Chapter 2 Conic Sections
Part1: Circle and Ellipse

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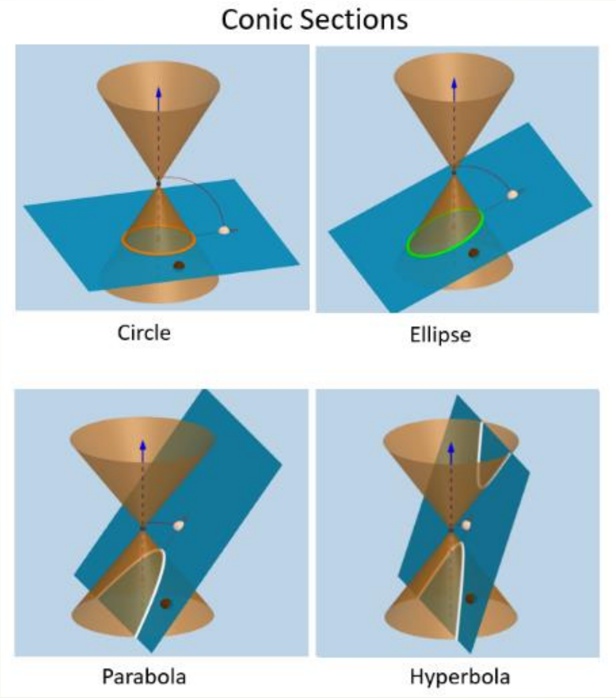
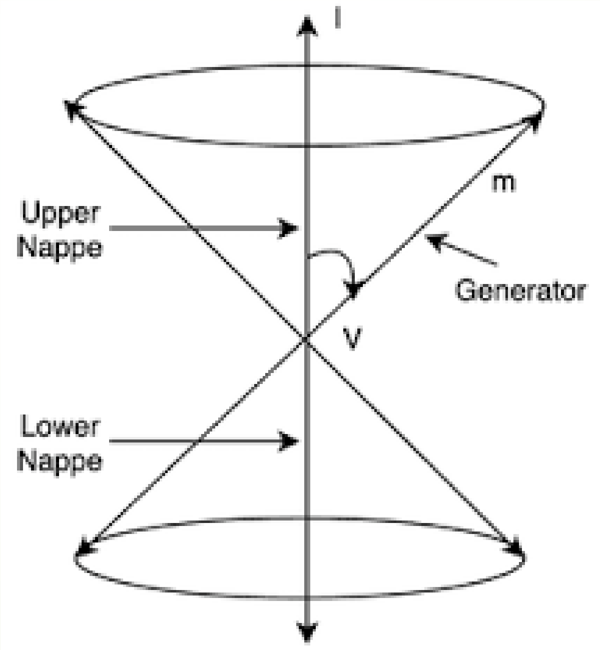
Conic Sections

A conic section is the intersection of a plane and a double right circular cone. There are four basic types of conic sections by changing the angle and location of the intersection without passing through the vertices of the cone. They are circle, ellipse, parabola, and hyperbola.



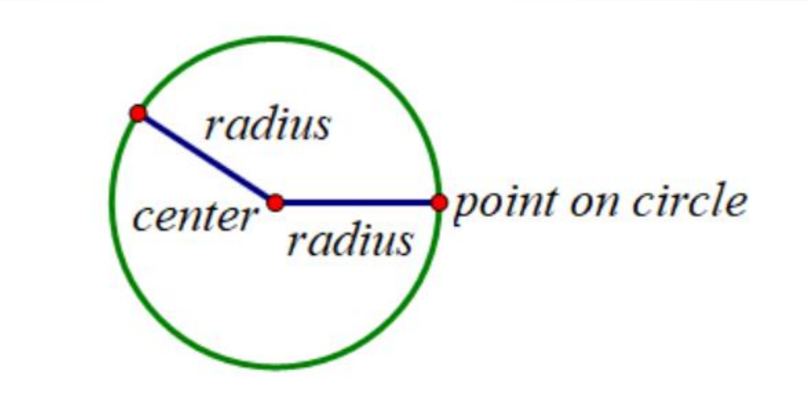
Cone

A cone can be described by the rotation of a line segment (*m*) called a generator of the cone around the vertical line (*l*) called the axis of a cone at a point (*V*) on this axis (called a vertex) as the following figure:



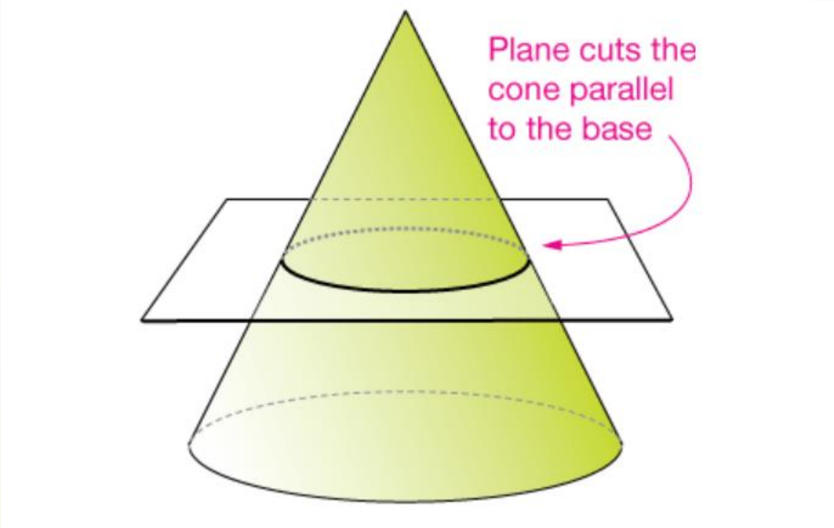
Circle

In geometry, a circle is the set of all points in the plane that are a fixed distance (the radius) from a fixed point (the center). Any line segment joining a point on the circle to the center is called a radius. By definition of a circle, any two radii have the same length.

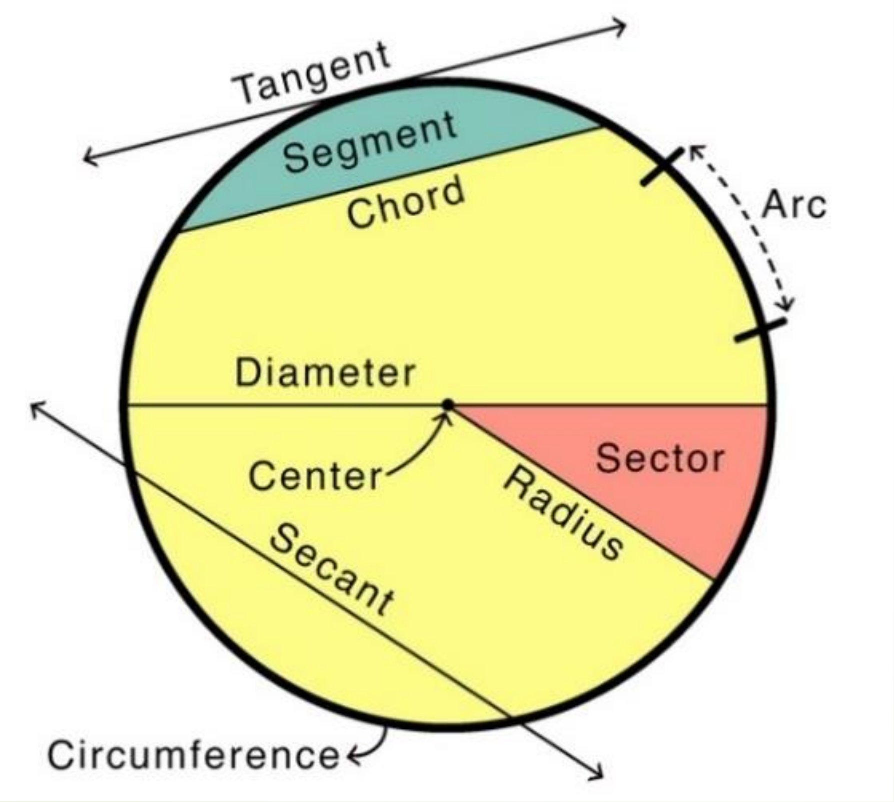


Circle

For conic section, the circle is a closed figure formed by the intersection of the surface of a right circular cone by a plane parallel to the base of the cone.



Parts of a Circle

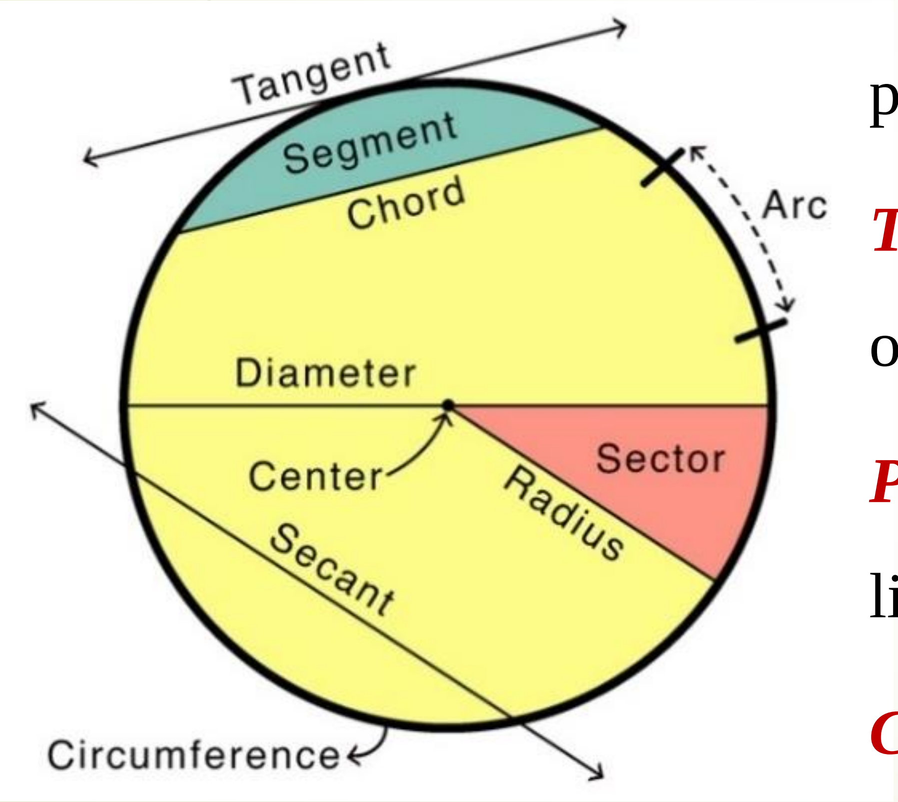


Radius: The distance from the center of the circle to its outer rim.

Chord: A line segment whose endpoints are on a circle.

Diameter: A chord that passes through the center of circle. The length of a diameter is two times the length of a radius.

Parts of a Circle (cont.)



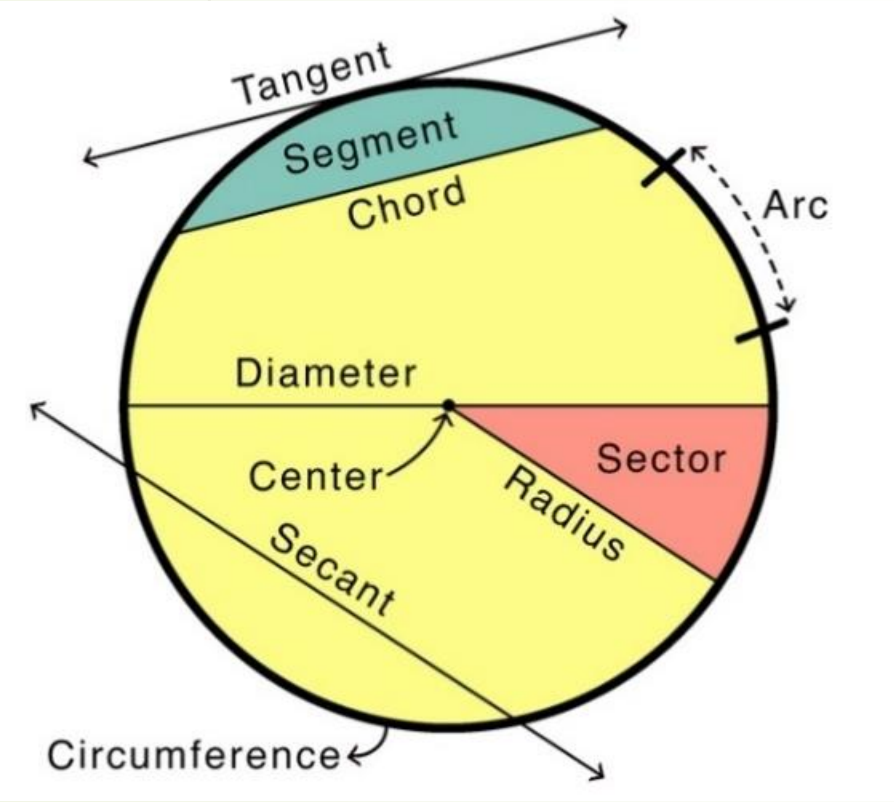
Secant: A line that intersects a circle in two points.

Tangent: A line that intersects a circle in exactly one point.

Point of Tangency: The point where a tangent line touches the circle.

Circumference: The distance around the circle is equal to $2\pi r$, where r is the radius of the circle.

Parts of a Circle (cont.)

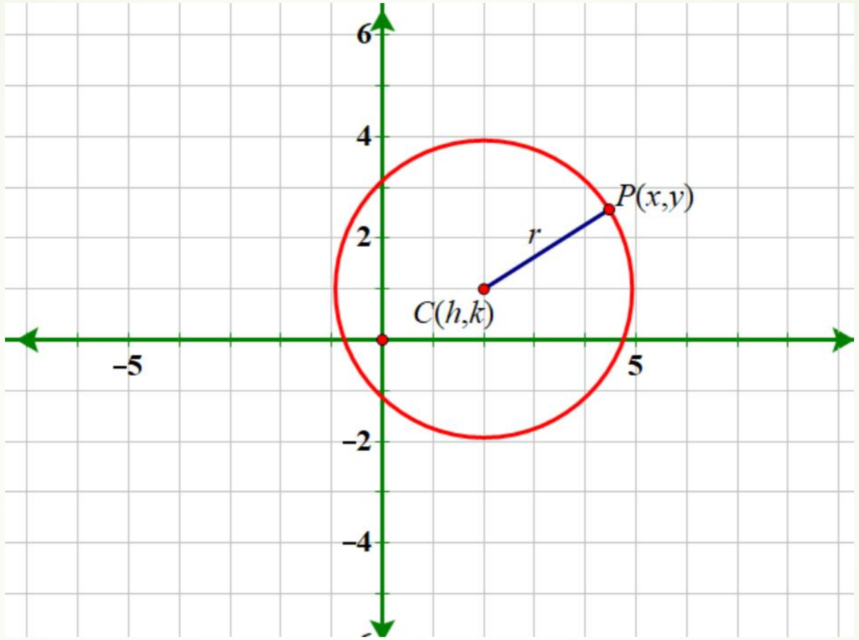


Arc: Any part of the circumference.

Segment: A region bounded by a chord of a circle and the intercepted arc of the circle.

Sector: A part of a circle made of arc of the circle along with its two radii.

Standard Form of Equation of a Circle: Center at (h, k)

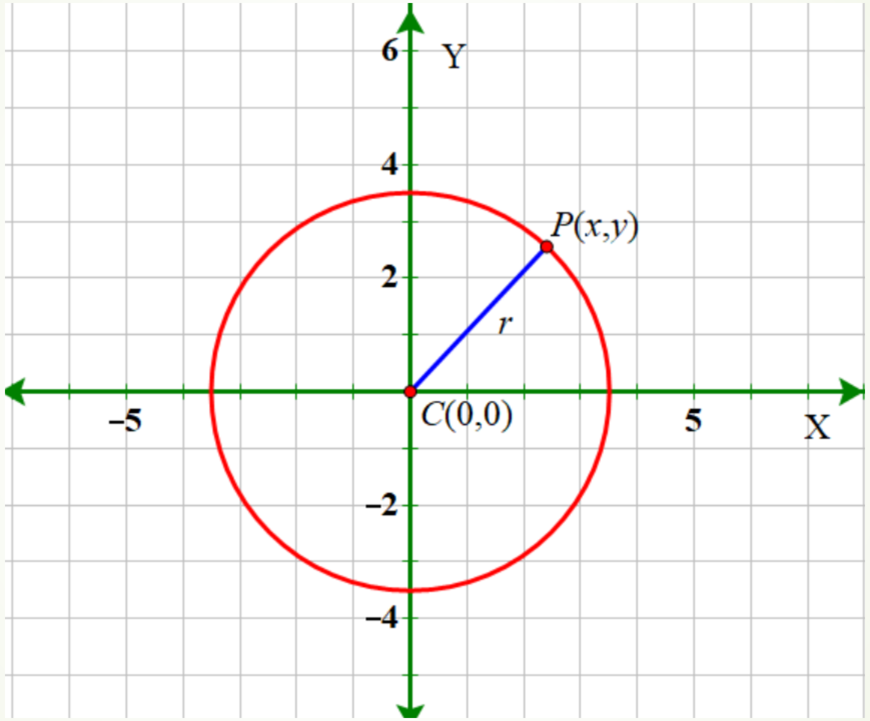


The distance between a point $P(x, y)$ on the circle to the center $C(h, k)$ is

$$PC = \sqrt{(x-h)^2 + (y-k)^2}$$

Since $PC = r$, then $r = \sqrt{(x-h)^2 + (y-k)^2}$ and $r^2 = (x-h)^2 + (y-k)^2$.

Standard form of Equation of a Circle: Center at (0, 0)



$$x^2 + y^2 = r^2$$

Example 1

(a) The circle with center (3,2) and the radius 3 has the equation

$$(x-3)^2+(y-2)^2=9 \qquad \#.$$

(b) The circle with center (0,0) and the radius 5 has the equation

$$x^2+y^2=25.\#$$

Example 2

(a) The graph of the equation $(x-4)^2+(y-5)^2=36$ is the circle with the center at (4,5) and radius 6.

(b) The graph of the equation $(x+3)^2+y^2=3$ is the circle with the center at (-3,0) and radius $\sqrt{3}$.

Example 3

Graph $(x-2)^2+(y+4)^2=9$

Solution:

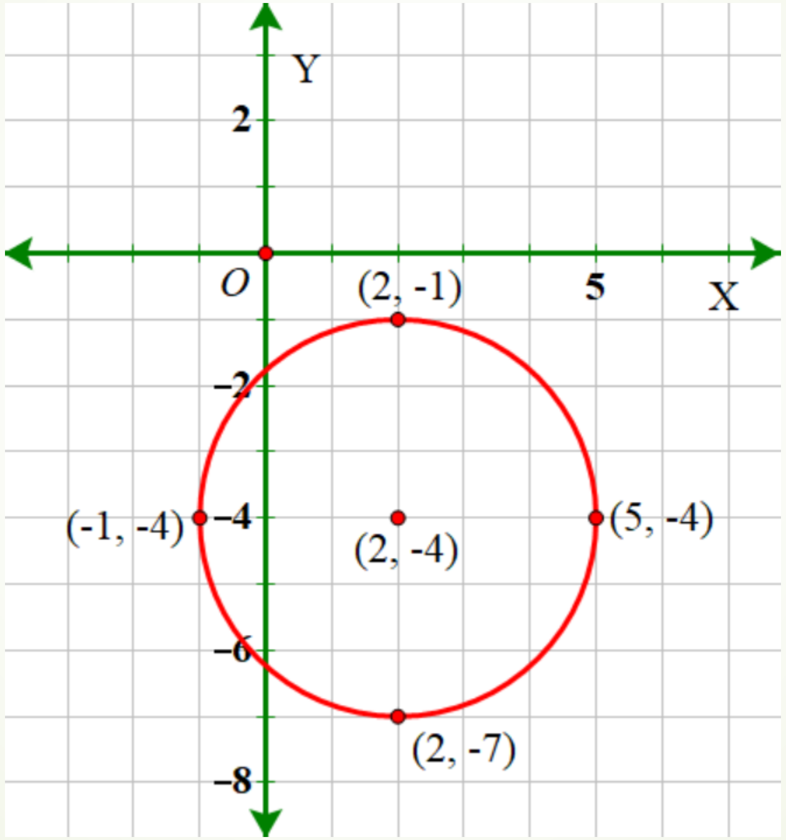
Step1: Compare the given equation to the standard form of an equation of a circle. The graph is a circle with center at $(h,k)=(2,-4)$ and radius $r=\sqrt{9}=3$.

Step 2: Plot the center. Then plot several points that are each 3 units from the center:

(2 + 3, -4) = (5, -4); (2 – 3, -4) = (-1, -4); (2, -4 +3) = (2, -1); (2, -4 -3) = (2, -7)

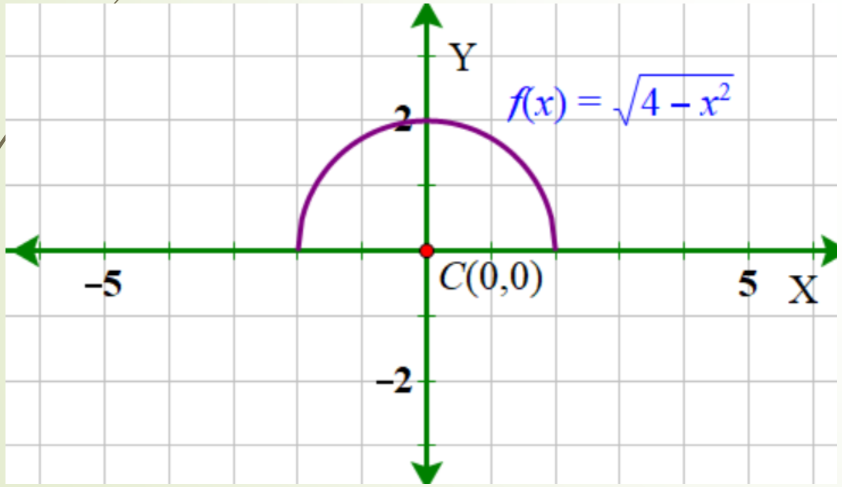
Step 3: Draw a circle through the points.

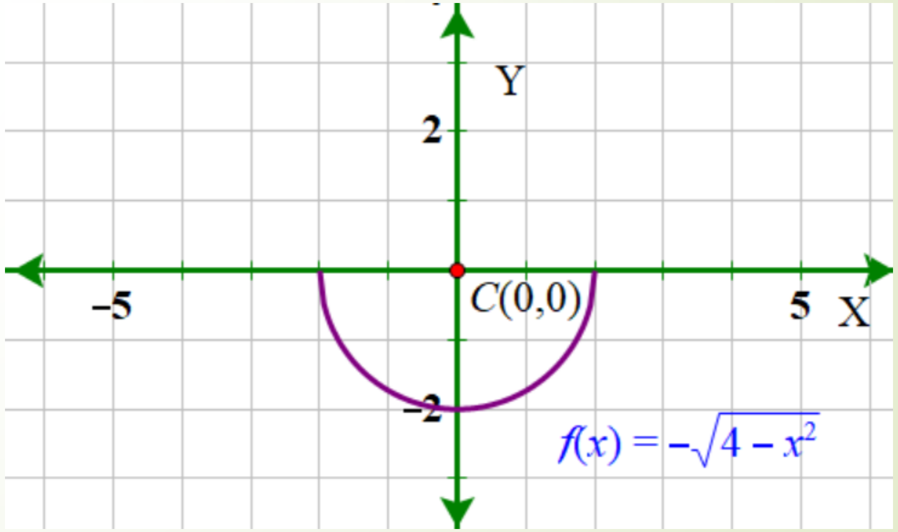
Example 3 (cont.)

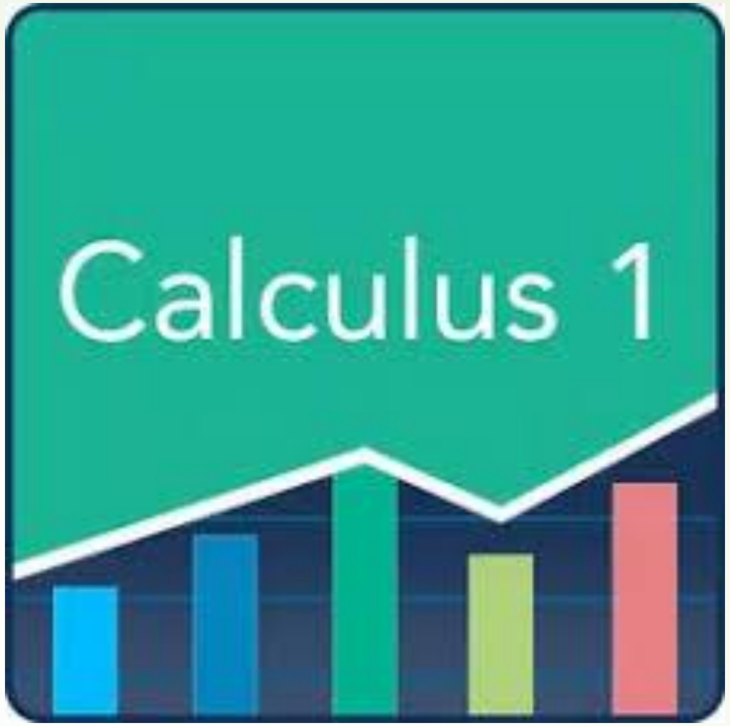


Circle Equation as Implicit Equation

An implicit equation is an equation which gives a relationship between the variables, but it does not specify in the form of $y = f(x)$.



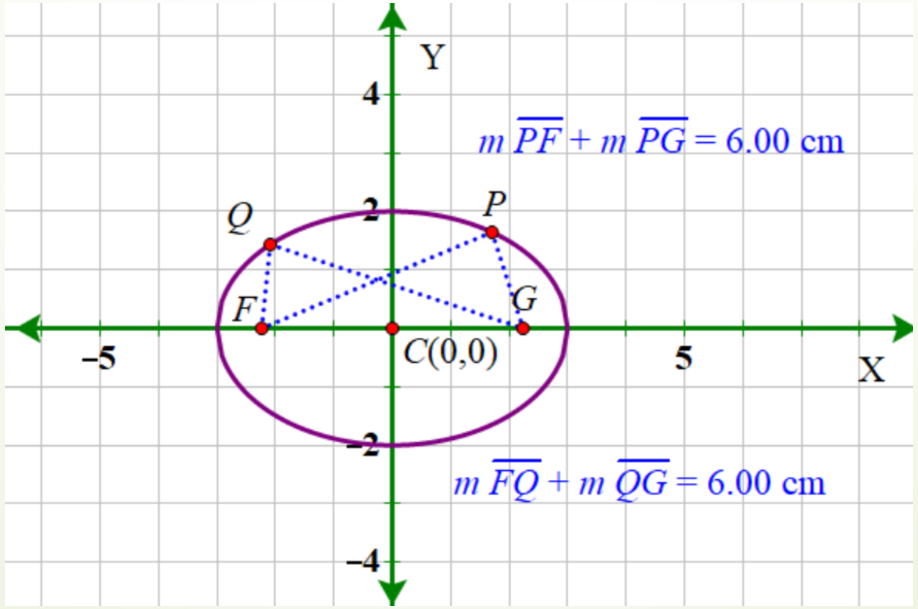




Practice 2.1

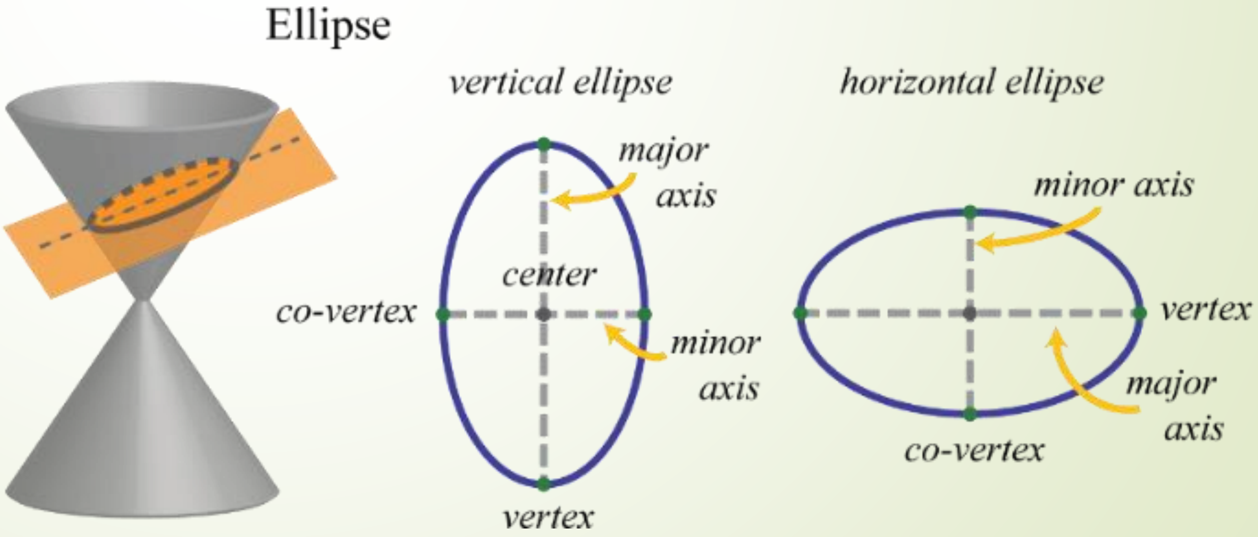
Ellipse

In mathematics, an ellipse is the set of points on a plane whose distance from two fixed points F and G have a constant sum. The two fixed points are called the foci (plural of focus) of the ellipse

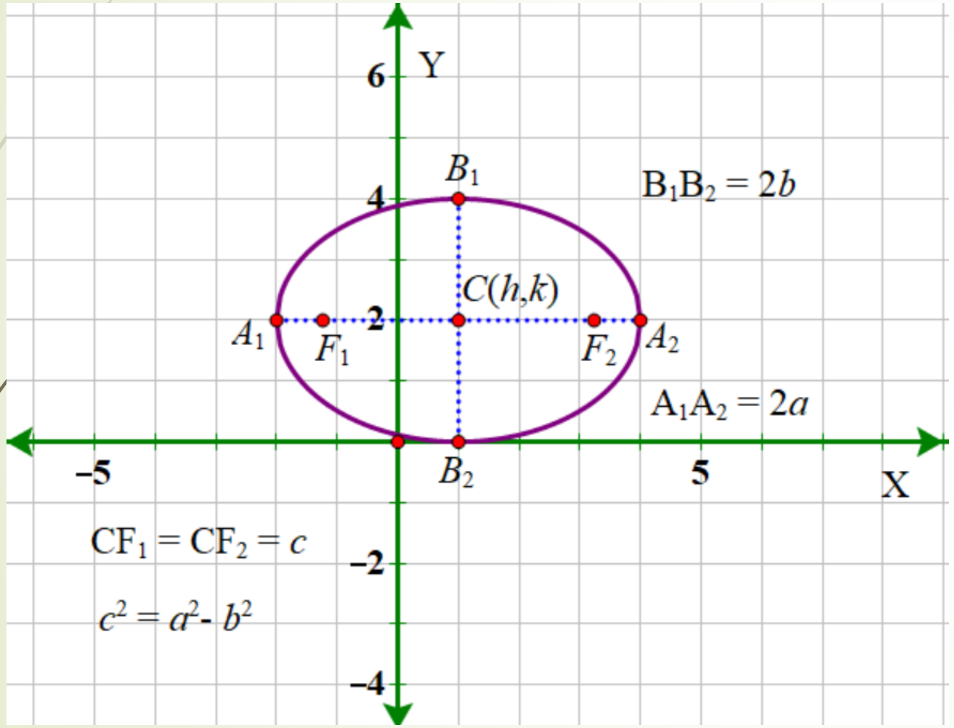


Ellipse (cont.)

The ellipse is one of the conic sections, that can be formed by the intersection of a cone with a plane that not parallel to the side of the cone and does not intersect the base of the cone.



Parts of Ellipse

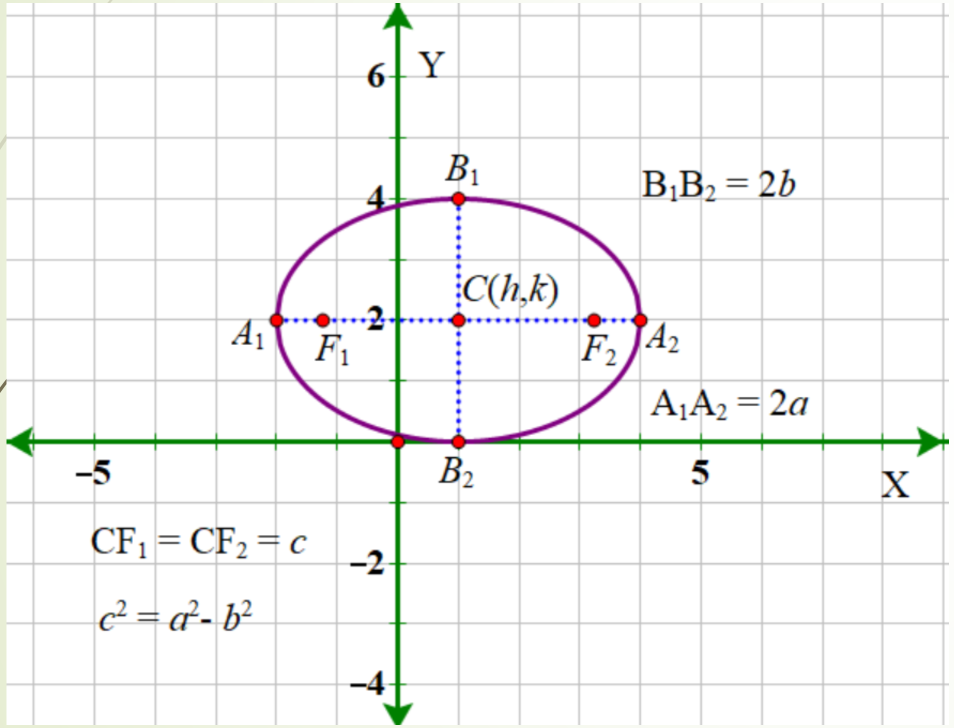


$C(h, k)$ is the ***center***,

F_1 *and* F_2 are the ***foci*** (plural of ***focus***) of the ellipse,

$CF_1 = CF_2 = c$ distance from the center to either focus,

Parts of Ellipse (cont.)



$\overline{A_1A_2}$ is the line segment lying on the ***focal axis*** (the line passes the foci) with endpoints A_1 and A_2 on the ellipse. Each point is called the ***vertex of the ellipse***,

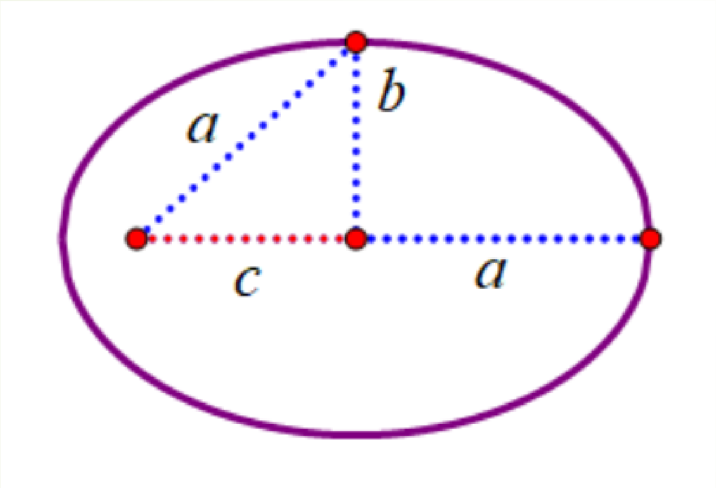
$\overline{A_1A_2}$ is the major axis with the length $2a$,

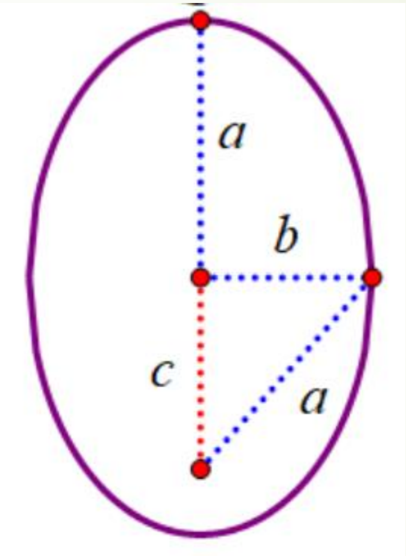
$\overline{B_1B_2}$ is the minor axis with the length $2b$,

Parts of Ellipse (cont.)

Relationships of $a, b,$ and c as follows:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$





$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

c is the distance from center to either focus.

$$c^2=a^2-b^2$$

Equation of an Ellipse

The equation of ellipse in standard form as follows:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Where: (h,k) is the center,

a is $\frac{1}{2}$ of the length of the major axis,

b is $\frac{1}{2}$ of the length of minor axis.

Example

Find the vertices and the foci of the ellipse $4x^2 + 9y^2 = 36$.

Solution: Given $4x^2 + 9y^2 = 36$.

Divide both sides of the equation by 36 yields the standard form.

Then, $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

Since the larger number is the denominator of $x^2$and the center is (0, 0), then the focal axis is the x-axis.

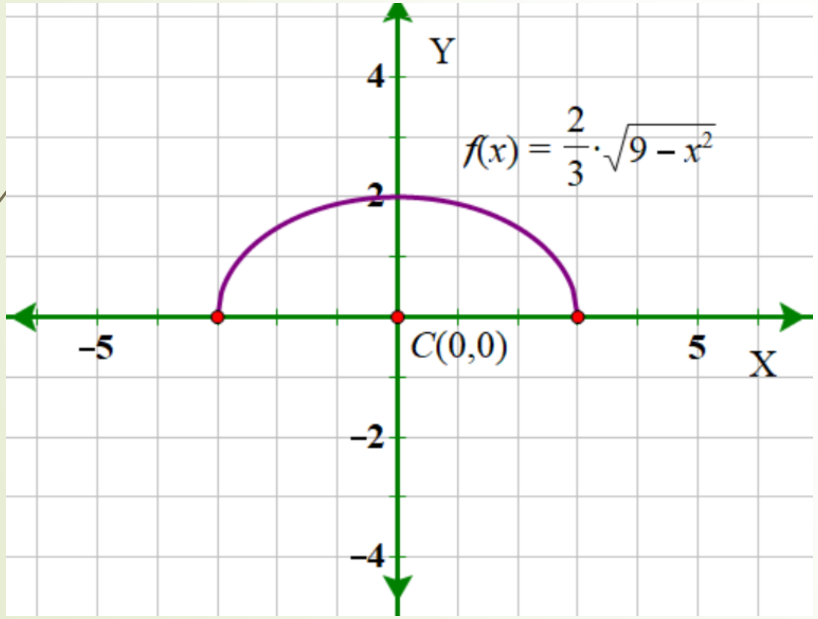
So, $a^2 = 9$,$b^2 = 4$, and $c^2 = a^2 - b^2 = 9 - 4 = 5$.

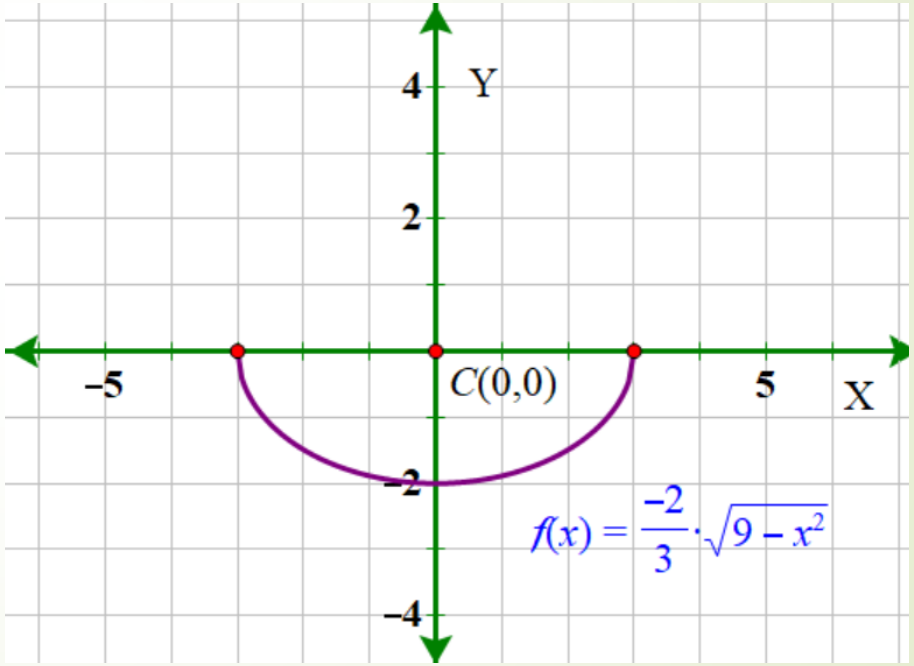
Thus, $a = \pm 3$ and $c = \pm\sqrt{5}$

Therefore, the vertices are $(\pm 3,0)$ and the foci are $(\pm\sqrt{5}, 0)$.#

Equation of an Ellipse as Implicit Equation

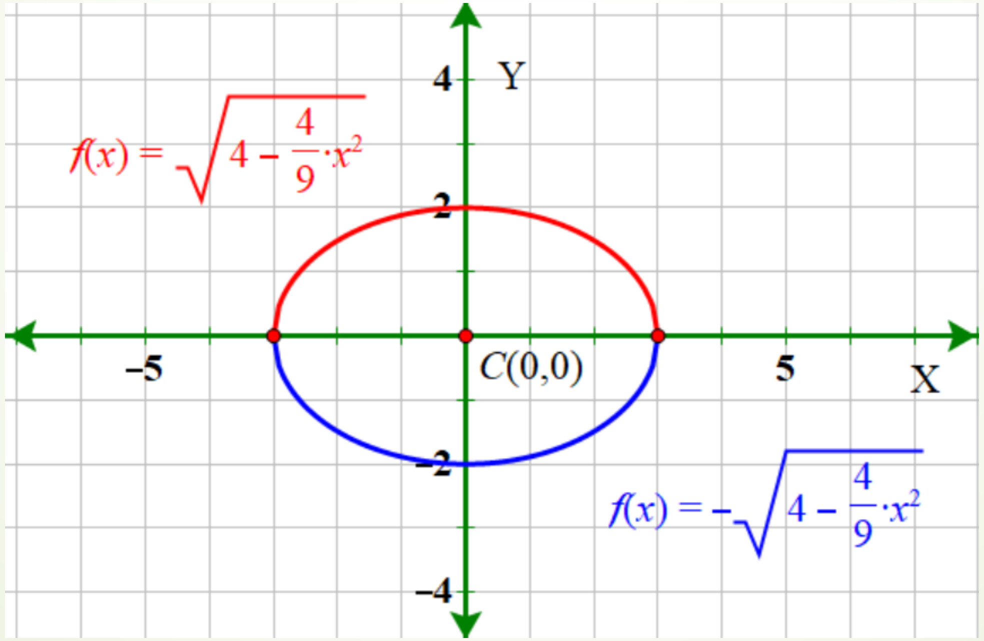
The equation of the ellipse is the implicit equation that cannot be written in the form of $y = f(x)$.





Equation of an Ellipse as Implicit Equation (cont.)

Sketch the ellipse by using $y = \pm\sqrt{4 - \frac{4}{9}x^2}$ on the same coordinate plane.



Calculus 1

Practice 2.2

Q & A