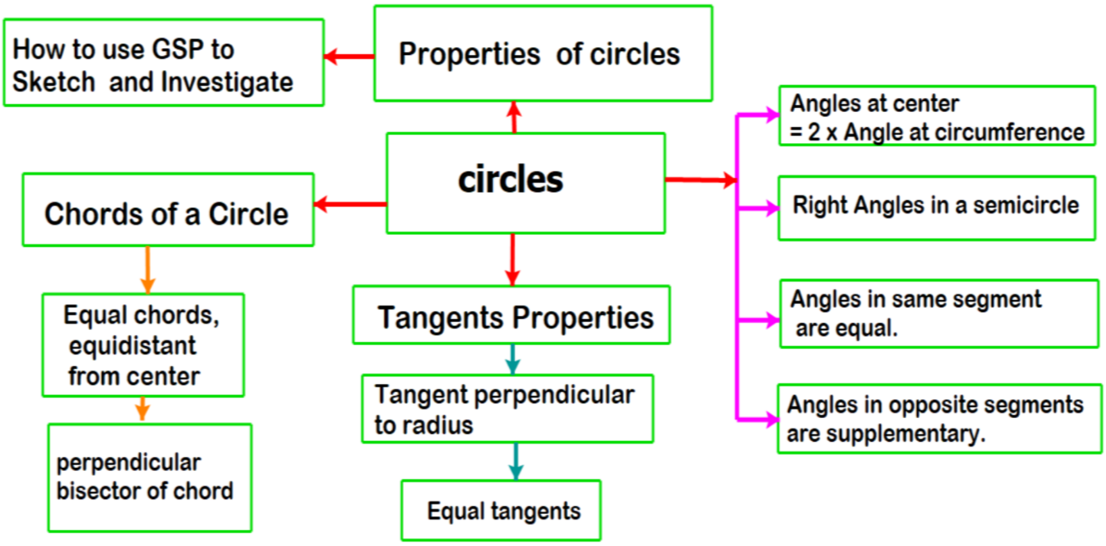
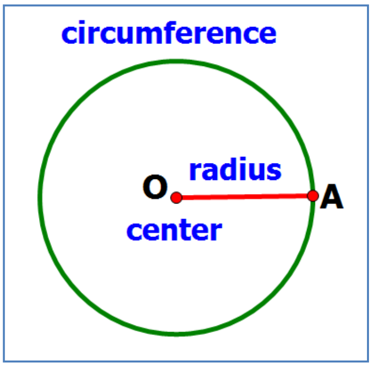
Properties of Circles

Concept Mapping



A circle

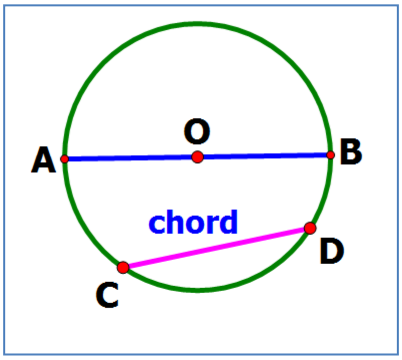
A circle consists of all points on a plan which are at the same distance from a given point. A circle is defined by a center and a radius.



Point ***O*** is called a **center** of a circle, and
OA is called a **radius** of a circle.

The **circumference** is the distance around the edge of the circle.

The relationships among circles and lines are as follows:



A **cord** is a line segment with both end points on the circumference.

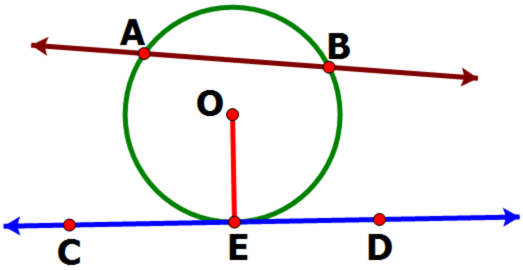
A **diameter** is a cord that contains the center.

In the figure, ***AB*** is a diameter of a circle. The line segment ***CD*** is called **a chord** of a circle.

Tangents

In the figure below, the straight line ***AB*** cuts the circle at two points, the straight line ***CD*** touches the circle at only one point. The line ***AB*** is called a secant line.

The line ***CD*** is called ***a tangent to the circle***, and point ***E*** is called the ***point of contact***. The radius ***OE*** is perpendicular to the line ***CD*** at point ***E.***

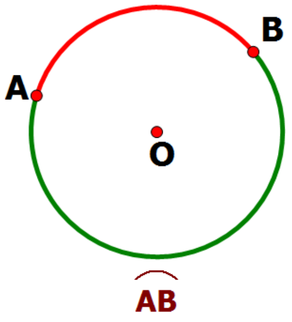


Note

1. A **secant** is a line that intersects a circle at **two** points.
2. ***A tangent*** is a straight line which touches the circle at only one point.
3. A tangent to a circle is ***perpendicular*** to the radius of the circle at the point of contact.

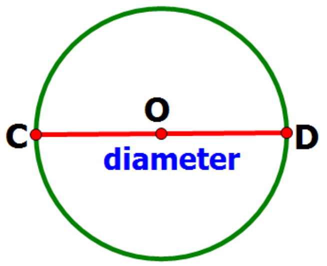
Properties of a Circle

Arcs of a Circle

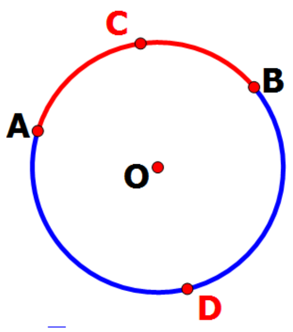


An **arc** ***AB*** of a circle consists of the points ***A*** and ***B*** of a circle together with a portion of the circle contained between the two points.

Points ***A*** and ***B*** are called the *endpoints* of the arc.

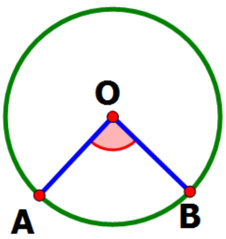


A **semicircle** is an arc whose endpoints are the endpoint of a diameter.



An arc (*ACB*) of a circle that is *shorter* than a semicircle is called a **minor arc** of the circle.

An arc (*ADB*) of a circle that is *longer* than a semicircle is called a **major arc** of the circle.



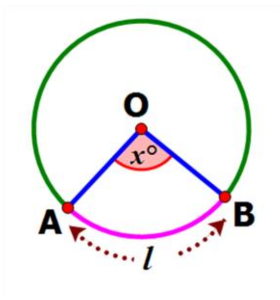
Central Angles in a Circle

An angle whose vertex is the center of a circle and whose sides are radii is called a **central angle**.

If ***O*** is the center of a circle containing points ***A*** and ***B***, then $\angle AOB$ is *a central angle.*

Length of an Arc of a Circle

The ratio of the length of an arc to the circumference of the circle equals the ratio of the central angle of the arc to 360°.

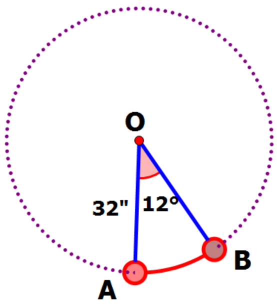


l = Length of arc
C = circumference of the circle
x = central angle of the arc
we have:

$$\frac{l}{C} = \frac{x}{360} \quad \text{or} \quad l = \frac{x}{360}(2\pi r)$$

Example 1 A clock has a pendulum 32 inches long. If the pendulum swings through an arc of 12°, how far does the pendulum travel?

Solution



The pendulum swings through an arc of a circle as shown in the figure.

The central angle of 12° means that

Arc AB is $\frac{12°}{360°} = \frac{1}{30}$ of a circle with radius 32″

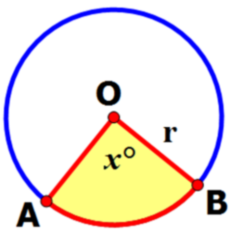
Therefore,

$$\text{Arc AB} = \frac{1}{30}(2\pi)(32) = 6.7 \text{ inches.}$$

The pendulum travel 6.7 inches.

Area of a Sector

The ratio of the area of a sector of a circle to the area of the circle is equal to the ratio of the central angle of the sector to 360°.



$$\frac{A_{sector}}{A_{circle}} = \frac{x}{360}$$

$$\text{or} \quad A_{sector} = \frac{x}{360}(\pi r^2)$$

Example 2

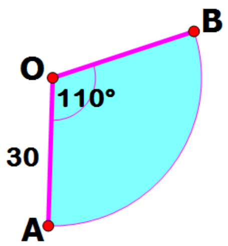
A lawn sprinkler sprays water over a radius of 30 feet. If the sprinkler is set to turn through an angle 110°, calculate the area that will be watered by the sprinkler.

Solution

The region watered by the sprinkler is a sector of a circle.
Because the central angle of the sector is 110° and the radius of the circle is 30 feet.

The area that will be watered by the sprinkler is

Area $= \frac{x}{360}(\pi \cdot r^2)$



Therefore:

Area $= \frac{110}{360}(\frac{22}{7} \cdot 30^2)$

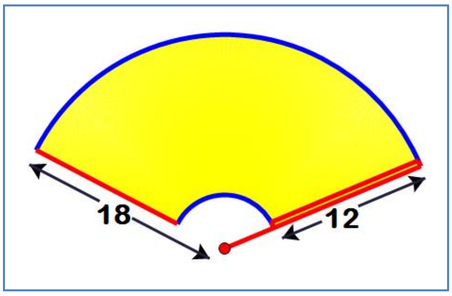
$= 864.3$ square feet

The area that will be watered by the sprinkler is 864.3 square feet

Exercises 1

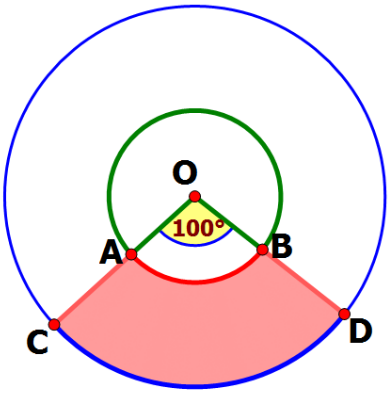
1. The pendulum of a clock is 40 inches long and swings through an arc of 8°. Find the length of arc that the pendulum traces out.

2. A windshield wiper is 18 inches long and has a blade 12 inches long. If the wiper sweeps through an angle of 130°, how large an area does the wiper blade clean? Round your answer to the nearest square inch.

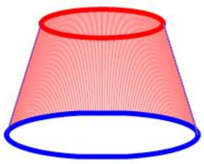


3. The diagram shows two concentric circles with center *O* and radii **50** cm and **68** cm respectively.
Given that ∠ AOB = 100°

A lampshade is made from the piece of material ABCD as shown.



Find:
a) The area of sector ***AOB***
b) The arc length ***AB*** and ***CD***
c) The surface area of the lampshade *ABCD.*



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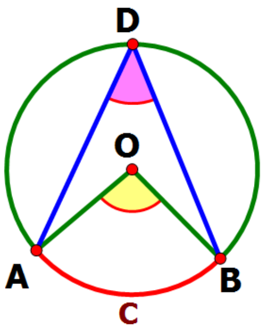
Angles in a Circle

Inscribed Angle in a Circle



In the figure, angle *ADB* is subtended by the **arc *ACB*** at the point D on the circumference of the circle. An angle *AOB* is subtended by the arc *ACB* at the center of the circle.

The angle *AOB* is called **a central angle** or angle at the center.

The angle *ADB* is called an **inscribed angle** in a circle.

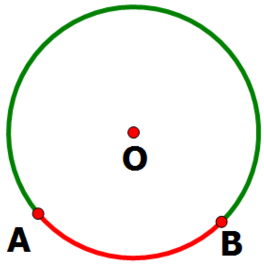
The angle subtended by an arc at the center of a circle is **twice** the angle subtended by the same arc at the circumference.
From the figure above:

$$\angle AOB = 2\angle ADB$$

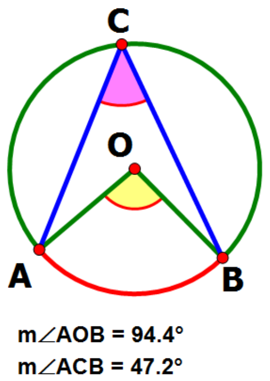
Using GSP to Sketch and Investigate

Open program GSP and select ***New Sketch*** from ***File*** menu

1. Construct a circle with center ***O***
2. Construct point ***A*** and ***B*** on the circle
3. Construct **minor arc *AB***
   - Select in order point ***O, A*** and ***B***
   - Choose *Arc on circle* in the Construct menu.
4. Construct **major arc**
   - Select in order point ***O, B*** and ***A***
   - Choose *Arc on circle* in the Construct menu, change color from Display menu.



5. Construct line segments ***OA*** and ***OB***
6. Construct point ***C*** on the circle
7. Construct line segments ***AC*** and ***BC***
8. Measure the angle at the center ∠***AOB***, and the inscribed angle ∠***ACB*** subtended by the arc ***AB***.
   - Select points ***A, O*** and ***B*** in order;
   - Choose *Angle* from Measure menu



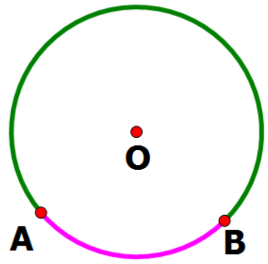
Exploring:

1. Calculate the ratio of ∠***AOB*** and ∠***ACB***
2. Drag the point ***C*** along the circle. What do you observe?
3. Drag the point ***A*** or ***B*** along the circle. What do you observe?
4. Describe the relationship between ∠***AOB*** and ∠***ACB***.

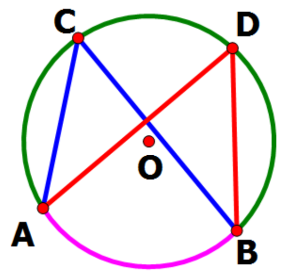
Angles in the same segment are equal.

Open program GSP and select ***New Sketch*** from ***File*** menu

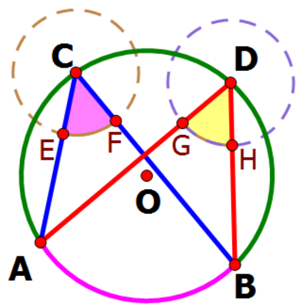
1. Construct a circle with center ***O***
2. Construct point ***A*** and ***B*** on the circle
3. Construct **minor arc *AB***
   - Select in order point ***O, A*** and ***B***
   - Choose *Arc on circle* in the Construct menu.
4. Construct **major arc**
   - Select in order point ***O, B*** and ***A***
   - Choose *Arc on circle* in the Construct menu, change color from Display menu.



5. Construct point ***C*** and ***D*** on the major arc
6. Construct line segments ***AC***, ***BC***, *AD* and *BD*

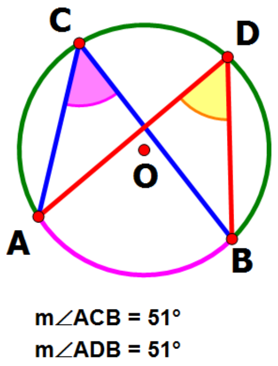


7. Construct sectors *ECF* and *GD*H
   - Construct sector *ECF*
     - Construct circle at point *C*
     - Construct minor arc EF, while this arc is selected, select Arc Interior and choose Arc Sector in Construct menu
   - Select circles and choose Hide in Display menu
   - Construct sector *GDH*

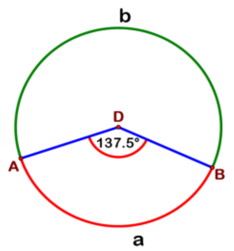


8. Measure angle *ACB* and *ADB*
    - Select in order point *A*, *C* and *B* in order
    - Select *Angle* from Measure menu

9. Drag or Animate point ***C*** and ***D*** to investigate measures of the angles.



Note:
If we divide the circumference of a circle into two arcs in the proportion of *Golden Ratio* (1.618),
the angle produced by the smaller arc is 137.5°, also known as the **Golden Angle.**



Golden Ratio:

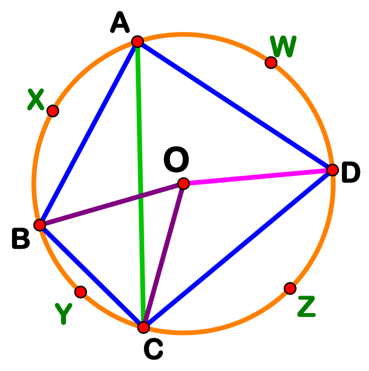
$$\frac{b}{a} = \frac{20.86\text{ cm}}{12.89\text{ cm}} = 1.618$$

Golden Angle = 137.5°

The Angle in a Semi-circle is a right angle.

Exercises 2

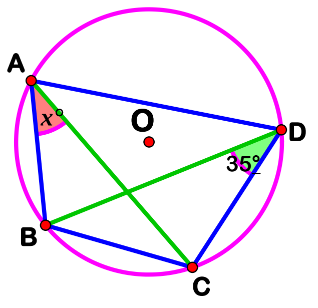
1. Given that ***O*** is the center of a circle, name the angle subtended by the given arc as following.



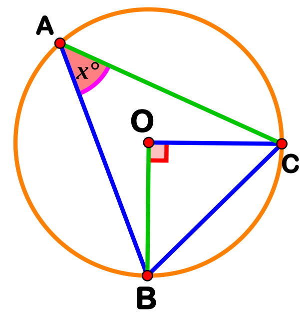
a)	Arc ***AXB*** at the circumference.
b)	Arc ***CZD*** at the circumference.
c)	Arc ***AWD*** at the circumference.
d)	Arc ***BYC*** at the center.
e)	Arc ***CZD*** at the center.
f)	Arc ***BYC*** at the circumference.

2. Given that ***O*** is the center of a circle, find the value of ***x*** in each case.

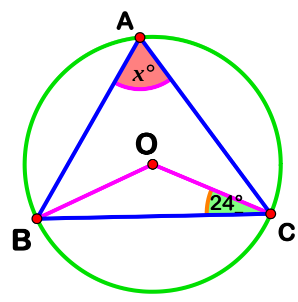
a)



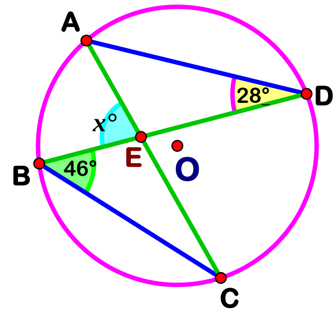
b)



c)

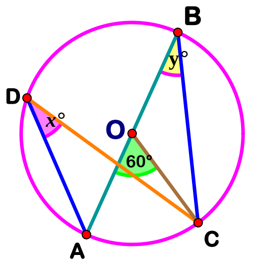


d)

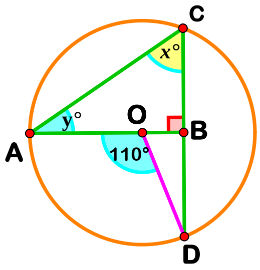


3. Given that ***O*** is the center of a circle, find the value of ***x*** and **y** in each case.

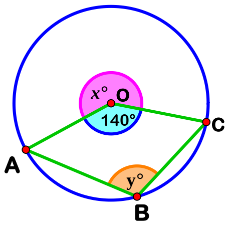
a)



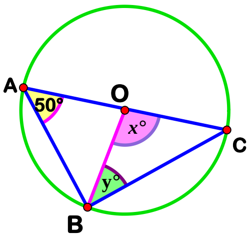
b)



c)

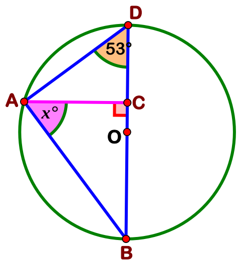


d)

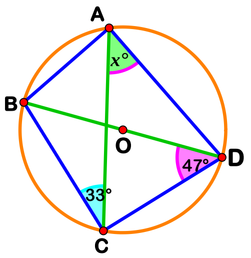


4. Given that ***O*** is the center of a circle, and BD is a diameter. find the value of ***x*** in each case.

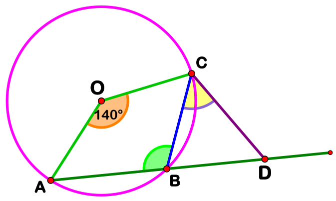
a)



b)



5. In the figure below, ***O*** is the center of the circle and *ABD* is a straight line. ∠ ***AOC*** = 140 °, and *BC* = *BD*. Find the value of ∠ ***ABC*** and ∠ ***BCD.***

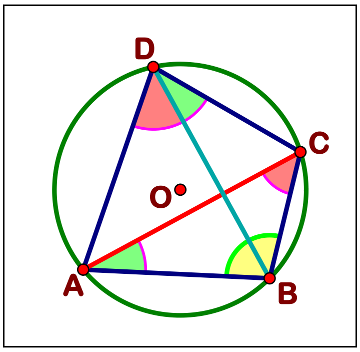


The sum of opposite angles of a quadrilateral inscribed in a circle is 180°.

A cyclic quadrilateral.

A quadrilateral is a *cyclic quadrilateral*, if there is a circle passing through all its four vertices.

The sum of opposite angles of a cyclic quadrilateral is 180°.



A triangle formed by 2 radii and a chord is always **an isosceles triangle**, as both radii are always equal.