

Stat 216 - Hypothesis Testing Worksheet

1. The EPA reports that the exhaust emissions for a certain car model has a normal distribution with a mean of 1.45 grams of nitrous oxide per mile and a standard deviation of 0.4. The car manufacturer claims their new process reduces the mean level of exhaust emitted for this car model. A SRS of 28 cars is taken and the mean level of exhaust emitted for this sample is 1.21 grams.

(a) State the null and alternative hypotheses.

$$H_0 : \mu = 1.45 \quad H_a : \mu < 1.45$$

(b) Calculate the test statistic.

$$z = \frac{\bar{x} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{1.21 - 1.45}{\frac{0.4}{\sqrt{28}}} = -3.17$$

(c) Calculate the p-value. Draw the picture.

$$P(Z < -3.17) = 0.0008$$

(d) What is the decision at the 0.01 significance level?

Reject H_0 because p-value (0.0008) $<$ α (0.01)

(e) What do you conclude about the mean level of exhaust emitted for this car model?

There is enough evidence to conclude that the true mean level of exhaust emitted for all cars of this model is less than 1.45 grams.

2. The amount of water consumed per week by Montana residences is normally distributed with an unknown mean μ and a standard deviation of 10 gallons. A simple random sample of ten residences has a mean value of $\bar{x} = 120.3$ gallons. The city of Bozeman claim that the average water consumed in the state of Montana is not 125 gallons.

(a) State the null and alternative hypothesis.

$$H_0 : \mu = 125 \quad H_a : \mu \neq 125$$

(b) Calculate the test statistic.

$$z = \frac{\bar{x} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{120.3 - 125}{\frac{10}{\sqrt{10}}} = -1.49$$

(c) Calculate the p-value. Draw the picture.

$$2 \times P(Z > |-1.49|) = 2 \times P(Z > -1.49) = 2 \times P(Z < -1.49) = 2 \times 0.0681 = 0.1362$$

(d) Make a decision at $\alpha = 0.10$.

Fail to reject H_0 because p-value (0.1362) $>$ α (0.10)

(e) What will the City of Bozeman conclude about the average amount of water consumed per week by Montana residences?

There is not enough evidence to conclude that the true average amount of water consumed per week in Montana is not equal to 125 gallons.

3. A credit card company wondered whether giving frequent flyer miles for every purchase would increase card usage, which has a current mean of \$2500 per year. They gave free miles to a SRS of 51 credit card customers and found the sample mean to be \$2542. Assume the population standard deviation is $\sigma = \$109$.

(a) State the null and alternative hypotheses.

$$H_0 : \mu = 2500 \quad H_a : \mu > 2500$$

(b) Calculate the test statistic.

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{2542 - 2500}{\frac{109}{\sqrt{51}}} = 2.75$$

(c) Calculate the p-value. Draw the picture.

$$P(Z > 2.75) = P(Z < -2.75) = 0.003$$

(d) Make a decision at $\alpha = 0.01$.

Reject H_0 because p-value (0.003) $<$ α (0.01)

(e) State your conclusion in terms of the problem.

There is enough evidence to conclude that the true mean credit card usage of people given frequent flyer miles is more than \$2500 per year.

4. Studies conducted in the 1970s indicated that the average age at which children take their first alcoholic drink is 14.6 years old. Sociologists believe that children are starting to drink at a younger age. A SRS of 144 young adults (18 years of age) is selected and the age at which each adult took their first alcoholic drink is recorded. The sample mean age was 13.3 years of age. The population standard deviation is known to be 5 years.

(a) State the null and alternative hypotheses.

$$H_0 : \mu = 14.6 \quad H_a : \mu < 14.6$$

(b) Calculate the test statistic.

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{13.3 - 14.6}{\frac{5}{\sqrt{144}}} = -3.12$$

(c) Calculate the p-value. Draw the picture.

$$P(Z < -3.12) = 0.0009$$

(d) What is the decision at the significance level 0.01?

Reject H_0 because p-value (0.0009) $<$ α (0.01)

(e) What do you conclude about the mean age at which children take their first alcoholic drink?

There is enough evidence to conclude that the true mean age at which all children take their first alcohol drink is younger than 14.6 years old.