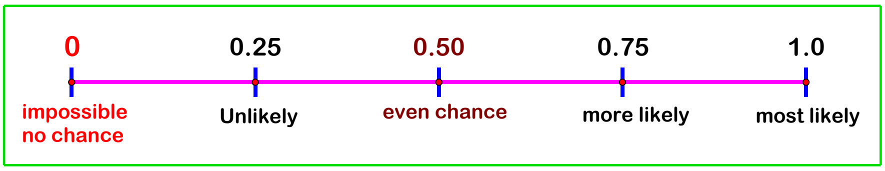
Probability

Chance

Chance is the occurrence of events in the absence of any obvious intention or cause. Chance is the unknown and unpredictable element in happening that seems to have no assignable cause. Calculating the chance of winning a game involves a branch of mathematics known as "probability". For example, how would you describe the chances of happening of the following events?

a) I will be late for school tomorrow.
b) It will rain in Bangkok tomorrow.
c) The new baby born in a hospital will be a girl.
d) We can fly to the sun.
e) The sun will rise in the east in the morning.

If you are to mark the events in a) to e) on the horizontal line below, where would you place each event on the line to show the likelihood they will occur?



Each of the above events may or may not happen. In our daily life, we use words such as *certainly*, *likely*, *unlikely*, or *impossible* to describe the chance of an event occurring. The mathematicians use the *probability* which is the values between 0 and 1 to measure of chance. In probability, the result or the outcome is not certain- it depends on chance.

Probability is a branch of mathematics that studies the likelihood, or chance, of an event happening. For example, the chance of wining the lottery is highly unlikely. Weather forecasters use probability to inform us of the likelihood or probability of storms, precipitation, temperature, and all weather patterns and trends. Probability has wide applications in the field of game strategy, the insurance industry, business and many more.

Probability refers to the likelihood for something to happen. When we speak of chance, we also refer to probability as a measure of chance.

Random Experiment

In probability, a random experiment is a process in which the result of the process cannot be predicted with certainty. The activity under consideration such as tossing a coin or throwing a die.

Outcome

A result of a random experiment is called *an outcome* of the experiment. An outcome may consist of more than one item of information.

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The following are some examples of *experiments* and their *possible outcomes*.

1) Tossing a coin

When a coin is tossed, there are *two possible outcomes*:

- Head (H) or Tail (T)

2) Throwing a die

A die is a special cube for which the following rule applies:

1. A die consists of six faces and dots. The dots are one, two, three, four, five and six as shown.
2. The *total number* of dots on the two opposite faces is always **seven**.

When a single die is thrown, there are *six possible outcomes*: 1, 2, 3, 4, 5 and 6.

3) Picking a ball

Three red balls and *two blue* balls of the same size are placed in a bag. One ball is picked up from the bag at random. There are *five possible outcomes*: R_1, R_2, R_3, B_1, and B_2.

Where ***R*** represents the outcome of getting a *red ball*, and

Where ***B*** represents the outcome of getting a *blue ball*.

Sample Space

The collection of all possible outcomes of the experiment is called the *sample space* and denotes by **S**.

The ***total number of possible outcomes*** is the number of elements in the sample space. It is denoted by **n(s).**

The **sample space** and the *total number of possible outcomes* of each of the three experiments above is:

Experiment 1: Tossing a coin

Sample space (S) consists of a head (H) and a tail(T)

$n(S) = 2$

Experiment 2: Throwing a die

Sample space (S) consists of 1, 2, 3, 4, 5 and 6

$n(S) = 6$

Experiment 3: *Three red* balls and *two blue* balls of the same size are placed in a bag. One ball is picked up from the bag at random.

Sample space (S) consists of balls R_1, R_2, R_3, B_1, B_2

$n(S) = 5$

Event

An event is a collection of outcomes of an experiment satisfying a given condition. Event consists of:

- *Simple event* or *An Elementary event* . An elementary event consists of a single outcome in the sample space.
- *Compound events*. Events which consist of more than one outcome in the sample space.

Type of Events:

1) ***Independent Event***: each event is not affected by the other events.
2) ***Conditional Event*** or *Dependent Event*: an event is affected by the previous events.
3) ***Mutually Exclusive Events***: Events can not happen at the same time.

Probability

Probability is a numerical statement about the chance that an event will occur.

Many events cannot be predicted with total certainty. The best way we can say is how **likely** it is that some event will happen.

Probability is just a guide. Probability does not tell us exactly what will happen, it is just a guide.

Sometimes we can measure a probability with a number, for example 25% chance of rain or we can use words such as impossible, unlikely, possible, likely, and certain.

When we perform a scientific experiment, we will get a certain result or outcome. But in probability, the result or outcome is not certain, it depends on chance.

Why study probability?

Because in business and economics we have to deal with Risk and Uncertainty.

Probability gives us a framework for dealing with Risk and Uncertainty, and provides us with methodologies to quantify risks and uncertainties, to compare them, and to make informed business decisions in uncertain environment.

Definition of Probability

Probability is the branch of mathematics concerning numerical description of how likely an event is to occur.

- The probability of an event A is written P(A).
- Probabilities are always numbers between 0 and 1, inclusively.

Probability of any event ***A*** occurring $= \dfrac{\text{Number of outcomes favourable to } A}{\text{Total number of possible outcomes}}$

In general, we can state the definitions of probability in two ways:

- Theoretical probability
- Empirical probability.

1.Theoretical Probability

Theoretical probability of an event is the number of favourable outcomes that event can occur, divided by the total number of outcomes. The probability of events that come from a sample space of known equally likely outcomes.

Theoretical Probability Formula

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of favourable outcomes for event E}}{\text{Total number of possible outcomes in sample space}}$$

P(E) = probability that an event E, will occur

n(E) = number of favourable outcome for event E

n(S) = total number of possible outcomes of sample space S

2. Empirical Probability

Empirical probability or *experimental probability* also known as relative frequency. Empirical probability is the ratio of number of outcomes in which a specified event occurs to the total number of trials in an actual experiment. In general sense, empirical probability estimates probabilities from observations or experience.

Empirical Probability Formula

$$P(\mathbf{A}) = \frac{n(\mathbf{A})}{n(S)} = \frac{\text{number of event } \mathbf{A} \text{ occurs}}{\text{Total number of observed occurrences}}$$

P(**A**) = probability that an event **A**, will occur

n(**A**) = number of event **A** occurs

Favorable Outcome

A favorable outcome is the equally likely outcomes of interest.

Complement

Complement of an event consists of all outcomes that are **not** in the event. For example, when we throwing **a die**.

The sample space or all possible outcomes S = {1, 2, 3, 4, 5, 6}

The favourable outcome or event A = {2, 4, 6}

The complement of event A or A' (not A) = {1, 3, 5}

In general:

$$\text{Probability of any event A occurring} = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$$

For example:

The probability of getting a "Head" when tossing a coin.

$$P\ (\text{Head}) = \frac{\text{Number of Head}}{\text{Total number of possible outcomes}}$$

$$P(H) = \frac{1}{2}$$

$$= 0.5$$

The probability of getting 6 when a die is rolled.

$$P\ (6) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(6) = \frac{1}{6}$$

$$= 0.16$$

Rules of Probability

The rules of probability:

1) For any event A, $0 \leq P(A) \leq 1$

2) P (certain event) = 1

We can write as

P(S) = 1

3) P(impossible event) = 0

P(empty set) = 0

4) **P(not A) = 1 – P(A)**

5) If two events ***A*** and ***B*** **are not** happen at the same time, then the probability of the occurrence of ***A*** **or** ***B*** is the **sum** of their individual probabilities.

P(A **or** B) = P(A) + P(B)

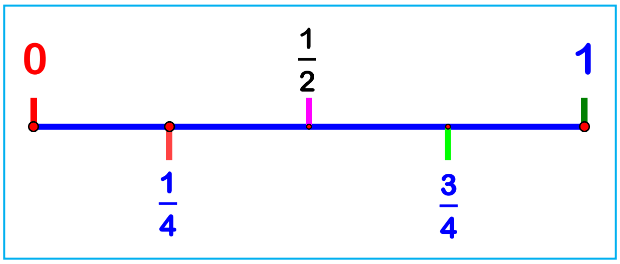
6) If ***A*** and ***B*** are two events, probability of ***A*** **and** ***B*** equals the probability of ***A*** **times** the probability of ***B***.

P (A ***and*** B) = P(A) × P(B)

Probability Line

Probability is the chance that something will happen. It can be shown on a line.

The probability of an event occurring is somewhere between impossible and certain. We can use numbers or words to show the probability of something happening. For example: The fractions on the probability line.



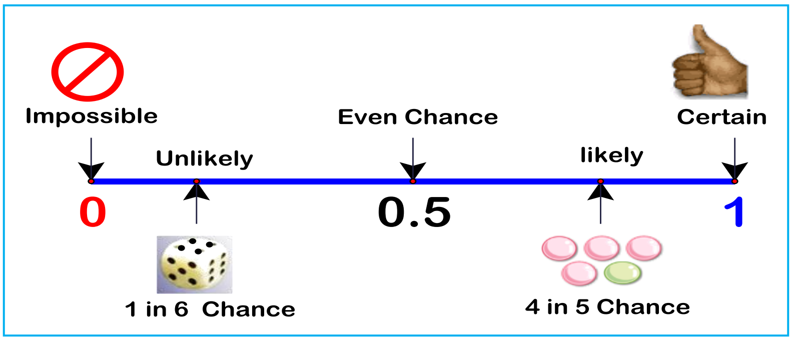
We can use numbers or words to show the probability of something happening as following:

Certain: When an event will always happen.

Impossible: When an event will never happen.

Likely : When an event has a good chance of happening.

Unlikely: When an event does not have a good chance of happening



Example 1

The probability of getting a "Head" when tossing a coin.

$$P\,(\text{Head}) = \frac{\text{Number of Head}}{\text{Total number of possible outcomes}}$$

$$P(H) = \frac{1}{2} = 0.5$$

The probability of getting 6 when a dice is rolled.

$$P\,(6) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(6) = \frac{1}{6} = 0.16$$

Example 2

Suppose that an experiment consists of drawing a card from a box containing 10 cards, each with a different number from 1 to 10 written on it. Find each of the following.

a) The possible outcomes for the experiment.

b) The event ***A*** consisting of outcomes which are numbers greater than 6.

c) The event ***B*** consisting of outcomes which are even numbers.

d) The probability of even numbers.

e) The probability of getting numbers less than 8.

Solution

a) The possible outcomes for the experiment.

All possible outcomes of an experiment or *sample space* **S**:

S = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

b) The event ***A*** consisting of outcomes which are numbers greater than 6.

A = 7, 8, 9, 10

c) The event ***B*** consisting of outcomes which are **even numbers**.

B = 2, 4, 6, 8, 10

d) The probability of even number.

A = favourable outcomes which are **even numbers**.

A = 2, 4, 6, 8, 10

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{5}{10} = 0.5$$

e) The probability of getting numbers less than 8.

A = favourable outcomes which are numbers less than 8

A = 1,2, 3, 4, 5, 6, 7

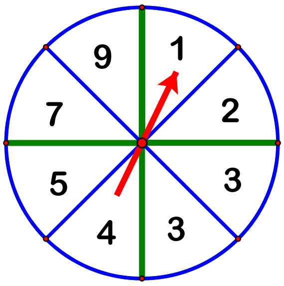
$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{7}{10} = 0.7$$

Example 3

The diagram shows a spinner divided into 8 equal sectors.



When the pointer is spun, what is the probability that:

a) the pointer will landing on a sector with number 3?

b) the pointer will landing on an odd numbered sector?

Solution

From the diagram shows a spinner divided into 8 equal sectors with number, each with a number written on it, the possible outcomes or sample space are:

Sample space is the number 1, 2, 3, 3, 4, 5, 7, and 9

Number of sample space = 8

a) What is the probability of the pointer landing on a sector with number 3?

A = favourable outcomes are sectors with number 3

Number of sectors with number 3 = 2 sectors

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{2}{8} = 0.25$$

b) What is the probability of the pointer landing on the odd numbered sector?

B = favourable outcomes are sectors with odd number

B = Sectors with number **1**, number **3**, number **3**, number **5**, number **7**, and number **9**

$n(B)$ = 6

$$P(B) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(B) = \frac{6}{8} = 0.75$$

Example 4

A shopkeeper observed 500 shoppers who walked into her shop over a period of time. She found that 350 of them made purchases while the rest did not buy anything. Using this observation, find the probability that a person who walked into the shop would not make a purchase.

Solution

Total number of shoppers = 500

Number of shoppers who did **not** make a purchase = 500 – 350

= 150

Probability that a shopper will **not** make a purchase

$$= \frac{\text{Number of shoppers who did not make a purchase}}{\text{Total number of shoppers}}$$

$$= \frac{150}{500}$$

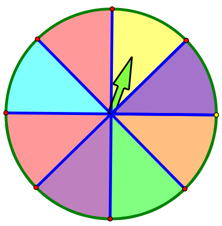
$$= 0.3$$

Example 5

When the pointer is spin, what is the probability that the pointer will landing at

a) red sector

b) green sector



Solution

From the diagram shows a wheel divided into 8 equal sectors with different colours, yellow, purple, orange, green, red, and blue.

Sample space is the sector with colour yellow 1 sector, purple 2 sectors, orange 1 sector, green 1 sector, red 2 sectors, and blue 1 sector.

Number of sample space = 8

a) What is the probability of the pointer landing on a sector with red colour?

A = favourable outcomes are sectors with red colour

Number of sectors with red colour = 2 sectors

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(A) = \frac{2}{8} = 0.25$$

b) What is the probability of the pointer landing on a sector with green colour?

B = favourable outcomes are sectors with red colour

Number of sectors with green colour = 1 sector

$$P(B) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(B) = \frac{1}{8} = 0.125$$

Exercise 1

1. There are 3 white stones and 2 pink stones in a box. In another box there are 2 white stones and 2 pink stones. A stone is selected at random from each box.
   a) Draw a possibility diagram to show the sample space.
   b) If A is the event "both stones are of the same color", find $P(A)$.
   c) If B is the event "the stones are of different colors", find $P(B)$.

2. A dice is rolled. Find the probability of getting
   a) an even number,
   b) a number greater than 4,
   c) a number which is even and greater than 4,
   d) a number which is even or greater than 4.

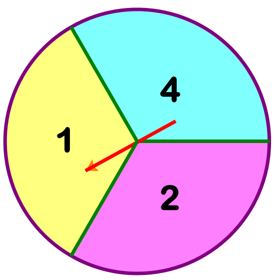
3. Two unbiased dice are rolled together. Drawing a tree diagram to show all possible outcomes and find the probability of obtaining:
   a) a sum which is less than 6
   b) a sum which is a prime number
   c) a product which is less than or equal to 5
   d) a product which is divisible by 3 and greater than 12

4. In a school there are three times as many female teachers as there are male teachers. A teacher is selected at random to represent the school in a mathematics club. Find the probability that the teacher selected is
   a) a female,
   b) a male.

5. The probability that a certain football team winning a match is 0.7. The probability of a tie is 0.1. What is the probability of the team losing?

6. Team ABC is playing a football match against team XYZ. If the probability that team ABC will win is $\frac{4}{11}$ and the probability that team XYZ will win is $\frac{2}{11}$, find the probability that the match will be draw.

7. The circular board is divided into the three equal sectors. An arrow on a spindle spins over the board with the numbers 1, 2 and 4 on it. It is spun twice and the numbers the arrow stops at are multiplied together. Find the probability of getting a product that is
   a) 4,
   b) odd,
   c) more than 2.



Mutually Exclusive Events (Disjoint):

- **Addition Rule of Probability**

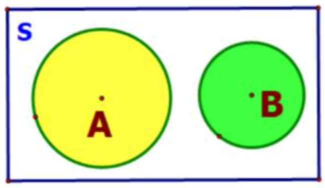
Mutually Exclusive Events

Two events are mutually exclusive if they cannot occur at the same time. For example:

- You cannot have Grade A and Grade B in the same subject.
- You cannot do both turning left and turning right at the same time. Turning left and turning right are mutually exclusive.
- Tossing a coin, the result will be heads or tails, but cannot happen both. Heads and tails are mutually exclusive.

Mutually Exclusive Events (Disjont): **Addition Rule of Probability:**

Two or more events are mutually exclusive if they cannot occur jointly.



P (Event A or Event B) = P(Event A) + P(Event B)

P (A or B) = P(A) + P(B)

A = number of boys in this class
B = number of girls in this class
S = Total number of students in this class

Notation: A, B are two Mutually Exclusive Events or disjoint events

Probability of event A or event B can be written as → P(A **or** B)

$$\mathbf{P\ (A\ or\ B) = P(A) + P(B)}$$

or $$\mathbf{P(A \cup B) = P(A) + P(B)}$$

Addition of Probability

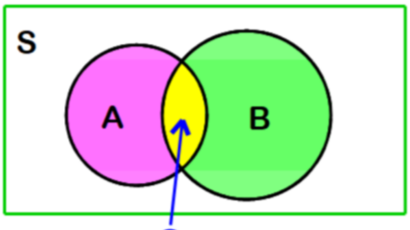
If two events ***A*** and ***B*** are mutually exclusive, then the probability of the occurrence of ***A*** or ***B*** is the **sum** of their individual probabilities.

$$\mathbf{P(\mathit{A} \cup \mathit{B}) = P(A) + P(B)}$$

Joint Events or Not Mutually Exclusive Events
Law of Addition for Events that are Not Mutually Exclusive

2. Law of Addition for Joint Events or two events are Not Mutually Exclusive

Two or more events are non exclusive or joint if the events can occur together.



A ∩ B
intersection of A and B

P (Event A or Event B) = P(Event A) + P(Event B) - P (Event A and Event B)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

Playing Cards

A standard deck of playing cards consist of 52 cards.

A playing cards consist of 4 suits: diamond, heart, club and spade, each suit has 13 cards. They are Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King. The diamond and heart cards are *red* colour cards and club and spade cards are *black* colour cards.

The Jack (J), Queen(Q) and King (K) in each suit are called **picture cards**.

diamond ♦	**A**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**J**	**Q**	**K**
heart ♥	**A**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**J**	**Q**	**K**
club ♣	**A**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**J**	**Q**	**K**
spade ♠	**A**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**J**	**Q**	**K**

Example 6

A card is randomly drawn from a standard deck of 52 playing cards. Find the probability that card drawn is

a) a heart
b) the queen of diamond
c) a club or a spade
d) not a heart

Solution

A standard deck of playing cards = 52 cards

Sample space of a playing cards consist of 4 suits, each suit has 13 cards. **Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King.**

n(s) = 52

a) Find the probability that card drawn is a heart

There are 13 hearts in the standard deck of playing card.

Let A = event of getting a heart

$\therefore \quad P(A) = \frac{13}{52} = \frac{1}{4}$ ▣

b) Find the probability that card drawn is the queen of diamonds
There is only 1 queen of diamonds in a standard deck of playing card.

Let B = event of getting the queen of diamonds

$\therefore \quad P(B) = \frac{1}{52} = \frac{1}{52}$ ▣

c) Find the probability that card drawn is a club **or** a spade.
There are 13 clubs in the standard deck of playing card.

Let C = event of getting a club

$\therefore \quad P(C) = \frac{13}{52} = \frac{1}{4}$

There are 13 spades in the standard deck of playing card.

Let D = event of getting a spade

$\therefore \quad P(D) = \frac{13}{52} = \frac{1}{4}$

The events C and D cannot happen at the same time.

Let P(C or D) = Probability that card drawn is a club **or** a spade

$\therefore$ P(C **or** D) = P(C) + P(D)

$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$ ▣

d) Find the probability that card drawn is not a heart

There are 13 hearts in the standard deck of playing card.

$\therefore$ The number of **not** hearts in the deck = $52 - 13 = 48$

Let A = event of getting a heart

A' = event of getting **not** a heart

$$\therefore \quad P(A') = \frac{48}{52} = \frac{12}{13}$$

Because $P(A) + P(A') = 1$

$$\therefore \quad P(A') = 1 - P(A)$$

$$= 1 - \frac{1}{13} = \frac{12}{13}$$

$$\therefore P(D) = \frac{6}{12} = \frac{1}{2}$$

Example 7

The probability that Pichai will win the 100 meters race is $\frac{1}{3}$ and the probability that Manat will win the same race is $\frac{1}{2}$. What is the probability that either Pichai or Manat will win?

Solution

Let A be the event "Pichai wins the 100 meters race".

$$P(A) = \frac{1}{3}$$

Let B be the event "Manat wins the 100 meters race".

$$P(B) = \frac{1}{2}$$

Let C be the event either Pichai or Manat will win.

Because Pichai and Manat cannot win in the same race.

$$P(C) = P(A) + P(B)$$

$$= \frac{1}{3} + \frac{1}{2}$$

$$= \frac{5}{6}$$

The probability that either Pichai or Manat will win is $\frac{5}{6}$