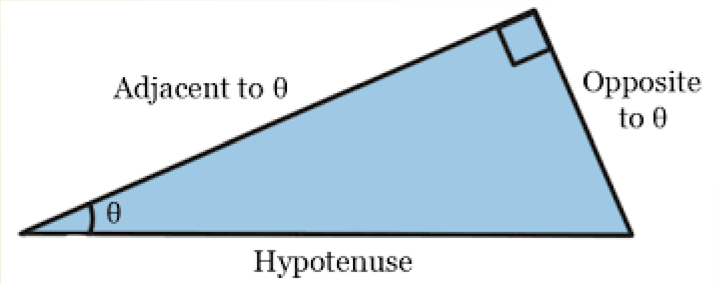
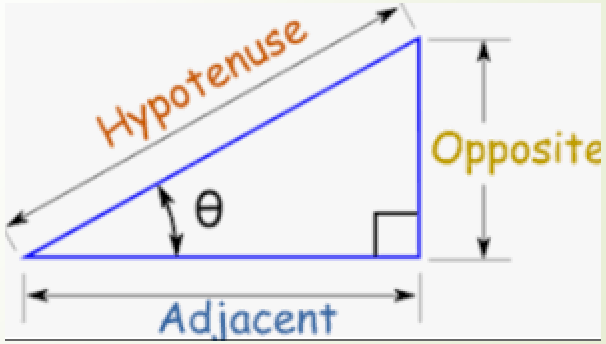
Chapter 1
The Trigonometry of Right Triangle





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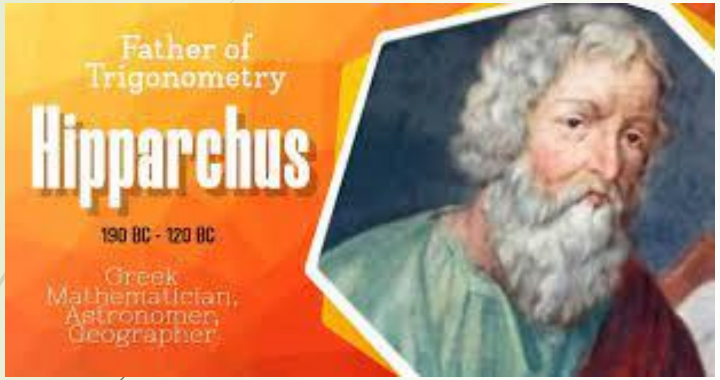
Father of Trigonometry

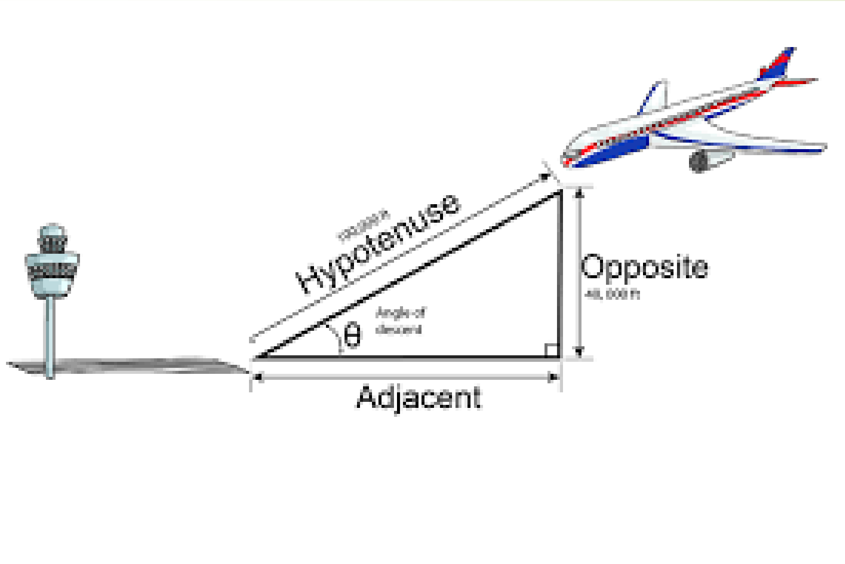


- Early study of triangles can be traced to the 2nd millennium BC, in Egyptian mathematics and Babylonian.
- Systematic study of trigonometric functions began in Hellenistic mathematics.
- Angles were measured in degrees and radius compiled by Greek Astronomer Hipparchus.
- Hipparchus was known as the father of trigonometry.

Source: https://vedicmathschool.org/hipparchus/

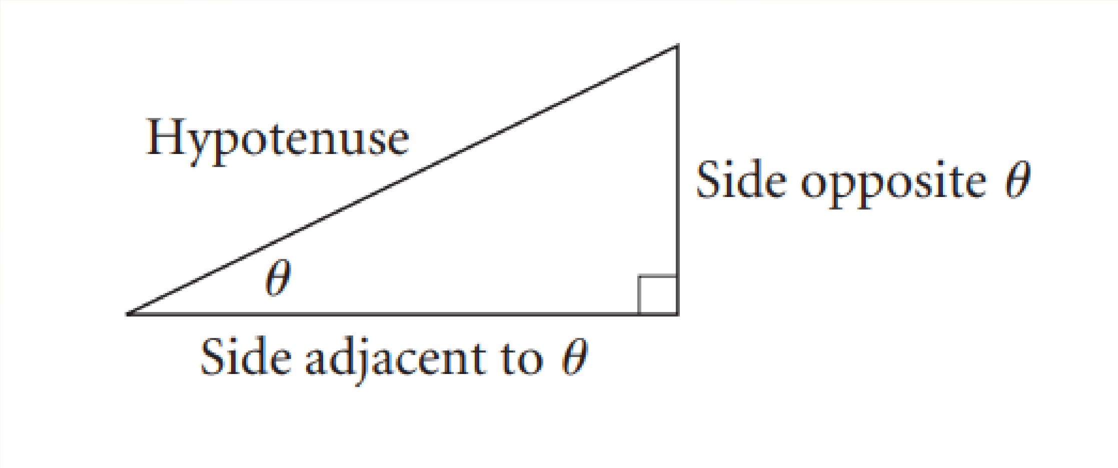
Measure of Right Triangles

Trigonometry is one of the branches of mathematics which deals with the relationship between the sides of a triangle (right triangle) with its angles.



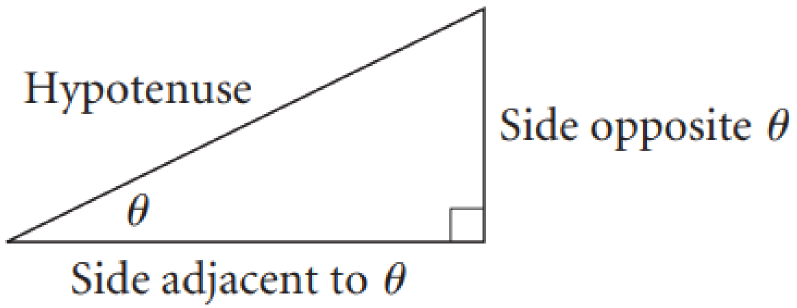
Source: https://numberdyslexia.com/real-life-applications-of-right-angle-triangle/

Trigonometric Ratios



Trigonometric ratios are the ratios of the length of sides of a right triangle. In trigonometry, there are six trigonometric ratios, namely, sine(sin), cosine(cos), tangent(tan), secant(sec), cosecant(csc), and cotangent(cot).

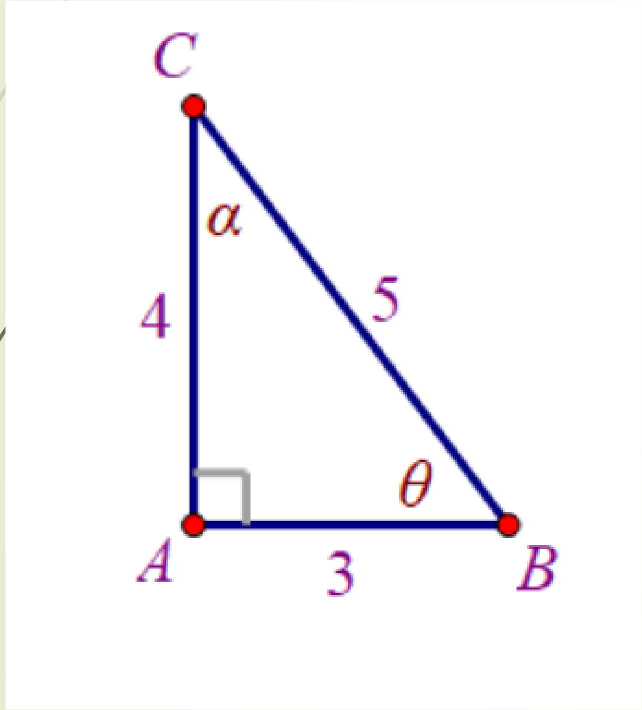
Trigonometric Ratio Values of an Acute Angle θ



$\sin\theta = \dfrac{\text{side opposite } \theta}{\text{hypotenuse}} = \dfrac{opp}{hyp}$	$\csc\theta = \dfrac{\text{hypotenuse}}{\text{side opposite } \theta} = \dfrac{hyp}{opp}$
$\cos\theta = \dfrac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \dfrac{adj}{hyp}$	$\sec\theta = \dfrac{\text{hypotenuse}}{\text{side adjacent to } \theta} = \dfrac{hyp}{adj}$
$\tan\theta = \dfrac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \dfrac{opp}{adj}$	$\cot\theta = \dfrac{\text{side adjacent to } \theta}{\text{side opposite } \theta} = \dfrac{adj}{opp}$

Example 1.1

In the right triangle shown below, find the values of six trigonometric ratios values of (a) θ and (b) α.



Solution:

a) $\sin\theta = \frac{opp}{hyp} = \frac{4}{5}$, $\csc\theta = \frac{hyp}{opp} = \frac{5}{4}$

$\cos\theta = \frac{adj}{hyp} = \frac{3}{5}$, $\sec\theta = \frac{hyp}{adj} = \frac{5}{3}$

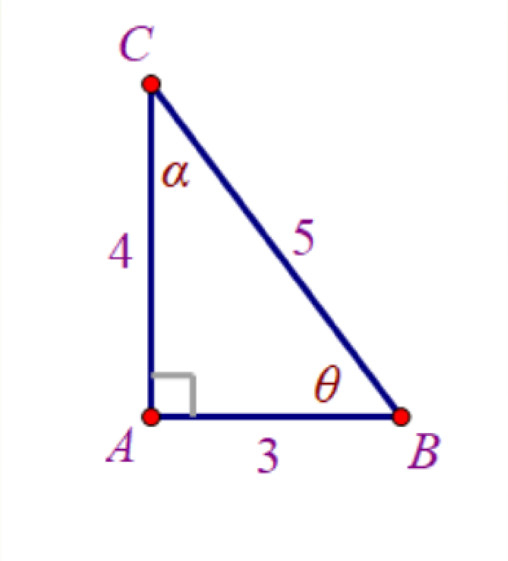
$\tan\theta = \frac{opp}{adj} = \frac{4}{3}$, $\cot\theta = \frac{adj}{opp} = \frac{3}{4}$

b) $\sin\alpha = \frac{opp}{hyp} = \frac{3}{5}$, $\csc\boldsymbol{\alpha} = \frac{hyp}{opp} = \frac{5}{3}$

$\cos\boldsymbol{\alpha} = \frac{adj}{hyp} = \frac{4}{5}$, $\sec\boldsymbol{\alpha} = \frac{hyp}{adj} = \frac{5}{4}$

$\tan\boldsymbol{\alpha} = \frac{opp}{adj} = \frac{3}{4}$, $\cot\boldsymbol{\alpha} = \frac{adj}{opp} = \frac{4}{3}$ #

Reciprocal Trigonometric Ratios



For any angle, the cosecant (csc), secant (sec), and cotangent (cot) are the reciprocals of the sine, cosine, and tangent ratio values, respectively. In above figure, there are the reciprocal relationship between the values of $\sin\theta$, $\frac{4}{5}$ and $\csc\theta$, $\frac{5}{4}$, the values of $\cos\theta$,$\frac{3}{5}$ and $\sec\theta$, $\frac{5}{3}$, and the values of $\tan\theta$, $\frac{4}{3}$ and $\cot\theta$, $\frac{3}{4}$.

Reciprocal Trigonometric Ratios Formula

The ***reciprocal sine*** is ***cosecant***:

$$\sin\theta = \frac{1}{\csc\theta} \quad and \quad \csc\theta = \frac{1}{\sin\theta}$$

The ***reciprocal cosine*** is ***secant***:

$$\cos\theta = \frac{1}{\sec\theta} \quad and \quad \sec\theta = \frac{1}{\cos\theta}$$

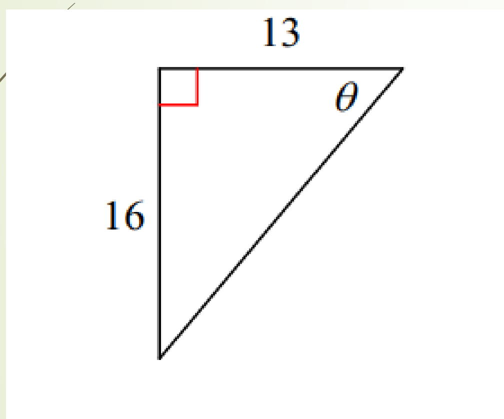
The ***reciprocal tangent*** is ***cotangent***:

$$\tan\theta = \frac{1}{\cot\theta} \quad and \quad \cot\theta = \frac{1}{\tan\theta}$$

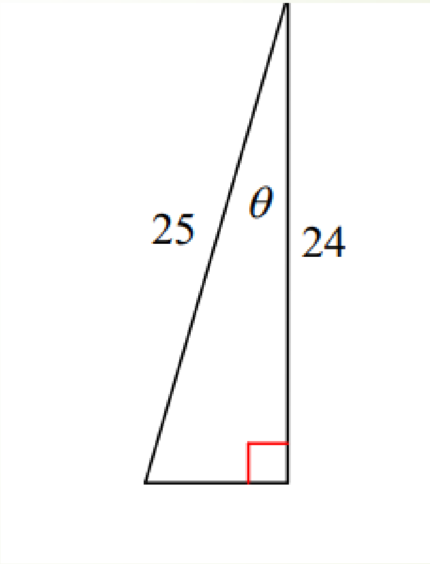
Example 1.2

Find the values of the trigonometric ratios indicated. (You will need to use Pythagorean theorem to find the missing side length).

a) $\csc\theta$

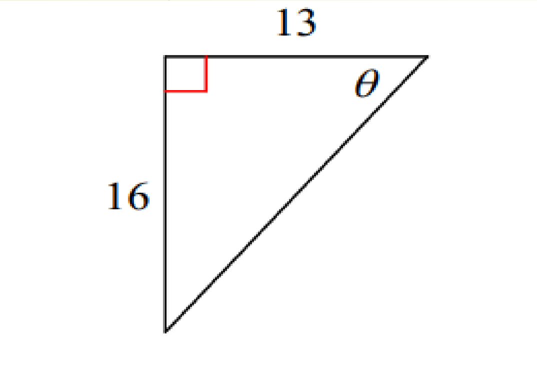


b) $\sec\theta$

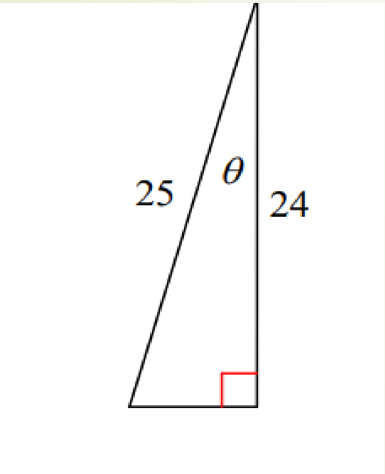


Example 1.2 (Cont.)

a) $\csc\theta$



b) $\sec\theta$



Solution:

a) Using Pythagorean theorem:

Hypotenuse $= \sqrt{16^2 + 13^2} = \sqrt{256 + 169} = 5\sqrt{17}$

Since $\csc\theta$ is the reciprocal sine, and $\sin\theta = \dfrac{16}{5\sqrt{17}}$

Therefore, $\csc\theta = \dfrac{5\sqrt{17}}{16}$ #

b) Using the reciprocal cosine, and $\cos\theta = \dfrac{24}{25}$

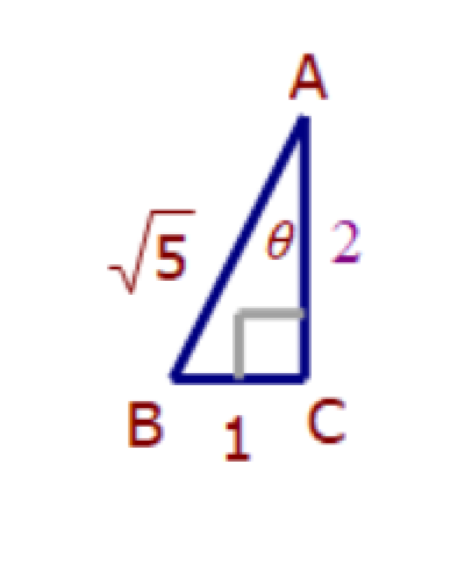
Therefore, $\sec\theta = \dfrac{25}{24}$ #

Example 1.3

Find the values of trigonometric ratios indicated. (You will need to draw a right triangle)

a) Find $\csc\theta$ if $\sec\theta = \frac{\sqrt{5}}{2}$ b) Find $\cot\theta$ if $\sec\theta = \frac{5}{3}$

Solution: a) Since $\sec\theta$ is the reciprocal cosine, and $\sec\theta = \frac{\sqrt{5}}{2}$



Then $\cos\theta = \frac{2}{\sqrt{5}}$

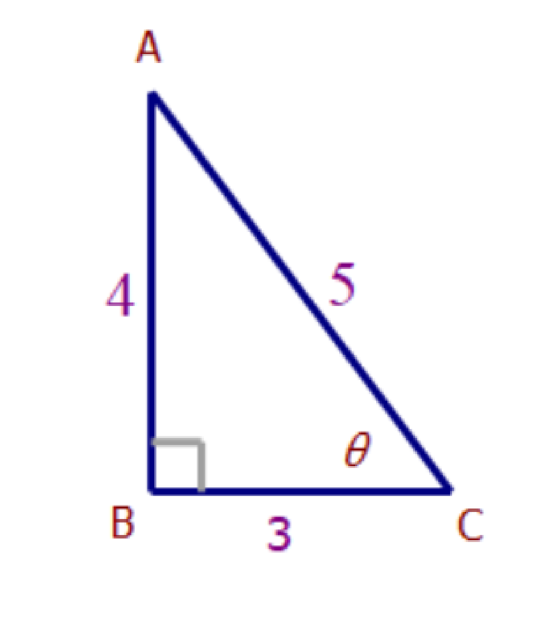
Using Pythagorean theorem:

$$\text{Opposite } \theta = \sqrt{(\sqrt{5})^2 - 2^2} = \sqrt{5-4} = 1$$

Since $\csc\theta$ is the reciprocal sine, and $\sin\theta = \frac{1}{\sqrt{5}}$

Therefore, $\csc\theta = \sqrt{5}$ #

Example 1.3 (Cont.)



Solution: b) Since $\sec\theta$ is the reciprocal cosine, and $\sec\theta = \frac{5}{3}$.

Then $\cos\theta = \frac{3}{5}$

Using Pythagorean theorem:

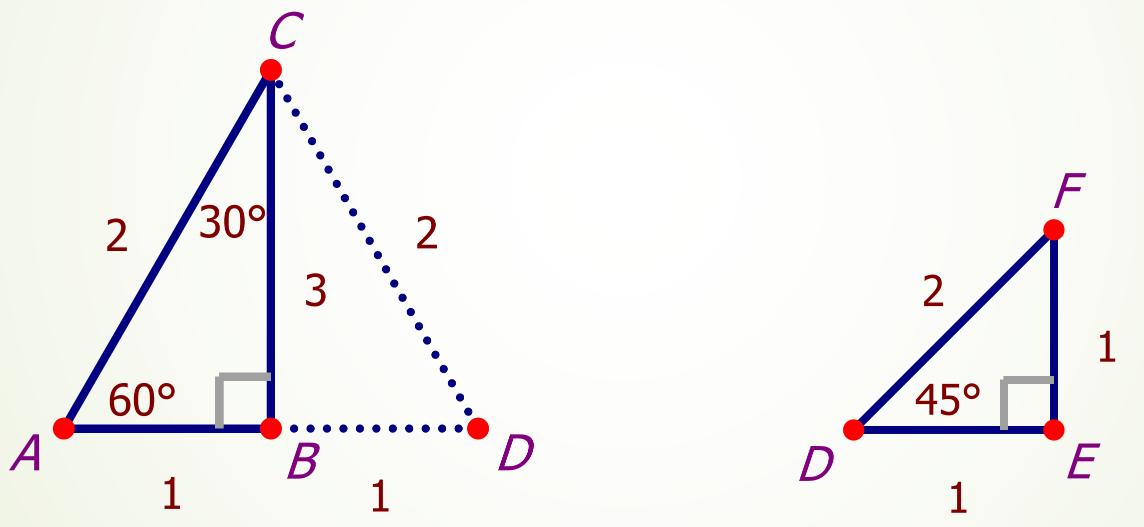
Opposite $\theta = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$

Since $\cot\theta$ is the reciprocal tangent, and $\tan\theta = \frac{4}{3}$

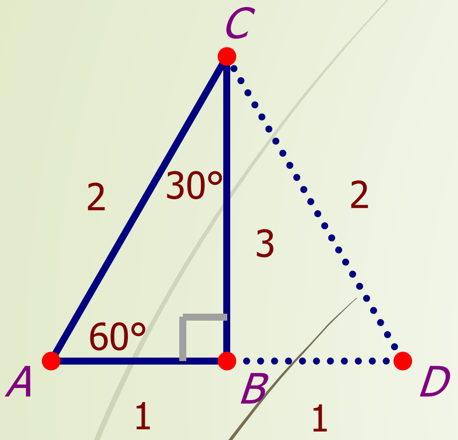
Therefore, $\cot\theta = \frac{3}{4}$ #

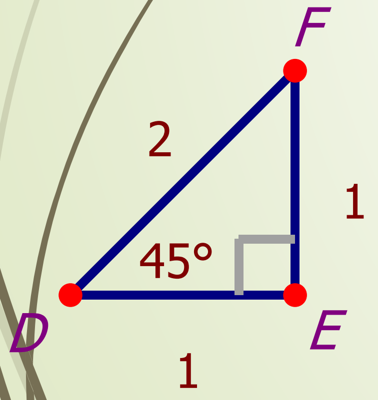
The Trigonometric Ratio for 30°, 45°, 60°

The trigonometric ratios for 30°,45°,"and" 60° are based on some standard triangles as follows:



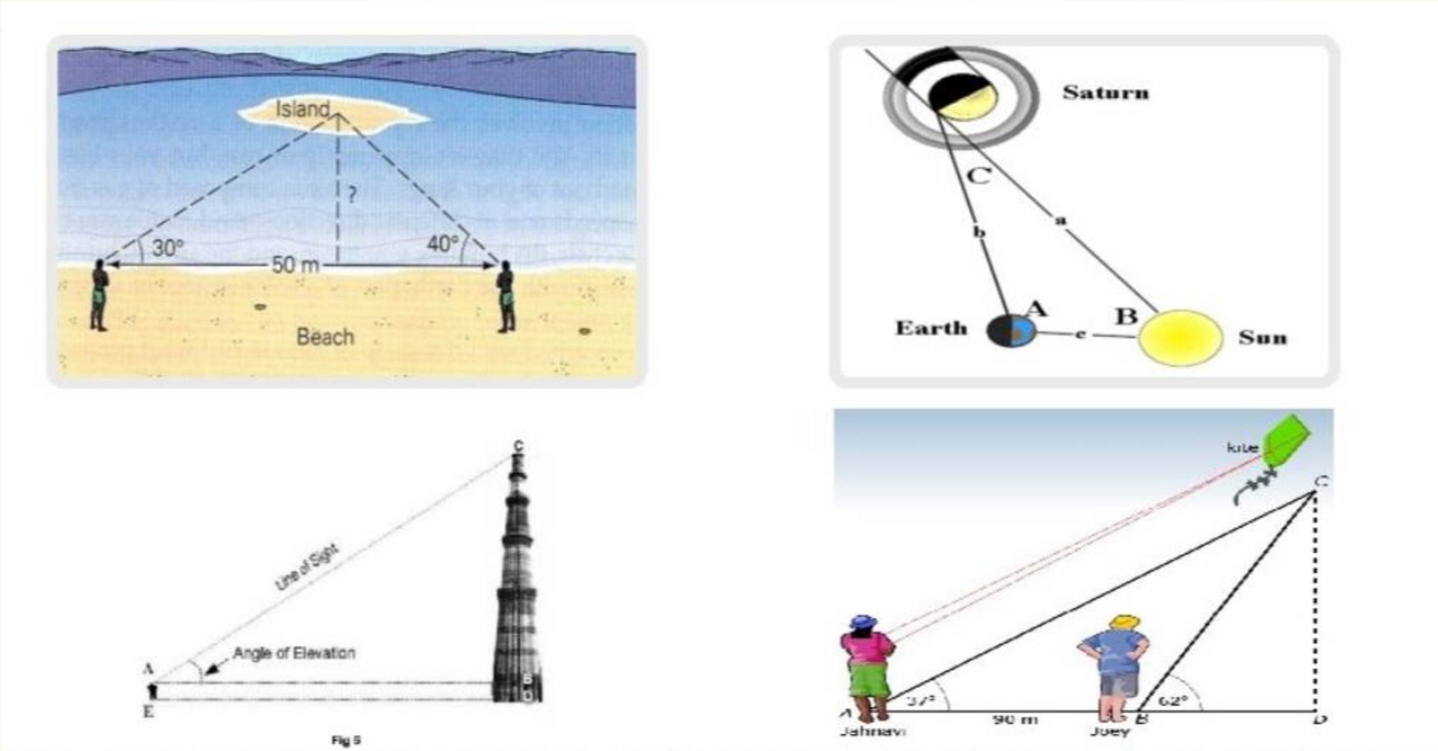
The Trigonometric Ratio for $30°, 45°, 60°$ (Cont.)





θ	$30°$	$45°$	$60°$
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
$\csc\theta$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$
$\sec\theta$	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2
$\cot\theta$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$

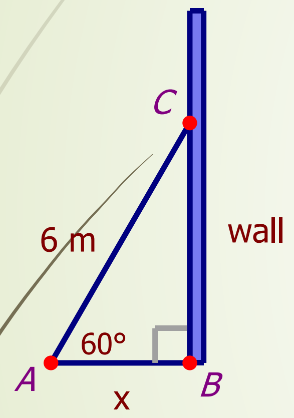
Applications of Right Triangle Trigonometry



Trigonometry can be used in navigation, satellite systems, measure the height of tower, decide the path of airplane, and others.

Applications of Right Triangle Trigonometry (Cont.)

Example 1.4 A 6-meter ladder leans against a brick wall forming angle θ of 60° with the ground. How far is the base of the ladder from the wall?



Solution: Let x be the distance along the ground to the wall, as in the left picture.

Since $\cos\theta = \frac{adj}{hyp}$, then, $\cos 60° = \frac{AB}{AC} = \frac{x}{6}$ (equation 1)

Since $\cos 60° = \frac{1}{2}$

Substitute $\cos 60°$ in equation 1 and solving for x:

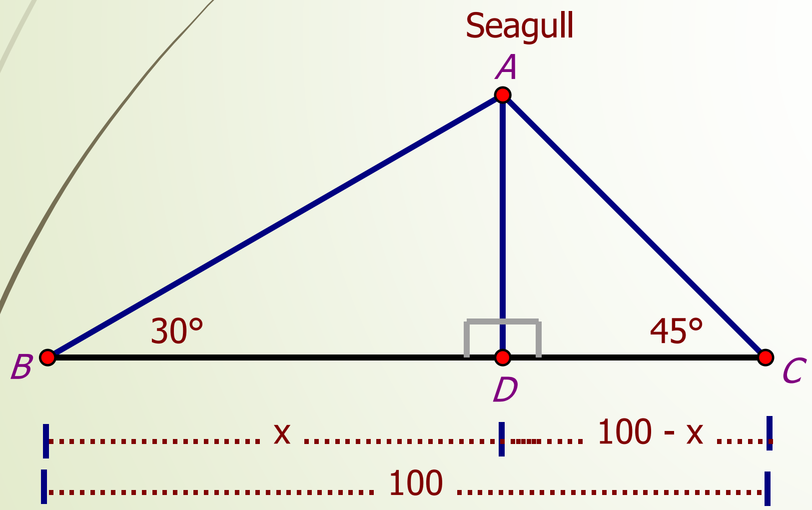
$$\frac{1}{2} = \frac{x}{6}$$

$$x = 3$$

Therefore, the base of the ladder 3 meters far from the wall. #

Applications of Right Triangle Trigonometry (Cont.)

Example 1.5 Two girls have the same height are standing 100 meters apart. They both see a beautiful seagull in the air between them. The angles of elevation from their eyes to the bird are 30° and 45°, respectively. How high up is the seagull?



Hint: In ΔABD, $\tan 30° = \frac{AD}{x}$ ………(eq. 1)

In ΔACD, $\tan 45° = \frac{AD}{100-x}$ ………(eq. 2)

……………..Solve for x……..

Answer: $AD \approx 36.60$ or about 37 meters high. #

Assignment

Detailed answers to the questions 1 to question 3 in Practice 1.1.

Q & A