

Measures of Central Tendency

A Measure of central tendency is a single value that reflects the average value of a set of data. There are three common measures of central tendency namely:

- arithmetic mean or mean;
- median; and
- mode.

- 1) **Arithmetic Mean** or **Mean**. The mean is the most commonly used type of average. It is easy to compute and tells us where most of data lie. However, it is easily affected by extreme values.
- 2) The **median** requires the data to be arranged in an *ascending* or *descending* order which may be tedious if the data set is large. However, the median is not easily affected by extreme values and thus may give a better indication of the data set than the mean. If there are no extreme values, the median is quite close to mean.
- 3) The **mode** can easily be read from any statistical diagram or from a frequency table. The mode tells us which datum in a set has highest frequency, it may be not useful for a data set with more than one mode.

Arithmetic Mean or Mean

Mean

For *data* the **arithmetic mean or mean is the sum of all the values divided by the total number of the values**. To find the mean for a sample, we use the following formula.

$$\text{Mean} = \frac{\text{Sum of all data}}{\text{Total number of data}}$$

Example 1

The salaries of 6 workers are given as follows:
5500, 7350, 6500, 7525, 5800, and 4900 Baht .
Find the mean salary.

Solution

$$\text{Mean salary} = \frac{\text{Sum of all data}}{\text{Total number of data}}$$

$$\text{Mean salary} = \frac{5500+7350+6500+7525+5800+4900}{6}$$

$$\text{Mean salary} = \frac{37575}{6}$$

$$\text{Mean salary} = 6262.50 \text{ Baht.} \quad \square$$

Example 2

The mathematics mean score of seven students is 42. Given that the scores of six students are 50, 39, 42, 58 and 46, find the score of the seventh student.

Let the score of the seventh student be x .

The mathematics mean score of seven students is 42.

$$\Rightarrow \text{Therefore } \frac{50 + 39 + 42 + 58 + 46 + x}{7} = 42$$

$$\frac{235+x}{7} = 42$$

$$235 + x = 294$$

$$x = 59$$

$$\text{The score of the seventh student} = 59 \quad \square$$

Example 3

The mean of eight number is 18. Six of numbers are 13, 19, 15, 25, 24, and 12. Find the mean of the remaining two numbers.

Solution

The mean of eight number is 18.

$$\text{Mean} = \frac{\text{Sum of all data}}{\text{Total number of data}}$$

$$\Rightarrow 18 = \frac{\text{Sum of the eight numbers}}{8}$$

$$\text{Sum of the eight numbers} = 8 \times 18 = 144$$

$$\begin{aligned} \text{Sum of the other two numbers} &= 144 - (13 + 19 + 15 + 25 + 24 + 12) \\ &= 36 \end{aligned}$$

$$\text{Mean of the other two numbers} = \frac{36}{2} = 18 \quad \square$$

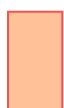


Exercise 1

- The mean of six numbers is 71. Two of the numbers are 96 and 46 and each of the other four numbers is equal to x .
 - Find the total value of the six numbers
 - Find the numerical value of x .
- From item 1, if each of the six numbers decreases by 5, find the mean of the resulting numbers.
- The mean height of 5 tennis male players is 178 cm. The mean height of 9 tennis female players is 165 cm. Find the mean height of all these tennis players.
- In Malinee's school, her mathematics teacher gives tests that are marked out of 100. Malinee has an average of 60 marks on her first four Mathematics tests. On the fifth test she got 80 marks. What is the average of Malinee's marks in Mathematics after all five tests?
- Some children were asked how many hours they had played on the internet during weekend. The table below shows the results.

Number of hours	0	1	2	3
Number of children	7	9	3	x

Find the value of x given that the mean is 1.



Median

Median

The median of a data set is the middle value when data is arranged in order from the smallest to largest.

The Median is calculated by placing all the observations in order (ascending or descending); the observation that fall in the middle is the median. The sample and population medians are computed in the same way.

For example: The prices of a hand phone (in baht) in 7 shops are:

7,900, 5,500, 2,400, 2,500, 4,650, 3,250, 6,425.

The above example, the prices of a hand phone in ascending order are as follows:

2,400, 2,500, 3,250, 4,650, 5,500, 6,425, 7,900



Middle position
Median price

The middle position is the 4th item.

∴ the median price = 4,650 baht.

In general, the calculation of the median of a data set with **N** items is as follows:

1. Arrange the **N** items in the data in ascending order (from the smallest to the largest).
2. Consider two cases as follows:
 - a) when **N** is **odd**, the median is the middle term of the ordered items.
 - b) when **N** is **even**, the median is the mean of the two middle terms of the ordered items.

Example 4

Find the median of the following data

5 4 1 3 3 6 5 2 4

Solution

Arranging the numbers in ascending order, we have

1 2 3 3 4 4 5 5 6

- b) Find the mean number of hours the students spent working out on their project work assignments.

Solution

- a) Find the median number of hours the students spent working out on their project work assignments.

Arranging the numbers of leaves in ascending order, we have

Number of hours	
Stem	Leaves
0	7, 8, 9, 9
1	3, 4, 5, 6, 7
2	2, 5, 7
3	4, 6
4	0

There were 15 students in the group.

∴ The median number of hours spent

= **hours** spent on the work by the 8th student

= 16 hours ■

- b) Find the mean number of hours the students spent working out on their project work assignments.

There were 15 students in the group.

∴ The mean number of hours spent

$$= \frac{7+8+9+9+13+14+15+16+17+22+25+27+34+36+40}{15}$$

$$= \frac{292}{15} = 19.46 \text{ hours} \quad \text{■}$$

Mode

The mode of a set of data is the value that occurs with the highest frequency. The sample and population modes are computed in the same way.

Example 7

Find the mode of the set of data

35, 32, 33, 40, 35, 39, 35, 38

Solution

Rearrange the data:

32, 33, **35, 35, 35**, 38, 39, 40

The number 35 occur 3 times which is the highest frequency.

\therefore Mode = 35 ▣

- It is possible that a data set has more than one mode.
- A data set will has no mode, if each value occurs only once.

Example 8

Find the mode of each of the following the set of data

a) 5, 3, 6, 4, 3, 9, 3, 7

b) 9, 4, 7, 1, 3

c) 11, 10, 9, 10, 13, 12, 14, 12

Solution

a) 5, 3, 6, 4, 3, 9, 3, 7

Rearrange the data:

3, 3, 3, 4, 5, 6, 7, 9

The number 3 occur 3 times which is the highest frequency.

\therefore Mode = 3 ▣

b) 9, 4, 7, 1, 3

Rearrange the data:

1, 3, 4, 7, 9

Each value occurs once.

\therefore This data set has no mode. ▣

c) 11, 10, 9, 10, 13, 12, 14, 12

Rearrange the data:

9, **10, 10**, 11, **12, 12**, 13, 14,

The number 10 and 12 occur twice. Thus this data set has more than one mode.

\therefore Mode = 10 and 12. ▣

Use and Purpose of Mean, Median and Mode

The **mean** is the most widely used *average*. It is usually preferred over the median and the mode because all the values in the data are used in calculating the mean. The mean is the most reliable measure *if there are no extreme values* in the data.

The **median** is the preferred *average* for describing data in social sciences, economic, and educational data. When *a set of data contains extreme values*, the *median* is a better indicator because it is **not affected** by extreme values.

The **mode** is an average that is useful in business planning as a measure of popularity that reflects opinion. The mode tells us which datum in a set has highest frequency, it may be not useful for a data set with more than one mode.

Example 9

The following data in the table below are monthly salaries earned by the sample group of employees of one company.

Position	Number of employees	Salary of each employee (baht)
Executive officer	1	32,000
Accountants	2	29,500
Secretary	1	28,000
Salesmen	3	27,500
Drivers	2	17,000

- What is the mean salary paid by the company?
- What is the median salary paid by the company?
- What is the mode salary paid by the company?
- Which is a good representative of the data on the salaries for this company? Compare the mean, median and the mode.

Solution

- a) **What is the mean salary paid by the company?**

From the data in the given table;

Let the number of employees = f
and the salary of each employee = x

Position	Number of employees (f)	Salary of each employee (x)	fx
Executive officer	1	32,000	32,000
Accountants	2	29,500	59,000
Secretary	1	28,000	28,000
Salesmen	3	27,500	82,500
Drivers	2	17,000	34,000
	$\Sigma f = 9$		$\Sigma fx = 235,500$

$$\begin{aligned} \text{Mean salary} &= \frac{235,500}{9} \\ &= 26,166.66 \text{ baht} \quad \square \end{aligned}$$

b) What is the median salary paid by the company?

Arranging the salary of each employee in ascending order, we have

17,000	}	4 numbers below
17,000		
27,500		
27,500		
27,500	←	Middle position
28,000	}	4 numbers above
29,500		
29,500		
32,000		

Median salary = 27,500 baht ▣

c) What is the mode salary paid by the company?

The salary number 27,500 occur 3 times which is the highest frequency.

∴ Mode salary = 27,500 baht ▣

d) Which is a good representative of the data on the salaries for this company? Compare the mean, the median, and the mode.

From a), b), and c) we have:

The mean salary = 26,166.66 baht

The median salary = 27,500 baht

The mode salary = 27,500 baht

The median and mode values are equal. The mean and median or mode values are close to each other. The mode or median values are good representative of data on the salaries for this company.

▣

Review

Data Presentation

- **Dot Plot:** A dot plot is a graphical display with dots representing data values that uses a number line and places a dot along an axis for each case to show frequency.
- **Stem-and-leaf plot:** Stem-and-leaf plot is a technique for summarizing all of data set collected from the observation. The advantages of the stem-and-leaf plot are that it provides an easy way to sort the data and the stem-and-leaf plot contains the original data.
- **Histograms:** Histograms are vertical bar graphs with no spaces in between the bars. The areas of the rectangular bars are proportional to the **frequencies** in each class. The bases of the rectangles equal to the class intervals.

Measure of Central Tendency

A Measure of central tendency is a single value that reflects the average value of a set of data. There are three common measures of central tendency:

- 1) **Mean:** Arithmetic mean or mean is the sum of all the values divided by the total number of the values.

$$\text{Mean} = \frac{\text{Sum of all data}}{\text{Total number of data}}$$

- 2) **Median:**

The median of a data set is the middle value when data is arranged in order from the smallest to largest value.

- 3) **Mode:**

The mode of a set of data is the value that occurs with the highest frequency. A data set has more than one mode or no mode, if each value occurs only once.

Mean	Median	Mode
<p>For ungrouped data x_1, x_2, \dots, x_n mean $\bar{x} = \frac{\text{Sum of } x_i}{n}$</p> <p>For grouped data mean $\bar{x} = \frac{\text{Sum of } fx_i}{\text{Sum of } f_i}$</p>	<p>Arrange n items in a data set in ascending order.</p> <p>If n is odd: median = the middle term</p> <p>If n is even: median = the mean of the two middle term</p>	<p>The mode of a data set is the value that occurs most often.</p>

Exercise 2

1. The number of bus passengers travelling on a certain route was recorded as shown below.

42, 29, 32, 45, 39, 37, 38, 38, 40, 36, 41 38

Find the mean, median and mode of passengers.

2. Find the median of the following set of data:

- a) 11, 9, 5, 6, 2, 6, 7
 b) 10, 10, 13, 18, 15, 11, 99, 10
 c) 31, 27, 22, 16, 18, 32

3. Find the mean, median and mode of the following set of data.

Stem	Leaves
2	1, 1, 2, 3, 3
3	7, 9, 9, 8
4	5, 5, 4, 3, 2, 7
5	3, 7, 3
6	1, 0, 0, 0

4. The table below shows the number of birthday cards received by 25 children last year.

Number of cards	1	2	3	4	5
Number of children	3	8	x	y	2

- a) Show that $x + y = 12$
 b) If the mean of the distribution is 2.76, show that

$$3x + 4y = 40$$

- c) Find the value of x and y .
 d) Determine the median.

5. The following table below shows the number of traffic accidents on the road during each month in a particular year in one province.

Month	Number of traffic accidents
January	230
February	175
March	150
April	258
May	192
June	186
July	168
August	170
September	155
October	134
November	167
December	224

- What is the *mean* for the number of traffic accidents in that year?
- What is the *median* for the number of traffic accidents in that year?
- What is the *mode* for the number of traffic accidents in that year?
- Which measure is the most appropriate as an average figure for public information? Explain your answer.

6.

The following table shows the monthly income per employee in XYZ Company.

Employee	Salary (baht)
A	12,500
B	27,500
C	26,000
D	25,500
E	19,000
F	31,000
G	19,000
H	25,000
I	19,000
J	22,500
K	28,000

- a) What is the *mean* salary of XYZ Company?
- b) What is the *median* salary of XYZ Company?
- c) What is the *mode* salary of XYZ Company?
- d) Which is a good representative of the data on the salaries for this company? Compare the mean, median and the mode.

7. Suppose the XYZ Company in **item 1** employs **Mr. L** as a new manager and pay him 55,000 baht per month.

- a) What is the mean salary of the employees now?
- b) How is the mean effected by the recruitment of the new manager?
- c) Is the mean salary a good representative of the earnings of the employees in this case?
- d) What is the median salary of the employees now?
- e) Does the median give a better indication of the average salary of these employees?
- f) What is the mode salary of the employees now?

8. The mean of a set of eight numbers is 4 and the mean of another set of fourteen numbers is x . When the two sets of numbers are combined, the new mean is 10. Find the value of x .

9. A class of students took a short test in which the highest possible score was 4. The scores of the class are recorded as follows:

Score	Number of students
0	2
1	8
2	10
3	16
4	x

- a) If the mode score is 3 marks, write down the inequality satisfied by x .
- b) If the mean score is 2.3 marks, find the value of x .
- c) If the median score is 2 marks, find the largest possible value of x .