

Heuristics or Heuretic (original Greek):

The aim of heuristic is to study the methods and rules of discover and invention.

Heuristics means "serving to discover or serving to know or understand".

The word problem is a task that is difficult for individual who is trying to solve it.

Problem:, or thought A doubtful or difficult question; a matter of inquiry, discussion; a question that exercises the mind.

## What is Heuristic Strategy?

A heuristic is a technique or suggestion designed to help you better understand a problem. The aim of heuristic is to study the methods and rules to discovery and invention.

#### Heuristics Strategies

*Heuristic Strategies* are techniques used by good problem solvers when they need to make progress on tasks that are problems for them.

*Heuristics Strategies* are rules for successful problem solving, general suggestions that help an individual to understand a problem better or to make progress toward its solution.

## Modeling a Control Strategy for Heuristic Problem Solving

A heuristic problem solving is a guide to use when the student did not know what to do next.

Heuristic strategies of problem solving are as follows:

- 1. *Drawing a diagram*, even when the problem appears amenable to a different kind of argument. Pictures always help you to see things.
- 2. *Exemplifying* the problem (examine special cases) with the result that you either solve it for special cases or see empirically determinable patterns.
- 3. Looking for preliminary simplification.

We now look at the following frequency used heuristics strategies.

### Analysis

- 1. Draw a diagram if at all possible.
- 2. Examine special cases:
  - Choose special values to exemplify the problem and get a feel for it;
  - Examine limiting cases to explore the range of possibilities;
  - Set any integer parameters equal to 1, 2, 3, ..., in sequence, and look for an inductive pattern.
- 3. try to simplify the problem by:
  - exploiting symmetry, or
  - generality arguments (including scaling).

### Exploration

- 1. Consider essentially equivalent problems:
  - Replace conditions by equivalent problems;
  - Re-combine the elements of the problem in different ways;
  - Introduce auxiliary elements;
  - reformulate the problem by:
    - change the perspective or notation,
    - considering argument by contradiction or contrapositive;
    - $\circ$  assuming you have a solution and determining its properties.
- 2. Consider slightly modified problems:
  - Choose subgoals;
  - Relax a condition and try to re-impose it;
  - Decompose the domain of the problem of the problem and work on it.
- 3. Consider broadly modified problems:
  - Construct an analogous problem with fewer variables.
  - Hold all but one variable fixed to determine that variable's impact.
  - try to exploit any related problems that have similar:
    - o form
    - o given
    - $\circ$  conclusion.

### **Verify Your Solution**

- 1. Does your solution pass these specific tests?
  - Does it use all the pertinent data?
  - Does it confirm to reasonable estimates or predictions?
  - does it withstand tests of symmetry, dimension analysis, and scaling?
- 2. Does it pass these general tests?
  - Can it be obtained differently?
  - Can it be substantiated by special cases?
  - Can it be reduced to know results?
  - Can it be used to generate something you know?

## Schematic Outline of Heuristic Problem-solving Strategy



Exploration is the heuristic heart of the strategy, for it is the exploratory phase that the majority of problem-solving heuristic come in.

# The Exploration Stage of the Process



### Phase 1: The problem and equivalents

- 1. What to look at equivalent problems.
  - Replace givens or goals with equivalent conditions;
  - Try to reformulate the problem using:
    - o a more convenient notation or different perspective;
    - a logically equivalent form (argument by contrapositive)
  - Reorganize the problem by:
    - arranging things in a different way (e.g., infinite series).
    - introducing something new (e.g., lines in a diagram).
- 2. What to try.
  - Consider standard procedures first:
    - (1) break up the problem by
      - trying to establish subgoals.
      - decomposing and then recombining the domain (e.g., vector analysis)
    - (2) Eliminate alternatives, by a systematic reduction of the search space.
    - (3) Build up a solution by
      - induction
      - synthesis techniques (e.g., integration)
  - Ask general questions about the given and goals:
    - (1) Do problem with the same or similar goals suggest appropriate subgoals, possible procedures or auxiliary elements you might introduce?
    - (2) What can you normally get from the givens? What is usually done with information like this? Can that help you?
    - (3) Do you know anything related to both the given and goals? Can it serve as a bridge between the two?

### **Phase 2: Slightly Modified Problems**

- 1. Try to solve an easier related problem, either by
  - adding a condition or piece of information to the givens, or
  - removing or trying to partly fulfill, a condition in the goals.
- 2. try to solve a harder related problem.

### Phase 3: Desperation Attempts (Examining any related problems for inspiration)

- 1. Can you think of any problem with similar givens or goals? What techniques were used? Do any of them suggest plausible approaches here?
- 2. Can you obtain the result you are interested in for some subclass of the class you are interested in?
- 3. Can you reverse some conditions? try to prove the converse, and see what happens

### Phase 4: Dealing with failure.

- Two squares, each s on a side, are placed such that the corner on one square lies on the center of the other.
  Describe, in terms of s the range of possible areas representing the intersections of the two squares.
- 2. Inscribe a square in a given triangle. Two vertices of the square should be on the base of the given triangle, the other two vertices of the square on the other two sides of the triangle, one on each.

#### **References:**

Schoenfeld, A.H. (1985). *Mathematical Problem Solving*. Academic Press, Inc. Polya, G. (1945). *How to solve it*. Princeton: Princeton university Press.